KILLARA HIGH SCHOOL
YEAR 11 MATHEMATICS – 2 UNIT
TERM 4 1997 ASSESSABLE TEST

Time allowed: 1 period

Instructions: **Attempt all questions**
Show all necessary working
Start each question on a new page

Question 1

a. Find the exact value (as a rational number in its lowest terms) of:

1. $\frac{1}{2}+\frac{2}{3}-\frac{4}{7}-\frac{5}{14}$

2. $\frac{A^4C}{B^4}$ where $A=\left(\frac{2}{3}\right)^2$, $B=\left(\frac{4}{3}\right)^4$ and $C=\left(\frac{8}{3}\right)^7$

b. What is the domain and range of the following function in terms of real numbers?

$$y=\frac{1}{x+1}$$

c. Find the values for x for which the equation $x^2+(m-2)x+4=0$ has

1. equal roots;
2. no real roots;

d. For the curve $y=x^2-4x$, what is the equation of the tangent to the curve at its vertex?

Question 2

a. On a number plane, mark the origin O and $A(5,0)$, $B(8,4)$, and $C(0,10)$. Join A to B. B to C, and C to A.

b. Show that the line AB has equation $3y=4x-20$.

c. Show that the gradient of the line BC is $-\frac{3}{4}$.

d. Show that AB and BC are perpendicular.

e. Show that the length of AB is 5 units.

f. Prove that triangles ACO and ACB are congruent. Give reasons.

g. Find the area of the quadrilateral $ABCO$.

h. If D is the point (8,0), calculate the perpendicular distance of D from the line AB.

Question 3

a. Differentiate the following functions.

1. $y = 6x^3$

2. $y = 3x^3 + \frac{1}{2}\sqrt{x}$

3. $y = (4x^3 + 1)(2x + 2)$

4. $y = \dfrac{\frac{2}{3}x^3 - 1}{4x}$

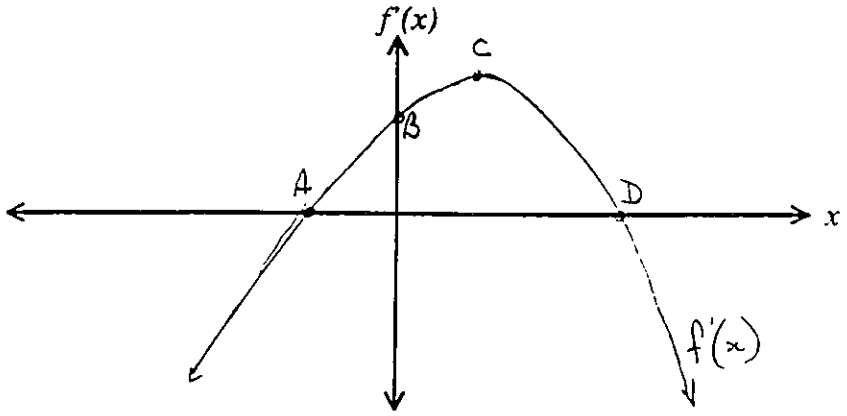
5. $y = \sqrt{3x^2 - 2}$

6. $y = \dfrac{6}{\sqrt{4x - 3}}$

b. If $dy/dt = 6t^2 - 2$ and $y = 1$ when $t = 1$, determine the equation for y.

Question 4

a.



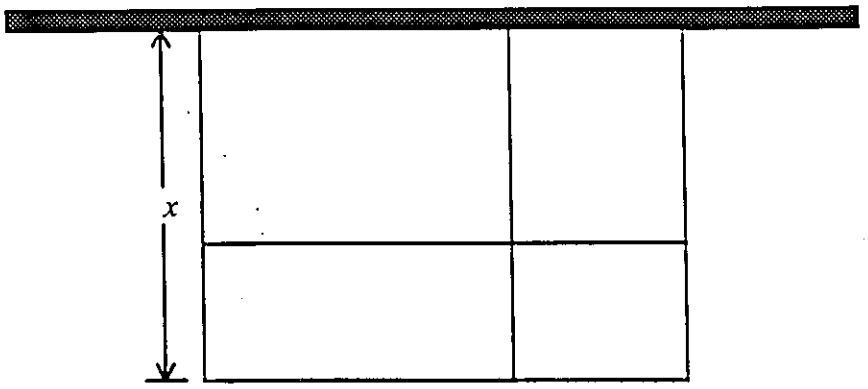
The above graph is that of $y = f'(x)$, answer the following questions which relate to the original function $y = f(x)$,

1. At which point/s does the graph $y = f(x)$ have stationary point/s?
2. At what point does the graph $y = f(x)$ have point/s of inflection?
3. Between which points is the graph $y = f(x)$ concave up?

b. For the curve $y = x^3 - 16x$

1. Find the stationary points and determine their nature.
2. Find any points of inflection.
3. Determine the x- and y- intercepts.
4. Neatly sketch the curve $y = x^3 - 16x$

c. A farmer wants to construct four rectangular holding paddocks for his stock against an existing straight fence (see figure).



If he has 1800 metres of fencing available,

1. Show that the total area of the field can be given as **Area** = $900x - \frac{3}{2}x^2$
2. Determine the maximum total area of holding paddocks he can construct?

