**YEAR 11 TERM 4 ASSESSMENT 2000**

# MATHEMATICS

Time Allowed: 70 minutes.
Calculators may be used.
**Set your work out clearly. Use a new page for each question.**

Question 1. *(12 marks)*

(a) Factorize $27a^3 - 125b^6$

(b) Expand $4(r-6)^2$

(c) Write the derivatives of:

(i) $\dfrac{1}{4x^4}$

(ii) $\dfrac{1}{2x+1}$

(iii) $\sqrt{x^3}$

(iv) $(3x^2 - 5x - 2)(1 + x^3)^{-3}$

Question 2. *(12 marks)*

(a) Write down the **functions** whose derivatives are, respectively,

(i) $\dfrac{1}{x^2}$

(ii) $3x^2 + 2$

(iii) $x^4$

(b) Find the equation of the line through the point **(2, 0)** and the intersection of the two lines : $x + y = 0$
$2x - y = 3$.

(c) Find the point of intersection of the parabola $x^2 = 4y$ with the line $y = 2x - 4$.

Question 3. *(12 marks)*
For the function $y = x^3 - 3x^2 - 9x + 30$.

(i) Find the stationary points of $y = x^3 - 3x^2 - 9x + 30$. Establish the nature of these points with the use of calculus

(ii) Sketch the curve, showing all important features.

(iii) Find the equations of the tangent and the normal to the curve $y = x^3 - 3x^2 - 9x + 30$ at the point **(2, 8)**.

(iv) Find the co-ordinates of the points (or point) where the line $y = 30 - 9x$ intersects (or touches) the curve.

Question 4. *(12 marks)*

(a) The first four terms of three sequences are given below:

A: **3, 1, 4, 5;** B: **4, -2, 1, -½;** C: **-3, -1, 1, 3.**

(i) In each case state whether the sequence is geometric, arithmetic or neither.

(ii) For the sequence C, assume that it is, in fact, the type you consider possible and write down :

1. the tenth term
and 2. the sum of the first ten terms.

**(b)** In the sequence **5, 11, 29, .....** the n*th* term is $3^n + 2$.

Write down (i) the fourth term
(ii) the sum of the first **n** terms.

*Continued over . . . .*

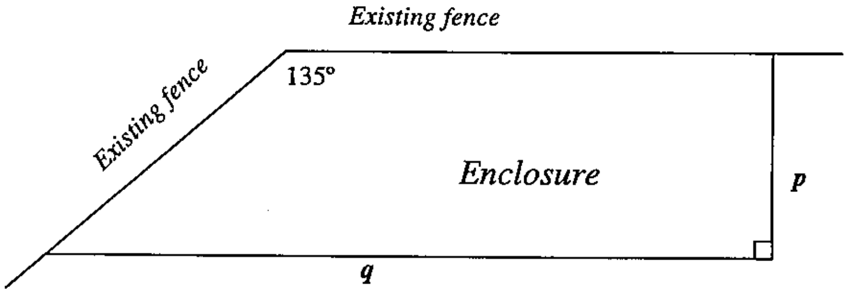
Question 5. *(12 marks)*

(a) (i) The series **1/3 - 1/6 + 1/12** is geometric.

Find the sum of the first ten terms . Give your answer as a **rational** number in its lowest terms.

(ii) Can there be an infinite geometric series with a limiting sum of **⅝** and the first term of **2**?
**All working and reasoning must be shown.**

(b)




At the corner of the property an enclosure is to be made for a chicken enclosure. The existing fence, which has an angle of $135^0$, is used for two side. 170 metres of fencing is available for use to make the enclosure.

Show that the area is $\mathbf{A = 170\,p - 3/2\,p^2}$ and show that the area of the enclosure is a maximum when $\mathbf{q = 2p}$.

