Killara High School.

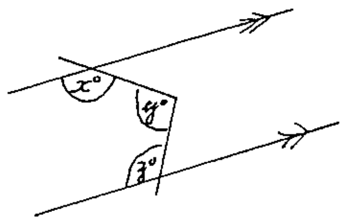
Year 11 2 Unit Mathematics Assessment

December 1996

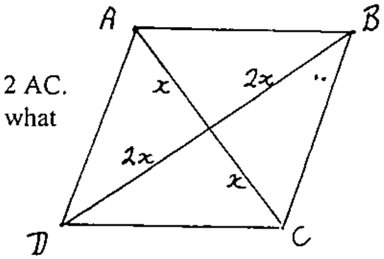
Question 1.

A. Find the number of sides of a regular polygon when each internal angle is 140 degrees

B. In the adjacent diagram. Find the value of $x + y + z$, given all necessary reasons.



C. ABCD is a rhombus, with diagonal BD = 2 AC. If each side of the rhombus is 45 cms, what is the area of the rhombus?



A. Differentiate the following:

a. $y = 4x^2$

b. $y = x^2(x + 5)$

c. $y = (3x + 4)^5$

d. $y = \dfrac{3x^2 + 1}{x^2}$

e. $y = \dfrac{2x^2 + 1}{x + 1}$

A. If the equation $3x^2 + Mx + 8$ has one root which is 6 times the other, find the value of M.

B. In the equation $kx^2 + 5x + k + 1 + 0$

i. Write an expression for the sum of the roots

ii. Write an expression for the product of the roots.

iii. If the product of the roots is twice the sum of the roots, find the value of k.

C. By using the substitution $M = 2^x$ find all values of x that satisfy the equation $4^x - 5.2^x + 4 = 0$

Question 4.

Consider the curve $y = x^3 - 3x^2 - 9x + 12$

Find i. Any stationary points and determine their nature (ie local maximum or minimum).

ii. What is the equation of the tangent to the curve at the point where $x = 2$.

Question 5.

A. Given that $f(x) = x^2 - 3x$, show <u>by 1st principles</u> that $f\ (x) = 2x - 3$

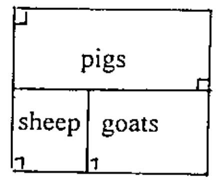
B. Consider the curve $y = x^3 - 3x^2 - 13x + 15$ Show that a point of inflection exists at a point where the curve cuts the x axis.

B. Sketch the curve $y = 2 + \dfrac{1}{x - 2}$

Question 6

A rectangular cattle yard has to be divided into 3 pens for sheep, goats and pigs, as in the adjacent diagram, by using a series of white steel gates The only restriction is that the width of the pig pen must be the same as the width of the other two pens .

If the total length of the white steel gates is 36 metres, find the maximum possible area of the pig pen.



Yr 11 : 2U : DECEMBER 1996

A. N^o of Sides $= 9$

B $180 - x + 180 - y + 180 - z = 180$

$x + y + z = 360$

C $(2x)^2 + (x)^2 = 45^2$

$5x^2 = 45^2$

$x^2 = 405$

$x = \sqrt{405}.$ 3

Area $= \frac{1}{2} 4\sqrt{405} . 2\sqrt{405}$

$= 4 \times 405$

$= 1620 \text{ cm}^2$

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2 A. $\frac{dy}{dx} = 8x$

B $\frac{dy}{dx} = 3x^2 + 10x$

C $\frac{dy}{dx} = 15(3x+4)^4$

D $\frac{dy}{dx} = -2x^{-3} = \frac{-2}{x^3}$

E. $\frac{dy}{dx} = \frac{(x+1).4x - (2x^2+1)}{(x+1)^2}$

$= \frac{2x^2+4x-1}{(x+1)^2}$

F. $\frac{dy}{dx} = x(x^2+4)^{-\frac{1}{2}}$

$= \frac{x}{\sqrt{x^2+4}}$

3A Let Roots be $\alpha, 6\alpha$

$7\alpha = -\frac{M}{3}$ $\quad 6\alpha^2 = \frac{8}{3}$

$M = -21\alpha$ $\quad \alpha = \frac{\pm 2}{3}$

$M = -21 \times \pm\frac{2}{3}$

$= \pm 14$ 3

B. i. Sum of Roots $= -\frac{5}{K}$

Prod of Roots $= \frac{K+1}{K}$

$\frac{K+1}{K} = \frac{-10}{K}$

$K = -11$

C $M = 2^x$ $\quad M^2 - 5M + 4 = 0$

$(M-4)(M-) = 0$

$M = 1, 4$

$2^x = 1 \quad x = 0$

$2^x = 4 \quad x = 2$

Q.4 i $y = x^3 - 3x^2 - 9x + 12$

$\frac{dy}{dx} = 3x^2 - 6x - 9$

$= 3(x^2 - 2x - 3)$

$= 3(x-3)(x+1)$

$= 0$ when $x = -1, 3.$

STAT. PTS ARE $-1, 17$ – MAX

AND $3, -15$ – MIN

ii At $x=2$, gradient $= -9$, $y = -10$ 3

EQTN $y + 10 = -9(x-2)$

$y = -9x + 8$

Q5. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - x^2 + 3x}{h}$

$= \lim_{h \to 0} \frac{2xh - 3h + h^2}{h}$

$= \lim_{h \to 0} 2x - 3 + h$ 3

$= 2x - 3$

B. $y = x^3 - 3x^2 - 13x + 15$

$y' = 3x^2 - 6x - 13$

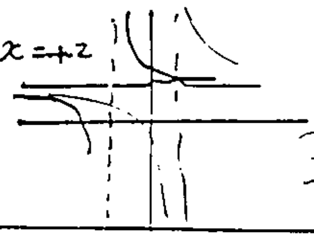
$y'' = 6x - 6 = 0$ when $x = 1$

When $x = 1$, $y = 1 - 3 - 13 + 15$ 3

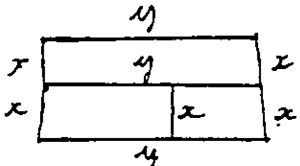
$= 0 \therefore$ PT ON x-AXIS

B. $y = 2 + \frac{1}{x+2}$

Asymptotes $y = 2$ $\quad x = -2$



Q.6



Total Fencing $= 5x + 3y = 36$

$3y = 36 - 5x$

$y = 12 - \frac{5}{3}x$

Area of Pig Pen $= x(12 - \frac{5}{3}x)$

$= 12x - \frac{5}{3}x^2$

$\frac{dA}{dx} = 12 - \frac{10}{3}x$

$= 0$ when $\frac{10}{3}x = 12$

$10x = 36$

$x = 3.6$

$y = 6$

Max Area $= 3.6 \times 6$

$= 21.6 \text{ m}^2$