/50

YEAR 11 2 UNIT EXAM – TERM 4

General 12 marks

(i) $\frac{3}{2^1}$ (1)

(ii) $\frac{\left(\left(\frac{2}{3}\right)^2\right)^4\left(\frac{3}{3}\right)^7}{\left(\left(\frac{4}{3}\right)^4\right)^6}$

$= \left(\frac{2^8}{3^8}\right)\left(\frac{8^7}{3^7}\right)\left(\frac{3^{16}}{4^{16}}\right)$ (1)

$= \frac{3}{8}$ (1)

) Domain: all real x x $x \neq -1$ (1)

Range: all real y - y: $y \neq 0$ (1)

) (i) $(m-2)^2 - 16 = 0$ (1)

$m = 6, -2$ (1)

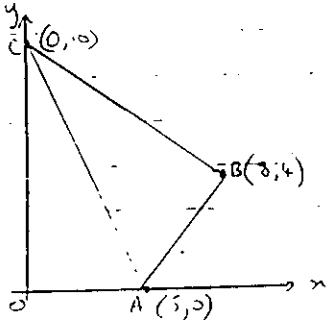
(ii) $(m-2)^2 - 16 < 0$ (1)

$-2 < m < 6$ (1)

) $V = (2, -4)$ $M = 0$ (1)

eqn is $y = -4$ as $M = 0$ (1)

QUESTION 2 12 marks



(1)

) $m(AB) = \frac{4}{3}$ (1)

$y - 0 = \frac{4}{3}(x - 3)$

$3y = 4x - 20$ (1)

c) $M(BC) = -\frac{3}{4}$ (1)

d) b if $M(BC) = -\frac{1}{M(AB)}$ (1)

$-\frac{3}{4} = -\frac{1}{\frac{4}{3}}$

$= -\frac{3}{4}$ (1)

e) $d(AB) = 5$ units (1)

f) Prove $\triangle ACO \equiv \triangle ACB$

$AO = AB = 5$ units (given) (½)

AC (common hypotenuse) (½)

$\angle ABC = \angle COA = 90°$ (see part d) (½)

$\therefore \triangle ACO \equiv \triangle ACB$ (RHS) (½)

g) Area $= 50$ units^2 (1)

h) $d = \frac{|4 \times 8 + 0 - 20|}{\sqrt{(4)^2 + (-3)^2}}$ (1)

$= 2\frac{4}{5}$ units (1)

QUESTION 3 12 marks

a) (i) $dy/dx = 18x^2$ (1)

(ii) $y = 3x^3 + \frac{1}{2}x^{-\frac{1}{2}}$ (1)

$dy/dx = 9x^2 + \frac{1}{4\sqrt{x}}$ (1)

or

$dy/dx = 9x^2 + \frac{1}{4}x^{-\frac{1}{2}}$

(iii) $dy/dx = 32x^3 + 24x^2 + 2$ (1)

(iv) $dy/dx = \frac{4x(2x^2) - 4(\frac{2}{3}x^3 - 1)}{16x^2}$ (1)

$= \frac{\frac{4}{3}x^3 + 1}{4x^2}$ (1)

(v) $dy/dx = \frac{1}{2}(3x^2 - 2)^{-\frac{1}{2}} \times 6x$ (1)

$= 3x(3x^2 - 2)^{-\frac{1}{2}}$

or

$\frac{3x}{\sqrt{3x^2 - 2}}$ (1)

(vi) $y = 6(4x - 3)^{-\frac{1}{2}}$ (1)

$dy/dx = -12(4x - 3)^{-\frac{3}{2}}$

or

$dy/dx = \frac{-12}{\sqrt{(4x-3)^3}}$ (1)

(2)

5) $y = 2t^3 - 2t + c$ (1)

$1 = 2 - 2 + c$

$c = 1$

$\therefore y = 2t^3 - 2t + 1$ (1)

(2)

QUESTION 4 14 marks

a) (i) A, D (1)

(ii) C (1)

(iii) A and C (1)

b) $y = x^3 - 16x$

$dy/dx = 3x^2 - 16$

$d^2y/dx^2 = 6x$

(i) Stat points at $dy/dx = 0$

$3x^2 - 16 = 0$

$x = \pm \frac{4}{\sqrt{3}}$

$x = \frac{4}{\sqrt{3}}$

$y = \frac{-128}{3\sqrt{3}}$ (1)

$d^2y/dx^2\left(\frac{4}{\sqrt{3}}\right) > 0$

$\therefore$ minimum point (1)

$x = -\frac{4}{\sqrt{3}}$

$y = \frac{128}{3\sqrt{3}}$ (1)

$d^2y/dx^2\left(-\frac{4}{\sqrt{3}}\right) < 0$

$\therefore$ maximum point

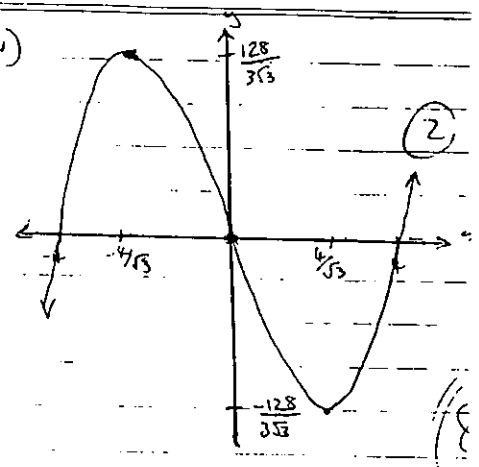
(ii) points of inflection at $d^2y/dx^2 = 0$

$x = 0$, $y = 0$ (1)

(iii) x-int $x = 0$ and $x = \pm 4$ (1)

y-int at $y = 0$ (origin) (1)

(iv)



(2)

c) (i) length $= \frac{1800 - 3x}{2}$

$= 900 - \frac{3}{2}x$

Area $= x\left(900 - \frac{3}{2}x\right)$

$= 900x - \frac{3}{2}x^2$ (1)

(ii) $dA/dx = 900 - 3x$ max at $\frac{dA}{dx} = 0$

$0 = 900 - 3x$

$x = 300$ (1)

$\therefore$ Area $= 135\,000$ m^2 (1)