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1(a) $27a^3 - 125b^6$

$= (3a)^3 - (5b^2)^3$

(2) $= (3a-5b^2)(9a^2+15ab^2+25b^4)$

(b) $4(r-6)^2$

$= 4(r^2-12r+36)$

(2) $= 4r^2-48r+144$

(c)(i) $\frac{d}{dx}\left(\frac{1}{4}x^{-4}\right)$

$= -x^{-5}$

(2) $= -\frac{1}{x^5}$

(ii) $\frac{d}{dx}(2x+1)^{-1}$

$= -1(2x+1)^{-2}.2$

(2) $= \frac{-2}{(2x+1)^2}$

(iii) $\frac{d}{dx}\left(x^{3/2}\right)$

$= \frac{3}{2}x^{1/2}$

(2) $= \frac{3\sqrt{x}}{2}$

(iv) $\frac{d}{dx}(3x^2-5x-2)(1+x^3)$

$= (1+x^3)^{-3}.(6x-5)+\frac{(3x^2-5x}{(1+x^3)}$

(2)

$= (1+x^3)^{-3}\left[6x-5-\frac{9x^2(3x^2-}{1+x^3}\right.$

$= \frac{(6x-5)(1+x^3)-27x^5+45x}{(1+x^3)^4}$

$\frac{6x-5+6x^4-5x^3-27x^5+45}{(1+x^3)^4}$

$\frac{-27x^5+6x^4+40x^3+18x^2+6}{(1+x^3)^4}$

2.(a)(i) $\frac{dy}{dx} = x^{-2}$

$y = -x^{-1}$

(2) $\therefore y = \frac{-1}{x} +$

(ii) $\frac{dy}{dx} = 3x^2+2$

(2) $y = x^3+2$

(iii) $\frac{dy}{dx} = x^4$

(2) $y = \frac{1}{5}x^5 +$

2.(b) $x+y=0$

$2x-y=3$

$2x-y-3=0$

$x+y+k(2x-y-3)=0$

$(2,0)$ $2+0+k(4-0-3)=0$

$2+k=0$

$\therefore k=-2$

$x+y-2(2x-y-3)=0$

$x+y-4x+2y+6=0$

③ $-3x+3y+6=0$

$x-y-2=0$

test $(2,0)$ $2-0-2=0$

(c) $x^2=4y$ ①

$y=2x-4$ ②

② into ① $x^2=4(2x-4)$

$x^2-8x+16=0$

$(x-4)^2=0$

$x=4$

in ② $y=8-4$
$=4$

check in ① $16=4.4$

③ Point of intersection is $(4,4)$

it actually touches parabola.

3. $y=x^3-3x^2-9x+30$

(i) $y'=3x^2-6x-9$
$=3(x^2-2x-3)$
$=3(x-3)(x+1)$

① $y'=0$, $x=3,-1$

$x=3, y=27-27-27+$
$=3$

① $(3,3)$

$x=-1, y=-1-3+9+3($
$=35$

① $(-1,35)$

$y''=6x-6$ $\quad x=1, y=1-3-9 =22$

$y''=6(x-1)$

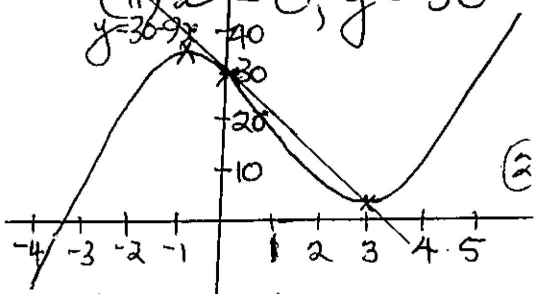
at $x=3$, $y''=6(3-1)>$

① $\therefore$ minimum at $(3,$

at $x=-1$, $y''=6(-1-1)<$

① $\therefore$ maximum at $(-1,35)$

(ii) $x=0, y=30$




$f(-2)=-8-12+18+30>0$

$f(-3)=-27-27+27+30>$

$f(-4)=-64-48+36+30<0$

(iii) $\frac{dy}{dx}=M=3(x-3)(x+1)$

at $x=2$, $M=3(2-3)(2+1)$
$=3.-1.3$

$M_T=-9$ ①

$\therefore M_n=\frac{1}{9}$

eqn of tangent: $y-8=-9(x-2)$

$y-8=-9x+18$

$9x+y-26=0$ ①

eqn of normal; $y-8=\frac{1}{9}(x-2)$

$9y-72=x-2$

$x-9y+70=0$ ①

(iv) $y=30-9x=x^3-3x^2-9x+30$

$0=x^3-3x^2$

$x^2(x-3)=0$

$x=0, x=3$

Line $y=30-9x$ intersects ①

with $y=x^3-3x^2-9x+30$

at $(3,3)$ and touches (two roots)

