KILLARA HIGH SCHOOL

TERM 1 EXAMINATION 2002

2 UNIT MATHEMATICS

Time Allowed: 3 hours

Instructions:

Attempt all questions
All questions are of equal value
Approved calculators may be used
Start your answer to each question on a new page

Question 1 *(14 marks)* Sequence + Series

(a) Which term of 16, 14, 12, 10, ... is -28? *(2 marks)*

(b) If the 10th term of an AP is 34 and the 20th term is 84 find a and d. Hence find the 30th term. *(4 marks)*

(c) Evaluate $\sum_{n=1}^{40} (10 - 3n)$ *(2 marks)*

(d) How many terms of $1 + 3 + 9 + 27$........ are needed for the sum to exceed 3000? *(3 marks)*

(e) If $x, 2, y$ are the first 3 terms of a GP and $x, y, 7$ are the first 3 terms of an AP, find x and y *(3 marks)*

Question 2 *(14 marks)* Circular Trigonometry

(a) Express in circular measure in terms of π *(2 marks)*

(i) 45° (ii) 240°

(b) Convert to degrees and minutes *(2 marks)*

(i) $\frac{\pi^c}{3}$ (ii) $1 \cdot 93^c$

(c) Find the area of the minor segment of a circle when $r = 12cm$, and $\theta = 60^\circ$ *(2 marks)*

(d) Without using graph paper and using $0 \le x \le 2\pi$ as domain, sketch the following functions on the one set of axes

(i) $y = \cos 3x$ (ii) $y = \cos \frac{x}{3}$ *(4 marks)*

(e) The longer "hand" of a clock is $1 \cdot 06m$ in length. How far does its tip move in 45 minutes? *(2 marks)*

(f) A sprinkler is set to water a sector of a circle with an angle of 75° at the centre. If the range of the sprinkler is 5m calculate the area watered. *(2 marks)*

Question 3 *(14 marks)* Calculus

(a) Expand and simplify (i) $\cos(x + \frac{\pi}{4})$ *(2 marks)*

(ii) $\tan(x - \frac{5\pi}{6})$ *(2 marks)*

(b) What is the period of $y = 3\sin 4x$? *(1 mark)*

(c) What is the amplitude of $y = 5\cos 2x$? *(1 mark)*

(d) Differentiate with respect to x

(i) $\sin(6 - x)$ *(1 mark)*

(ii) $3x - \cos 3x$ *(2 marks)*

(e) Integrate with respect to x

(i) $\cos(5 + x)$ *(1 mark)*

(ii) $x^3 - \sin(x + 2)$ *(2 marks)*

(f) Find the primitive function of $3\cos x + 2\sin 2x$ *(2 marks)*

Question 4 *(14 marks)* Calculus and Trigonometry

(a) Give the primitive functions of

(i) $5x^2$ (ii) $\frac{5}{x^2}$ *(2 marks)*

(b) Evaluate

(i) $\int_{-2}^{1} x^3 dx$ (ii) $\int_{1}^{3} (x^2 - 2x + 3)dx$ *(4 marks)*

(c) Find the first derivative of

(i) $(x + \frac{1}{x})^3$ *(2 marks)*

(ii) $\frac{\cos x}{3 - \sin x}$ *(2 marks)*

(iii) $(5x + 3)\tan x$ *(2 marks)*

(iv) $\frac{\tan(x^2 - 5)}{(x^2 - 5)}$ *(2 marks)*

Question 5 *(14 marks)* Integration

(a) Find the area enclosed between the y axis the curve $x = y^2 + 1$ and the lines $y = 1$ and $y = 3$ *(3 marks)*

(b) Find the volume of the solid created when the curve $y = x^2 + x$ is rotated about the x axis between $x = 1$ and $x = 2$. *(3 marks)*

(c) Apply the Trapezoidal rule using three function values to calculate $\int_{1}^{3} \frac{2}{x}dx$ *(4 marks)*

(d) Find the area enclosed between $y = x^2 - 9$ and $y = x - 3$ *(4 marks)*

Question 6 *(14 marks)* Quadratic + Parabola

(a) What values of m will make the expession $x^2 + 6x + m$ positive definite *(2 marks)*

(b) Write $3x^2 + 2x + 5$ in the form $A(x-1)^2 + B(x-1) + C = 0$ *(2 marks)*

(c) If α and β are the roots of the equation $2x^2 - 3x + 4 = 0$ find

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\alpha^2 + \beta^2$

(iv) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ *(4 marks)*

(d) Given $2\tan^2\theta - \tan\theta - 6 = 0$

(i) Substitute $x = \tan\theta$

(ii) Factorise and solve for x

(iii) Solve for θ if $0 \le \theta \le 2\pi$ *(4 marks)*

(e) Find the equation of the parabola with vertex (-3, 4) and directrix with equation $y = 1$. *(2 marks)*

