KILLARA HIGH SCHOOL

YEAR 11 - MATHEMATICS - 2 UNIT

TERM IV ASSESSMENT TEST 1998

Time Allowed: 65 minutes

INSTRUCTIONS:

- All questions may be attempted.
- It is in your best interests to show all necessary working.
- Use your own writing paper.
- Approved calculators may be used.
- The value of each question is shown in brackets adjacent to it.

QUESTION 1: (11 Marks)

i) Use your calculator to evaluate:

$$\sqrt{\frac{(24{\cdot}609)^5}{27{\cdot}8-19{\cdot}66}}$$

(a) correct to 1 decimal place **(1)**

(b) in scientific notation correct to 3 significant figures **(1)**

ii) Find 'x' and 'y' if $\left(3-\sqrt{5}\right)^2 = x-\sqrt{y}$ **(2)**

iii) (a) Solve $x^2-7x+12=0$ **(2)**

(b) Hence, or otherwise, solve $m^4-7m^2+12=0$ **(2)**

iv) Solve $\cos^2\theta=\frac{1}{2}$ given $0^0 \le \theta \le 360^o$ **(3)**

QUESTION 2: (12 Marks)

i) (a) Sketch the parabola which has its focus at (2,2) and its directrix $y=-4$, showing these features on your diagram. (x intercepts etc are not necessary). **(2)**

(b) What is the equation of this parabola? **(2)**

ii) (a) Find the coordinates of the vertex of the parabola

$y=x^2-2x-8$ **(2)**

(b) Sketch the graph of $y=x^2-2x-8$ showing the coordinates of the vertex and the x and y intercepts. **(2)**

(c) Hence, or otherwise, solve $x^2-2x-8<0$ **(1)**

(d) State the domain and range of $y=x^2-2x-8$ **(2)**

(e) On your diagram shade the region where $y \le x^2-2x-8$ **(1)**

QUESTION 3: (12 Marks)

i) Differentiate the following functions:

(a) $f(x)=4x^3-7+\sqrt{x}$ **(2)**

(b) $f(x)=(3x^2+4)^5$ **(2)**

(c) $f(x)=\frac{6x^3+7x^2}{x^2}$ **(2)**

(d) $f(x)=\frac{3x}{x^2-1}$ **(2)**

(ii) If α and β are the roots of the quadratic equation $5x^2-x-15=0$, find

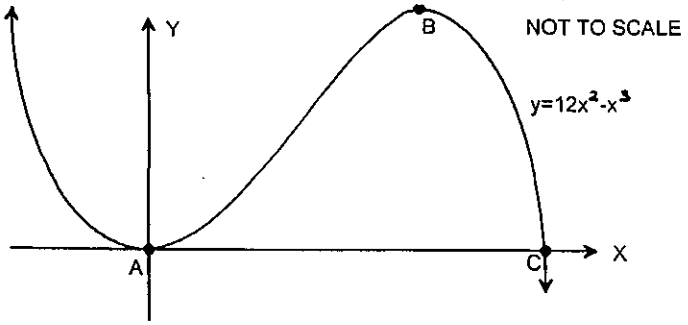
(a) $\alpha+\beta$ **(1)**

(b) $\alpha\beta$ **(1)**

(c) $\alpha\beta^2+\alpha^2\beta$ **(2)**

QUESTION 4: (16 Marks)

(i)



The diagram shows a sketch of $y = 12x^2 - x^3$

(a) Find the coordinates of

α) A and C **(2)**

β) B **(2)**

(b) Describe the nature of the points A and B **(2)**

(c) Find the equation of the tangent to the curve at the Point (-1, 13) **(2)**

ii) An arithmetic series is given by:

$1250 + 1230 + 1210 + ...$

(a) Find the 125th term of the series. **(2)**

(b) Find the sum of the first 125 terms. **(2)**

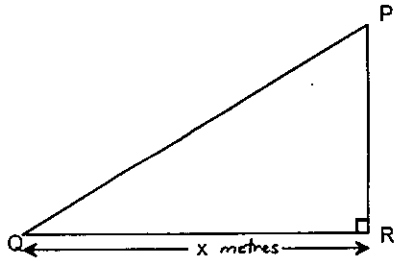
iii) Show that the following series is geometric:

$\frac{2}{5} - \frac{2}{15} + \frac{2}{45} - .....$ **(2)**

iv) The n'th term of a geometric sequence is given by $T = 4^{1-n}$. Show that $\frac{1}{256}$ is a term of this sequence. **(2)**

QUESTION 5: (14 Marks)

(i)



A wire of length 5 metres is to be bent to form the hypotenuse and base of a right angled triangle PQR as shown. The base, QR, is denoted 'x' metres. **(1)**

(a) Write an expression for PQ in terms of 'x'.

