# 2U MATHEMATICS ASSESSMENT

2 periods - Week 8, Term 4 - 1993.

Answer all questions.
Show all working.
Non-programmable calculators may be used.

Question 1 (16 marks)

(a) Evaluate $2\pi\sqrt{\frac{c}{f}}$ when $c = 6.3$

and $f = 7.9$, to 2 significant figures.

(b) The line $3x - py = 5$ passes through the point $(3,2)$, what is the value of p?

(c) Simplify $3\sqrt{27} + \sqrt{12}$

(d) Express $\frac{3}{2 + \sqrt{5}}$ as a single fraction with a rational denominator.

(e) Find the value of $\frac{3.67^2 + 10.06^{-3}}{-7.8}$ correct to 2 decimal places.

(f) Express $0.2\dot{1}$ as a rational number.

(g) Expand and simplify $(3x-2)^3$

(h) Express $6ab + a^2 - 2b^2 - 2ab$ in its simplest form.

(i) If $a = -2$, $b = 8$, $c = -1$

evaluate $\frac{(a + c)^2}{b^2}$

QUESTION 2. (12 marks)

(i) Solve (a) $5(a-3) = 2(a+6)$

(b) $|5x+2| < 12$

(ii) Find the exact roots of the quadratic equation $5x^2-3x-1 = 0$

(iii) Two numbers x and y are connected by the equations $x-y = 2$ and $xy = 5$. Find the values of x and y.

(iv) Find $\lim_{x \to 2} \frac{x^3 - 8}{2x - 4}$

(v) If $\sin \theta = \frac{2}{\sqrt{29}}$ and $\theta$ is obtuse

find the exact values of

(a) $\tan \theta$, (b) $\cos \theta$

QUESTION 3. (12 marks)

(a) Find the derivatives of the following functions of x.

(i) $5x^3 - 4x^2 + \sqrt{x}$

(ii) $x^2 + \frac{3}{x^2}$

(iii) $(x-1)(x^2+x+1)$

(b) (i) Sketch $y = \sqrt{25 - x^2}$

(ii) State the domain and range of this function.

(c) What is the exact value of $\sec 135^\circ$?

(d) Show that $\frac{x^2}{9} + \frac{y^2}{4} = 1$

if $x = 3 \cos A$ and $y = 2\sin A$

QUESTION 4 (12 marks)

(a) Sketch the function defined by the rule

$$f(x) = \begin{cases} 2x - 1 & \text{if } x \geqslant 1 \\ 2 & \text{if } x < 1 \end{cases}$$

(b) Find the equation of the tangent to the curve $y = 6x - 2x^2$ at the point where $x = 2$.
What is the equation of the normal at this point?

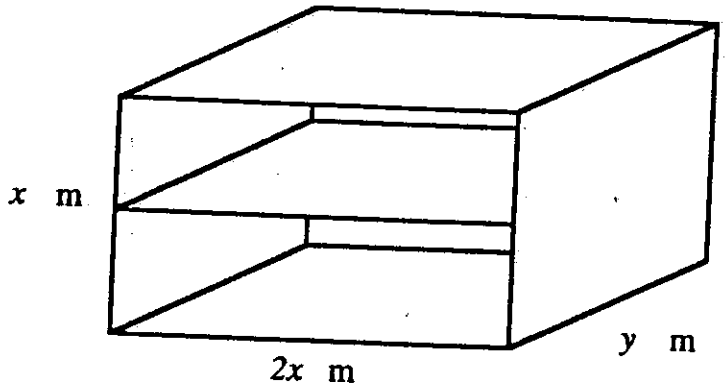
(c) For the curve $y = x^3 - 12x$

(i) What are the intercepts on the x and y axes?

(ii) Find the stationary points and determine their nature.

(iii) Thus sketch the curve showing all relevant details.

QUESTION 5 (12 marks)
(a)




A rectangular storage unit, closed on all sides except the front, is to be constructed, with a shelf, as shown. It is to be built with $24m^2$ of pineboard (disregard thickness)

(i) Show that $2x^2+8xy = 24$

(ii) Form a function, V(x), that measures the volume of the storage unit.

(iii) Determine the maximum volume the storage unit can have.

(b) PQRS is a rhombus in which the side lengths are 4 cm., the angle PQR is $60^\circ$ and the diagonals intersect at M. The angle PMQ is denoted by $\phi$.

(i) Draw a diagram to represent this information.

(ii) Explain why $\phi$ is a right angle

(iii) Calculate the exact length of the diagonal PR.

(iv) Calculate the exact length of the other diagonal SQ.

QUESTION 6 (12 marks)

(a) Differentiate the following with respect to x.