it at $(0,30)$ ①

4.(a) A $3,1,4,5$

(i) ① neither

B. $4,-2,1,-\frac{1}{2}$

$r=-\frac{1}{2}$

① $\therefore$ geometric

C. $-3,-1,1,3$

$d=2.$

① $\therefore$ arithmetic

(ii) $a=-3$

$d=2$

1. $T_n=a+(n-1)d$

$T_{10}=a+9d$

$=-3+18$

② $=15$

2. $S_n=\frac{n}{2}(a+l$

$=\frac{10}{2}(-3+1$

$=5\times12$

② $=60$

~~if geometric $a=-3$~~

~~$r=\frac{1}{3}$~~

~~1. $T_n=ar^{n-1}$~~

~~$T_{10}=-3(-\frac{1}{3})^9$~~

~~$=\frac{1}{3^8}=$~~

~~2. $S_n=\frac{-3(1-\frac{1}{3}}{(1-\frac{1}{3})}$~~

4(b) 5, 11, 29, .... $3^n+2$

$T_4 = 3^4+2$

$= 81+2$ (2)

$= 83$

$a=5$ $r=$ ? not arith/geom but both!

$3^1+2+3^2+2+3^3+2+\dots+3^n+2$

$=3^1+3^2+3^3+\dots+3^n+2\times n$

$S_n$ of $3^1+3^2+\dots$ (1)

$=\frac{3(3^n-1)}{3-1}$

$=\frac{3^{n+1}-3}{2}$

$S_n$ of $2+2+\dots = 2n$ (1)

Total sum $= \frac{3^{n+1}-3}{2}+2n$

$\sum_{n=1}^{n} 3^n+2 =$ or $\frac{3}{2}(3^n-1)+2n$ (1)

5.(a)(i) $\frac{1}{3}-\frac{1}{6}+\frac{1}{12}$

$r=\frac{-1/6}{1/3}=\frac{-1}{2}$

$r=\frac{1}{12}\div-\frac{1}{6}$

$=-\frac{1}{2}$ (1)

$S_{10}=\frac{\frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^{10}\right)}{1+\frac{1}{2}}$ (1)

$\frac{\frac{1}{3}\left(1-\frac{1}{1024}\right)}{\frac{3}{2}}$

$\frac{1}{3}\times\frac{2}{3}\left(\frac{1023}{1024}\right)$

$=\frac{2}{9}\left(\frac{1023}{\cancel{1024}\ 512}\right)$

(2) $=\frac{1023}{4608}=\frac{341}{1536}$

(ii) $\frac{a}{1-r}=\frac{5}{8}$

$\frac{2}{1-r}=\frac{5}{8}$

$16=5-5r$

$5r=-11$ (2)

$r=-\frac{11}{5}$

No, r must be between $-1$ and $+1$ ie $|r|<1$

5(b)

$p+q=170$

$q=170-p$ (1)

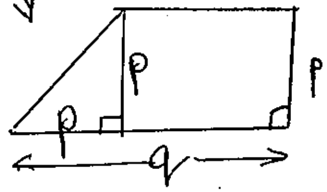
Area $= p(q-p)+\frac{1}{2}p^2$

$=p(170-p-p)+\frac{1}{2}p^2$

$=p(170-2p)+\frac{1}{2}p^2$

$=170p-2p^2+\frac{1}{2}p^2$

$=170p-\frac{3}{2}p^2$ (1)




Area $= \triangle + \square$

$=\frac{1}{2}p^2+(q-p)$

$=\frac{1}{2}p^2+pq-p$

$A'=170-3p$

$A'=0$, $3p=170$

$p=\frac{170}{3}$ (1)

$A''=-3<0$ (1)

$\therefore$ at $p=56\frac{2}{3}$ a maximum occurs

when $p=56\frac{2}{3}$

$q=170-56\frac{2}{3}$

$=113\frac{1}{3}$. (1)

ie. $q=2p$ (1)