KILLARA HIGH SCHOOL

2 UNIT MATHEMATICS

TERM IV
1999

Time : 65 minutes

1 (i) Differentiate

(a) $y = 3x^2 + 6x - 7x^{-1}$

(b) $y = (2x+3)(x+2)$

(c) $y = \sqrt{3x^2 + 6x}$

(d) $y = \frac{x}{2} + \frac{2}{x}$

(e) $y = \frac{x}{x+1}$

(f) $y = \frac{4x^2+5x}{2x}$

(ii) Find the equation of the tangent to $y = x^3 + 3x^2 - 8x$ at $x = 1$

(iii) At what points on the curve $y = x^3 - 12x$ is the tangent parallel with the x axis

2 (i) Given a sequence 2, 5, 8, ...

(a) Find the 40th term.

(b) Find the sum of the first 40 terms.

(ii) The tenth term of an A.P. is 29 and the fifteenth is 44. Find the sum of the first 75 terms.

(iii) The sum to n terms of a certain Arithmetic series is $34n - n$.

Find the

(a) First term

(b) Common difference

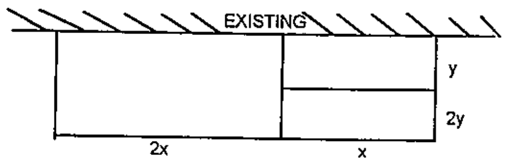
(iv) Express $9 \cdot 8\dot{7}\dot{6}$ as an infinite series, hence calculate its value as a mixed numeral.

(v) Can there be an infinite series with a limiting sum of $\frac{5}{8}$ and a first term of 2. All working and reasoning must be shown.

3 (i) The first three terms of a sequence are $x-2, x+1, 3x-3$. Find x if the sequence is Geometric.

(ii) The series $\frac{1}{3} - \frac{1}{6} + \frac{1}{12} - \ldots$ is geometric. Find the sum of the first 10 terms as a fraction in its simplest form.

(iii) Show by first principles that the derivative of $x^2 - 4x$ is $2x - 4$.

(iv) 

Farmer Dan of the O'H Ranch wants to erect an enclosure for his farm animals. The enclosure is to have 3 separate sections and dimensions as shown. An existing wall is to be used and 30 metres of fencing.

1) Show that the area of the enclosure is given by $A = 30x - 4x^2$.

2) Find the dimensions of the enclosure in order to have maximum Area.

... MATHS. TERM IV 1999

1)(i)

(a) $y = 3x^2 + 6x - 7x^{-1}$.
$\frac{dy}{dx} = 6x + 6 + 7x^{-2}$
$= 6x + 6 + \frac{7}{x^2}$ 3

) $y = (2x+3)(x+2)$.
$= 2x^2 + 7x + 6$.
$\frac{dy}{dx} = 4x + 7$ 3

$y = \sqrt{3x^2+6x}$
$= (3x^2+6x)^{\frac{1}{2}}$
$\frac{dy}{dx} = \frac{1}{2}(3x^2+6x)^{-\frac{1}{2}} \times (6x+6)$.
$= \frac{3x+3}{\sqrt{3x^2+6x}}$. 3

$y = \frac{x}{2} + \frac{2}{x}$.
$= \frac{1}{2}x + 2x^{-1}$.
$\frac{dy}{dx} = \frac{1}{2} - 2x^{-2}$
$= \frac{1}{2} - \frac{2}{x^2}$. 3

$y = \frac{x}{x+1}$. $\frac{u}{v}$.
$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$
$= \frac{1(x+1) - x(1)}{(x+1)^2}$.
$= \frac{1}{(x+1)^2}$. 3

1). $y = \frac{4x^2+5x}{2x}$.
$= 2x + \frac{5}{2}$.
$\frac{dy}{dx} = 2$. 3

/18

ii) $y = x^3 + 3x^2 - 8x$
$\frac{dy}{dx} = 3x^2 + 6x - 8$. 1
At $x=1$, $m = 3 + 6 - 8$.
$= 1$. 1
At $x=1$, $y = 1 + 3 - 8$
$= -4$. $\therefore (1,-4)$. 1
Eqn tangent $y - y_1 = m(x - x_1)$.
$y + 4 = 1.(x-1)$
$y + 4 = x - 1$.
$y = x - 5$. 1

/4

iii) $y = x^3 - 12x$.
$\frac{dy}{dx} = 3x^2 - 12$.
When // x axis $m = 0$. 1
$0 = 3x^2 - 12$
$= 3(x^2 - 4)$
$0 = (x+2)(x-2)$
$\therefore$ $x = -2$ or 2. 1
When $x = -2$, $y = -8 + 24$
$= 16$. $\therefore (-2, 16)$ 1
$x = 2$, $y = 8 - 24$. $\therefore (2, -16)$ 1

/4

(2). $2, 5, 8, \ldots$
i) a) A.P. where $a = 2$, $d = 3$.
$T_n = a + (n-1)d$.
$T_{40} = 2 + 39 \times 3$
$= 2 + 117$
$T_{40} = 119$. 2

b). $S_{40} = \frac{n}{2}\{a + l\}$
$= \frac{40}{2}(2 + 119)$
$= 20(121)$
$= 2420$ 2.