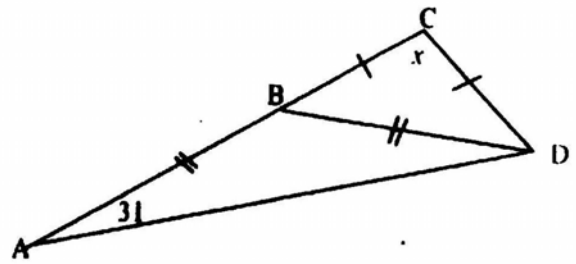
Question 7 *(14 marks)* Identities and Geometry

(a) Simplify (i) $\frac{3}{\sec^2\theta} + \frac{3}{\operatorname{cosec}^2\theta}$ *(1 mark)*

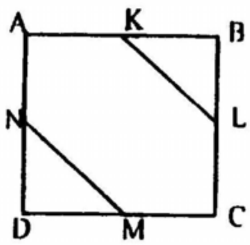
(ii) $\frac{\sec\theta}{\cos\theta} + \frac{\tan\theta}{\cot\theta}$ *(1 mark)*

(b) A and B are two gun emplacements on the coastline $2km$ apart. C is a target at sea such that $\angle BAC = 60^\circ 37'$ and $\angle ABC = 64^\circ 18'$. What is the range from A to target at C at sea. *(4 marks)*

(c) Find the value of the pronumeral giving reasons for your answer *(3 marks)*



(d)



A gift box with a square face (ABCD) is tied with ribbon crossing at K, L, M, N the midpoints of each respective side. Prove that KL = NM. *(5 marks)*

Question 8 *(14 marks)* Applications Derivative

(a) For the function $f(x) = 2x^3 - 8x$ find

(i) the first derivative *(1 mark)*
(ii) the second derivative *(1 mark)*
(iii) hence find any stationary point(s) and explain their nature(s). *(2 marks)*

(b) Find the equation of the normal, at $x = \frac{\pi}{3}$, to the curve $y = \tan(3x)$ *(3 marks)*

(c) If the gradient of the tangent at the point (x, y) on a given curve is $y' = 2\sin 3x$ and the curve passes through the point $(\frac{\pi}{3}, \frac{8}{3})$ find the equation of the curve. *(2 marks)*

(d) A new high school is to be built in a suburb where one side faces a road and the other 3 sides create a rectangular block. If the fencing available for these 3 sides is 700m find the length of road frontage that will give a maximum area. *(3 marks)*

(e) Find the area enclosed by the curve $y = 3\cos(x + 2)$ the x axis and the lines $x = \frac{\pi}{3}$ and $x = \frac{3\pi}{4}$ *(2 marks)*

Question 9 *(14 marks)*

(a) Write $1\cdot\dot{2}\dot{3}$ as a mixed fraction *(2 marks)*

(b) Anna borrows \$30000 from her Credit Union @ $1\cdot6\%$ per month reducible interest. She repays the loan over 3 years in equal monthly instalments. What are her repayments each month? *(3 marks)*

(c) Find the perpendicular distance from the point (3, 4) to the line $5x - 3y + 2 = 0$ *(2 marks)*

(d) On a number plan sketch the lines

(i) $2x + y = 4$ and (ii) $y = 3x + 6$

(iii) Clearly label the region in which $2x + y < 4$ and $y \geq 3x + 6$ *(4 marks)*

(e) Express $x^2 - 2x = 8y - 25$ in standard form and hence find

(i) the co-ordinates of the vertex

(ii) the co-ordinates of the focus *(3 marks)*

END OF PAPER

TERM 1 EXAM 2002.

2U MATHS SOLUTIONS.

QUESTION 1.

(a) $a = 16$ $d = -2$. $T_n = a + (n-1)d$

$-28 = 16 + (n-1)^{-}2.$ 1 ...

$-44 = -2(n-1)$

$n - 1 = 22$

$n = 23$ $\therefore -28$ is 23^{rd} term. 1 mk

(b) $T_{10} = a + 9d = 34$

$T_{20} = a + 19d = 84$ 1

$\therefore 10d = 50$

$\therefore d = 5$ 1

$\therefore a + 45 = 34$

$a = -11$ 1

$\therefore T_{30} = a + 29d$

$= -11 + 29 \times 5$

$= 134$ 1 mk

OR

$T_{20} - T_{10} = 50$ — 2 mk

$\therefore T_{30} = 84 + 50$ — 1 mk

$= 134$. 1 mk

(c) $\sum_{n=1}^{40} (10 - 3n) = 7 + 4 + 1 + \ldots + {}^{-}110$ i.e. $a = 7$ $d = {}^{-}3$ 1 mk

$S_{40} = \frac{n}{2}[2a + (n-1)d]$ OR $\frac{n}{2}[a + l]$

$= \frac{40}{2}[14 + 39 \times {}^{-}3]$ $\Rightarrow 20[7 - 110]$ 1 mk

$= 20 \times -103$

$= -2060$ $= -2060$ 1 mk

(d) $1 + 3 + 9 + 27 + \ldots$ $a = 1$ $r = 3$ i.e. GP 1 mk

$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{1(3^n - 1)}{3 - 1} > 3000$ 1 mk

$\therefore 3^n - 1 > 6000$

$3^n > 6001$

$3^8 = 6561$ 8 terms 1 mk

(e) $x, 2, y = GP$ $\quad$ $xy, 7 = AP$

$\therefore \frac{2}{x} = \frac{y}{2}$ $\quad$ $\therefore y - x = 7 - y$

$\therefore xy = 4$ —— (1) 1 mk $\quad$ $\therefore 2y - 7 = x$ (2) 1 mk

Sub (2) into (1) $\quad (2y-7)y = 4$

$2y^2 - 7y - 4 = 0$

$(2y+1)(y-4) = 0$

$y = -\frac{1}{2}$ or $y = 4$

$\therefore x = -8$ or $x = 1$.