(i) $\frac{1}{4\sqrt{x}}$

(ii) $\frac{5x-3}{2x^3+1}$

(iii) $(3x - 1)^6$

(b) A set of 11 concentric circles is such that the circumference of each is $2\pi$cm. more than the one immediately inside it. The innermost circle has a diameter of 20 cm.

Find (i) the circumference of the outermost circle.

(ii) the diameter of a circle whose circumference is equal to the total of the circumferences of the 11 circles.

# Year 11 Term 4 Maths Assessment Solutions.

## Question 1:

(a) $5 \cdot 6$ (2.s.f.) (2)

(b) $3x - py = 5$

$9 - 2p = 5$

$p = 2$ (1)

(c) $3\sqrt{27} + \sqrt{12}$

$= 3 \times 3\sqrt{3} + 2\sqrt{3}$.

$= 11\sqrt{3}$ (2)

(d) $\frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$

$= \frac{6 - 3\sqrt{5}}{4 - 5}$

$= 3\sqrt{5} - 6$ (2)

(e) $-1 \cdot 73$ (2d.p.) (2)

(f) $x = 0 \cdot 2\dot{1} = 0 \cdot 2111\ldots$

$10x = 2 \cdot 11\overline{\ \ \ \ }$.

$100x = 21 \cdot 11\overline{\ \ \ \ }$.

$90x = 19$

$x = \frac{19}{90}$ (2)

(g) $(9x^2 - 12x + 4)(3x - 2)$

$= 27x^3 - 36x^2 + 12x - 18x^2 + 24x - 8$

$= 27x^3 - 54x^2 + 36x - 8$. (2)

(h) $6ab + a^2 - 2b^2 - 2ab$

~~$= a(6b + a) - 2b(b + a)$~~

$= 4ab + a^2 - 2b^2$.

$= a^2 + 4ab - 2b^2$. (1)

(i) $\frac{9}{64}$. (2)

## Question 2

(1) (a) $5(a-3) = 2(a+6)$

$5a - 15 = 2a + 12$.

$3a = 27$

$a = 9$. (2)

(b) $|5x+2| \geq 12$.

$5x + 2 \geq 12$ or $5x + 2 \leq -12$

$5x \geq 10$ $\quad$ $5x \leq -14$

$x \geq 2$ $\quad$ $x \leq -\frac{14}{5}$

$x \leq -2\frac{4}{5}$ (2)

$-2\frac{4}{5} \leq x \leq 2$

(ii) $5x^2 - 3x - 1 = 0$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{3 \pm \sqrt{9 + 4 \times 5 \times 1}}{10} = \frac{3 \pm \sqrt{29}}{10}$ (2)

(iii) $x - y = 2$. $-①$ $\Rightarrow y = x - 2$

$xy = 5$ $-②$

$\therefore x(x-2) = 5$

$x^2 - 2x - 5 = 0$

$x = \frac{2 \pm \sqrt{4 + 4 \times 1 \times 5}}{2} = \frac{2 \pm \sqrt{4 + 20}}{2}$.

$= \frac{2 \pm 2\sqrt{6}}{2} = 1 \pm \sqrt{6}$. (1)

$\therefore x = 1 + \sqrt{6}$ or $1 - \sqrt{6}$. (1)

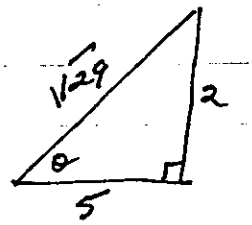
$y = -1 + \sqrt{6}$ or $-1 - \sqrt{6}$. (1)

(using $y = x - 2$)

(iv) $\lim_{x \to 2} \frac{x^3 - 8}{2x - 4} = \lim_{x \to 2} \frac{(x-2)(x^2 + 2x + 4)}{2(x-2)}$

$= 6$ (2)

(v)




(a) $\tan\theta = -\frac{2}{5}$ (1)

(b) $\cos\theta = \frac{-5}{\sqrt{29}}$ (1)

## Question 3

(a) (i) $y = 5x^3 - 4x^2 + \sqrt{x}$

$= 5x^3 - 4x^2 + x^{\frac{1}{2}}$

$\frac{dy}{dx} = 15x^2 - 8x + \frac{1}{2}x^{-\frac{1}{2}}$

$= 15x^2 - 8x + \frac{1}{2\sqrt{x}}$ (1)

(ii) $y = x^2 + \frac{3}{x^2}$

$= x^2 + 3x^{-2}$

$\frac{dy}{dx} = 2x - 6x^{-3}$

$= 2x - \frac{6}{x^3}$ (2)