/4

ii) AP. T_{10}: $a + 9d = 29$ — (1)
T_{15}. $a + 14d = 44$. — (2)
(2)-(1): $5d = 15$
$\therefore d = 3$. 1
Sub in (1). $a = 29 - 9d$.
$= 29 - 27$
$a = 2$. 1
$\therefore$ Sequence: $2, 5, 8 \ldots$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$.
$S_{75} = \frac{75}{2}\{4 + 74 \times 3\}$. 1
$= \frac{75}{2}\{226\}$.
$S_{75} = 8475$ 1

/4

$S_n = 34n - n^2$.
$= S_1 = 34 - 1$
$= 33$.
S_2: $34 \times 2 - 4$ 1
$= 64$. $\therefore T_2 = S_2 - S_1 = 64 - 33 = 31$. 2
S_3 $34 \times 3 - 9$
$= 93$. $T_3 = S_3 - S_2$.
$= 93 - 64 = 29$.
$T_3 - T_2 = T_2 - T_1$.
$29 - 31$ $31 - 33$
-2. $= -2$.
$\therefore$ A.P. 33, 31, 29 ...
$a = 33$. $d = -2$.

(3)

$9.8\dot{7}\dot{6}$.
9.8767676.
$9.8 + \left(\frac{76}{1000} + \frac{76}{100000} + \frac{76}{10000000} + \cdots\right)$ 1
G.P. $a = \frac{76}{1000}$
$r = \frac{1}{100}$ 1
$S_\infty = \frac{a}{1-r}$.
$= \frac{\frac{76}{1000}}{1 - \frac{1}{100}}$
$= \frac{76}{1000} \times \frac{100}{99}$
$= \frac{76}{990}$. 1
$9.8\dot{7}\dot{6} = 9\frac{4}{5} + \frac{76}{990}$.
? 434 1

(4)

v). $S_\infty = \frac{5}{8}$. $a = 2$. ?
$S_\infty = \frac{a}{1-r}$.
$\frac{5}{8} = \frac{2}{1-r}$. 1
$5(1-r) = 16$.
$5 - 5r = 16$.
$5 - 16 = 5r$.
$-11 = 5r$.
$\frac{-11}{5} = r$
$r = -2\frac{1}{5}$. 1

No, as $|r| > 1$ there cannot be an infinite series.. 1

(3)

(3) $x-2,\ x+1,\ 3x-3, \ldots$

i) If G.P, $r = \dfrac{3x-3}{x+1} = \dfrac{x+1}{x-2}$

$(3x-3)(x-2) = (x+1)(x+1)$

$3x^2 - 9x + 6 = x^2 + 2x + 1$

$r =$ $\quad 2x^2 - 11x + 5 = 0$

$(2x-1)(x-5) = 0$

$\therefore x = \frac{1}{2}$ or $\underline{5}$

(3

ii) $\frac{1}{3} - \frac{1}{6} + \frac{1}{12} \ldots$

G.P. $a = \frac{1}{3}$ $\quad r = \dfrac{-\frac{1}{6}}{\frac{1}{3}} = -\frac{1}{2}$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_{10} = \dfrac{\frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^{10}\right)}{1 - -\frac{1}{2}}$

$= \dfrac{\frac{1}{3}\left(1 - \frac{1}{1024}\right)}{\frac{3}{2}}$ $\quad \frac{1}{3} \times \frac{2}{1}$

$= \frac{2}{9}\left(\frac{1023}{1024}\right)$

$= \underline{\underline{\frac{341}{1536}}}$

(4

iii) $y = x^2 - 4x$

$y + \delta y = (x+\delta x)^2 - 4(x+\delta x)$

$= x^2 + 2x\delta x + \delta x^2 - 4x - 4\delta x$

$\delta y = 2x\delta x + \delta x^2 - 4\delta x$

$\frac{\delta y}{\delta x} = 2x + \delta x - 4$

$\therefore \delta x \to \frac{dy}{dx} = 2x - 4$

(3

iv) $(2x + x) + x + 3(3y) = 30$

$4x + 9y = 30$

$9y = 30 - 4x$

$y = \left(\dfrac{30-4x}{9}\right)$

area $= 3x \times 3y$

$= \cancel{9}x \times \left(\dfrac{30-4x}{\cancel{9}}\right)$

1) $\underline{A = 30x - 4x^2}$

2) $A = 30x - 4x^2$

$\frac{dA}{dx} = 30 - 8x$

Max when $\frac{dA}{dx} = 0$

$\therefore 0 = 30 - 8x$

$8x = 30$

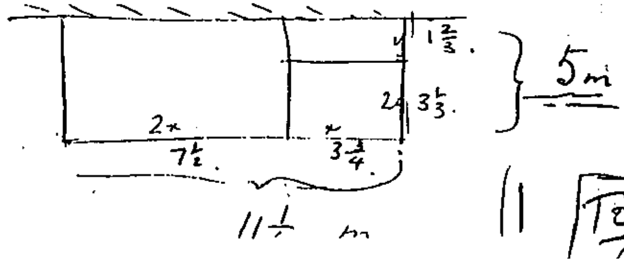
$x = \underline{3.75 \text{ m}}$

$y = \dfrac{30 - 4 \times 3.75}{9}$

$= \frac{15}{9}$

$= \underline{1\frac{2}{3} \text{ m}}$ $\quad (1.\dot{6})$

$\therefore$ Enclosure:



$\therefore$ Dimensions

$5\text{m} \times 11\frac{1}{4}\text{m}$

(5

~~(8)~~

Tota