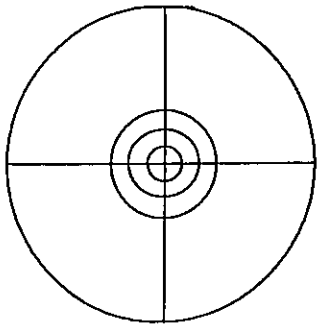
(b) Show that the height, PR, can be expressed as $\sqrt{25-10x}$. **(2)**

(c) Hence, show that the area of ΔPQR, in terms of x, is

$\frac{x\sqrt{25-10x}}{2}$ square metres. **(1)**

(d) Use calculus to find the maximum possible area of the triangle. **(4)**

ii) The figure shows a wire frame of concentric circles (not all shown) held in place by two crosswires. The radius of the inner circle is 2·5 cm and the radii of circles increases by 2cm per circle. The radius of the outer circle is 52·5 cm.



(a) How many circles are in the framework? **(2)**

(b) By using an arithmetic series find the total length in exact form (ie.in terms of π) of the wire used to make the framework. ($c = 2\pi r$) **(4)**

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SOLUTIONS

QUESTION 1. (11 marks)

i) a) 1053.0 (1)

b) 1.05×10^3 (1)

ii) $(3-\sqrt{5})^2 = 9 - 6\sqrt{5} + 5$

$= 14 - \sqrt{36 \times 5}$

$= 14 - \sqrt{180}$

$\therefore x = 14, y = 180$ (2)

iii) a) $x^2 - 7x + 12 = 0$

$(x-4)(x-3) = 0$

$x = 4, 3$ (2)

b) $m^4 - 7m^2 + 12 = 0$

let $x = m^2$,

$\therefore x^2 - 7x + 12 = 0$

$x = 4, 3$ so $m^2 = 4, 3$

$\therefore m = \pm 2, \pm\sqrt{3}$. (2)

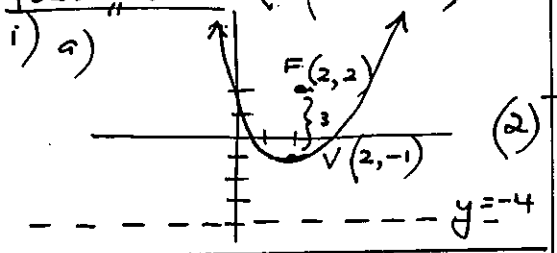
iv) $\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

$\therefore \theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$ (3)

QUESTION 2 (12 marks)

i) a)



(2)

b) equn. $(x-h)^2 = 4a(y-k)$

$(x-2)^2 = 4 \times 3 (y+1)$

$(x-2)^2 = 12(y+1)$ (2)

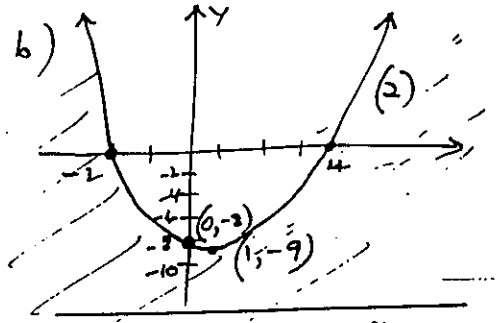
ii) a) $y = x^2 - 2x - 8 = (x-4)(x+$

$x = \frac{-b}{2a} = \frac{-2}{2\times1} = \frac{2}{2} = 1$

$x = 1, y = 1 - 2 - 8 = -9$

$\therefore V = (1, -9)$ (2)

b)



(2)

c) $-2 < x < 4$ (1)

d) ~~range~~ $y \geq -9$ (1)

e) on diagram (1) test (0,0) is $0 \not\geq -8$

QUESTION 3 (12 marks)

i) a) $\frac{d}{dx} 4x^3 - 7 + x^{1/2}$ (2)

$= 12x^2 + \frac{1}{2}x^{-1/2} = 12x^2 + \frac{1}{2\sqrt{x}}$

b) $y' = 5(3x^2+4)^4 . 6x = 30x(3x^2+4)^4$ (2)

QUES. 3 cont:

c) $f(x) = \frac{6x^3}{x^2} + \frac{7x^2}{x^2}$

$= 6x + 7$

$\therefore y' = 6$ (2)

d) $y' = \frac{Vu' - uV'}{V^2}$

$= \frac{(x^2-1).3 - 3x.2x}{(x^2-1)^2}$

$= \frac{3x^2 - 3 - 6x^2}{(x^2-1)^2}$

$= \frac{-3x^2 - 3}{(x^2-1)^2}$ (2)

$= \frac{-3(x^2+1)}{(x^2-1)^2}$

ii) $5x^2 - x - 15 = 0$

a) $\alpha + \beta = \frac{-b}{a} = \frac{1}{5}$ (1)

b) $\alpha\beta = \frac{c}{a} = \frac{-15}{5} = -3$ (1)

c) $\alpha\beta^2 + \alpha^2\beta = \alpha\beta(\beta+\alpha)$ (2)

$= \frac{1}{5} \times -3 = \frac{-3}{5}$.