(iii) $y = (x-1)(x^2+x+1)$

$\Rightarrow u \cdot v$

$u = x - 1 \qquad v = x^2 + x + 1$

$u' = 1 \qquad v' = 2x + 1$

$\frac{dy}{dx} = vu' + uv'$

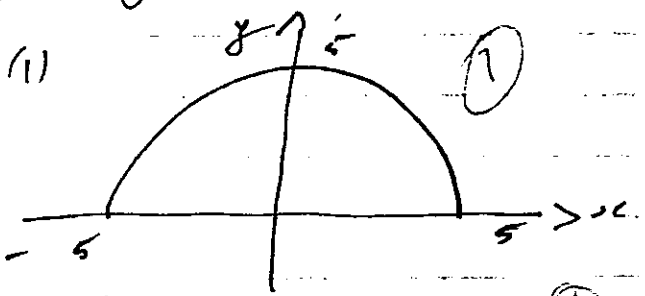
$= (x^2+x+1).1 + (x-1)(2x+1)$

$= x^2 + x + 1 + 2x^2 + x - 2x - 1$

$= 3x^2$. (2)

(b) $y = \sqrt{25 - x^2}$

(i)




(1)

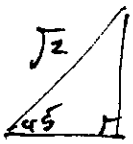
(ii) D: $-5 \le x \le 5$ (1)

R: $0 \le y \le 5$ (1)

(c) $\sec 135 = \sec(180 - 45)$

$= -\sec 45$

$= -\frac{1}{\cos 45}$.




$= -\sqrt{2}$ (2)

(d) $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

LHS: $\frac{(3\cos A)^2}{9} + \frac{(2\sin A)^2}{4}$

$= \frac{9\cos^2 A}{9} + \frac{4\sin^2 A}{4}$ (1)

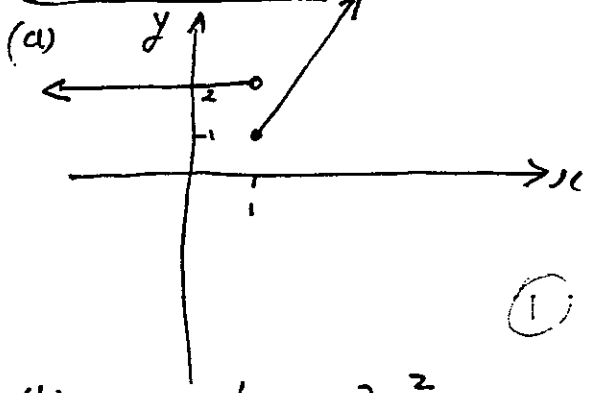
$= \cos^2 A + \sin^2 A$

$= 1$

$=$ RHS. (1)

## Question 4

(a)




(1)

(b) $y = 6x - 2x^2$, $x = 2$.

Tangent: $y' = 6 - 4x$ (1)

$x = 2$, $m = -2$. (1)

$y = 12 - 8 = 4$.

$y - y_1 = m(x - x_1)$

$y - 4 = -2(x - 2)$

$y - 4 = -2x + 4$.

$2x + y - 8 = 0$ (1)

Normal $y - 4 = \frac{1}{2}(x - 2)$.

$2y - 8 = x - 2$

$x - 2y + 6 = 0$ (1)

(c) $y = x^3 - 12x$.

(i) $y = x(x^2 - 12)$

$= x(x - \sqrt{12})(x + \sqrt{12})$ (1)

intercepts $x = 0, 2\sqrt{3}, -2\sqrt{3}$ (2)

(ii) $\frac{dy}{dx} = 3x^2 - 12$ (1)

S.P. when $dy/dx = 0$

$3x^2 - 12 = 0$

$x^2 - 4 = 0$

$(x - 2)(x + 2) = 0$.

S.P. at $x = 2, -2$

$y = -16, 16$

$(2, -16)$ $(-2, 16)$ (3)

Test

| $x$ | $2^-$ | $2$ | $2^+$ |
|---|---|---|---|
| $\frac{dy}{dx}$ | $-$ | $0$ | $+$ |
| | \ | — | / |

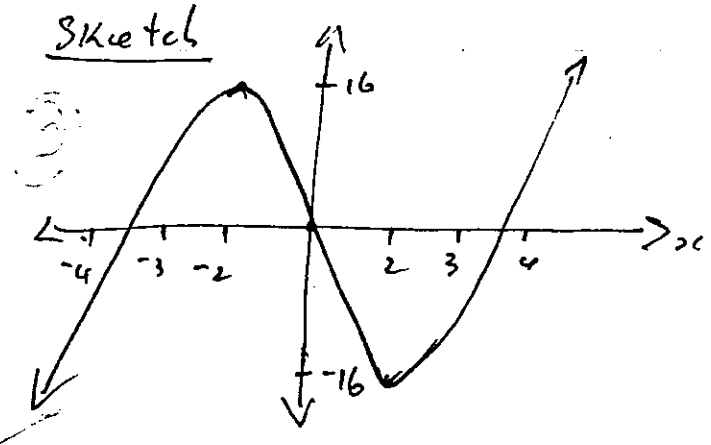
$(2, -16)$ is a Min. T.P.

(1)

| $x$ | $-2^-$ | $-2$ | $-2^+$ |
|---|---|---|---|
| $\frac{dy}{dx}$ | $+$ | $0$ | $-$ |
| | / | — | \ |

$(-2, 16)$ is a Max T.P.

Sketch




(3)

## Question 5

(i) Total Surface Area.

$= 2xy + 3 \times 2xy + 2x^2$

$= 2x^2 + 8xy$

$\therefore 2x^2 + 8xy = 24$. — (1)

(ii) $V = 2x^2 y$

(iii) $V = 2x^2\left(\frac{24 - 2x^2}{8x}\right)$ (1) sub in (1)

$= 2x^2\left(\frac{12 - x^2}{4x}\right)$

$= x^2\left(\frac{12 - x^2}{2x}\right)$.

(2) $= \frac{12x^2 - x^4}{2x} = 6x - \frac{1}{2}x^3$

(1) $\frac{dV}{dx} = 6 - \frac{3}{2}x^2$.

$\frac{dV}{dx} = 0$

$6 = \frac{3}{2}x^2$.

$12 = 3x^2$.

$u = x^2$

$x = \pm 2$.

$x = 2$.

| $x$ | $2^-$ | $2$ | $2^+$ |
|---|---|---|---|
| $\frac{dy}{dx}$ | + | 0 | − |

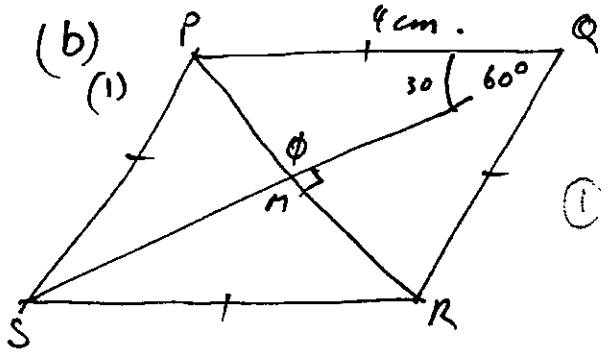
MAX at $x = 2$ (1)

$\therefore V_{MAX} = 6 \times 2 - \frac{1}{2} \cdot 2^3$ (1)

$= 12 - 4$

$= 8\ m^3$ (1)

(b) (i)




(1)

(ii) Diagonals of a rhombus (1) bisect at right angles

(iii) $\sin 30 = \frac{PM}{4}$

$PM = 4 \sin 30$.

$= 4 \times \frac{1}{2}$

$= 2$.

$PR = 2\,PM$

$= 4$ units (1)

$\cos 30 = QM/4$ (1)

$QM = 4 \cos 30$

$= 2\sqrt{3}$ $\therefore QS = 4\sqrt{3}$

## Question 6

(i) $y = \frac{1}{4\sqrt{x}} = \frac{1}{4} \times x^{-\frac{1}{2}}$.

$\frac{dy}{dx} = -\frac{1}{8} x^{-3/2}$

(2) $= -\frac{1}{8} \times \frac{1}{\sqrt{x^3}} = \frac{-1}{8\sqrt{x^3}}$ #

(ii) $y = \frac{5x-3}{2x^3+1} \Rightarrow \frac{u}{v}$

$u = 5x - 3$

$u' = 5$

$v = 2x^3 + 1$

$v' = 6x^2$

$v^2 = (2x^3+1)^2$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$= \frac{(2x^3+1)5 - (5x-3)(6x^2)}{(2x^3+1)^2}$

$= \frac{10x^3 + 5 - 30x^3 + 18x^2}{(2x^3+1)^2}$

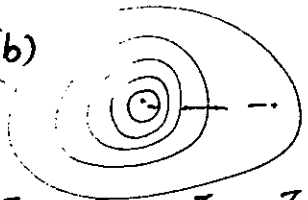
(2) $= \frac{5 + 18x^2 - 20x^3}{(2x^3+1)^2}$ #

(iii) $y = (3x-1)^6$.

$\frac{dy}{dx} = 6(3x-1)^5 \cdot 3$

(2) $= 18(3x-1)^5$ #

(b)



$C = 2\pi r$

$= \pi d$.

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \ldots \quad T_{11}$.

$20 \quad 20+2\pi \quad 20+4\pi \ldots \quad 20+20\pi$ (2)

(i) $20\pi + 20\pi = 40\pi$ (2)

(ii) $S_n = \frac{n}{2}(a+l)$

$= \frac{11}{2}(40\pi + 20\pi)$

$= 11 \times 30\pi = 330\pi$

diameter $= \frac{330\pi}{\pi} = 330$ cm (2)