QUESTION 4 (16 marks)

a) α) $y = 12x^2 - x^3$

x intercepts, $y = 0$

$12x^2 - x^3 = 0$

$x^2(12-x) = 0$

$\therefore x = 0, 12$

$x = 0, y = 0 \therefore A = (0,0)$ (1)

$x = 12, y = 0 \therefore C = (12,0)$ (1)

β) $y = 12x^2 - x^3$

$y' = 24x - 3x^2$

st. pt when $y' = 0$

$\therefore 3x(8-x) = 0$

$\therefore x = 0, 8$. (2)

$\therefore x = 8, y = 256 \therefore B = (8,256)$

b) A is a min S.P. (1)

B is a ~~max~~ S.P. (1)

c) $y' = 24x - 3x^2$ (-1, 13)

$f'(-1) = 24(-1) - 3(-1)^2$

$= -24 - 3 = -27$

$\therefore m = -27$

equn. $y - y_1 = m(x - x_1)$

$y - 13 = -27(x+1)$

$y - 13 = -27x - 27$

$27x + y + 14 = 0$. (2)

ii) a) $a = 1250, d = -20$

$T_{125} = a + 124d$

$= 1250 + 124 \times -20$

$= -1230$. (2)

b) $S_{125} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{125}{2}[2 \times 1250 + 124 \times -20]$

$= 1250$. (2)

iii) G.P. when $\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\frac{-2/15}{2/5} = \frac{-2}{15} \times \frac{5}{2} = \frac{-1}{3}$

$\frac{2/45}{-2/15} = \frac{2}{45} \times \frac{-15}{2} = \frac{-1}{3}$ $\therefore$ G.P. (2)

Ques 4 cont:

iv) 4^{1-n}

$\frac{1}{256} = \frac{1}{4^4} = 4^{-4}$.

$\therefore 4^{1-n} = 4^{-4}$

$\therefore 1-n = -4$

$5 = n$ (2)

$\therefore \frac{1}{256}$ is the 5th term.

Question 5 (14 marks)

a) $PQ = 5 - x$ (1)

b) $PR^2 + x^2 = (5-x)^2$

$PR^2 = 25 - 10x + x^2 - x^2$

$= 25 - 10x$

$\therefore PR = \sqrt{25-10x}$. (2)

c) $\therefore A = \frac{1}{2}$ base x height.

$= \frac{1}{2} \times \sqrt{25-10x}$

$A = \frac{x\sqrt{25-10x}}{2}$ (1)

d) $A = \frac{1}{2} x \cdot (25-10x)^{1/2}$

$A' = (25-10x)^{1/2} \cdot \frac{1}{2} + \frac{1}{2} x \cdot \frac{1}{2}(25-10x)^{-1/2} \cdot -10$

$= (25-10x)^{1/2} \cdot \frac{1}{2} - \frac{5}{2} x (25-10x)^{-1/2}$

$= \frac{\sqrt{25-10x}}{2} - \frac{5x}{2\sqrt{25-10x}}$

St. pt when $A' = 0$

$\frac{\sqrt{25-10x}}{2} - \frac{5x}{2\sqrt{25-10x}} = 0$

$(\times 2\sqrt{25-10x})$

$25 - 10x - 5x = 0$

$25 = 15x$

$\frac{5}{3} = x$

check max.

$f'(1) > 0$

$f'(5/3) = 0$ $\therefore$ max, $x = 5/3$.

$f'(2) < 0$

$\therefore A = \frac{\frac{5}{3}\sqrt{25 - 10 \cdot \frac{5}{3}}}{2}$

$= \frac{5}{6}\sqrt{25 - 16\frac{2}{3}}$

$= \frac{5}{6}\sqrt{\frac{25}{3}}$

$A = \frac{25}{6\sqrt{3}}$ m^2. (4

ii) a) $r_1 = 2.5$ $r_2 = 4.5$, $r_3 = 6.5$

$r_n = 52.5$

$\therefore a = 2.5$ $d = 2$, $n = ?$

$52.5 = 2.5 + (n-1) \times 2$

$50 = 2n - 2$

$n = 26$ $\therefore$ 26 circles (2

b) $L = \sum_{26}$ + 4x 52.5

$= \frac{26}{2}[25\pi + 25(4\pi)]$

$= 13(110\pi) + 210$.

$= (1430\pi + 210)$ m

$C_1 = 2 \times \pi \times 2.5 = 5\pi$

$C_2 = 2 \times \pi \times 4.5 = 9\pi$

$C_3 = 2 \times \pi \times 6.5 = 13\pi$

$\therefore$ A.P $a = 5\pi$, $d = 4\pi$. (