ANS $(-8, -\frac{1}{2})$ or $(1, 4)$. 1 mk

QUESTION 2.

(a) (i) $45° = \pi/4$ 1 mk $\quad$ (ii) $240° = 4\pi/3$ 1 mk

(b) (i) $\pi/3 = 60°$ 1 mk $\quad$ (ii) $1.93^c = 110°35'$ 1 mk

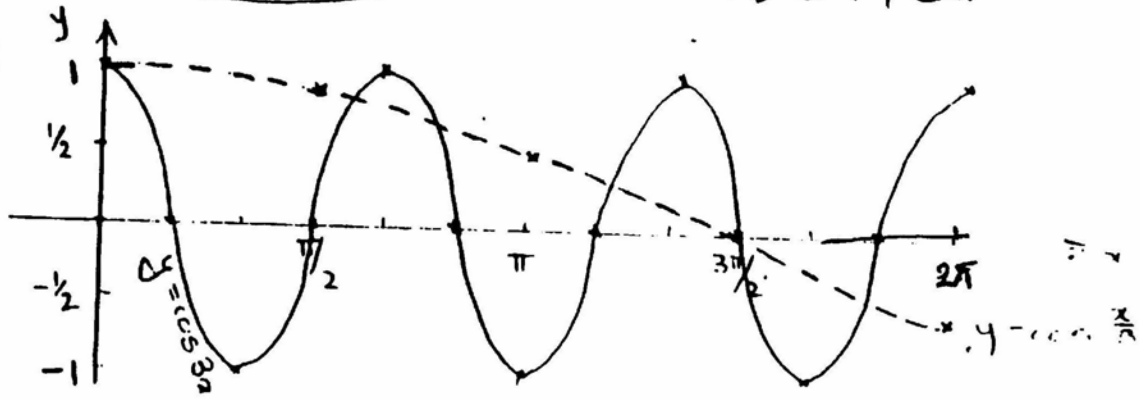
(c) If in degrees
$A = \frac{1}{2} \times 12^2 (60 - \sin 60)$ ✗
BUT $= 4257.6$ Allow 1

$A = \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{1}{2} \times 12^2 (\pi/3 - \sin \pi/3)$ 1 mk

$= 13.044 \text{ cm}^2$ 1 mk

(d)



(e) $45 \text{mins} = 3/4 \text{ hr}$ $\therefore$ $3/4$ revolution or $\frac{3\pi}{2}$ radians

$l = r\theta = 1.06 \times \frac{3\pi}{2}$

$= 4.9951$ m.

$\doteqdot 5$ m. $\quad$ 500 cm $\quad$ 4995 mm

QUESTION 2 CONTINUED

f) $A = \pi r^2 \frac{\theta}{360}$ or $\frac{1}{2} r^2 \theta$

$= \pi 5^2 \frac{75}{360}$ or $\frac{1}{2} \times r^2 \times \frac{75\pi}{180}$ — 1 · $\frac{5\pi}{12}$

$= 16 \cdot 362\ m^2$ or $\frac{125\pi}{24}\ m^2$ — 1

QUESTION 3.

NOT IN 4 UNIT — 4 marks. Allow

a) (i) $\cos(x + \pi/4) = \cos x \cos \pi/4 - \sin x \sin \pi/4$ — 1 mk

$= \frac{\cos x - \sin x}{\sqrt{2}}$ or $\frac{1}{2}(\cos x - \sin x)$ — 1 mk

(ii) $\tan(x - 5\pi/6) = \frac{\tan x - \tan(5\pi/6)}{1 + \tan x \tan 5\pi/6}$ — 1 mk

$= \frac{\tan x + 1/\sqrt{3}}{1 + \tan x \times -\frac{1}{\sqrt{3}}}$

$= \frac{\sqrt{3}\tan x + 1}{\sqrt{3} - \tan x}$ — 1 mk

$= \frac{\sqrt{3}\tan^2 x + 4\tan x + 1}{3 - \tan^2 x}$

(b) Period $= \frac{2\pi}{4} = \frac{\pi}{2}$ or $90°$ — 1 mk

(c) Amplitude $= 5$ — 1 mk

(d) (i) $f(x) = \sin(6 - x)$

$f'(x) = -\cos(6 - x)$ — 1 mk

(ii) $f(x) = 3x - \cos 3x$

$f'(x) = 3 + 3\sin 3x$ — 1 mk, 1 mk

(e) (i) $\int \cos(5 + x)\,dx$

$= \sin(5 + x) + C$

(ii) $\int (x^3 - \sin(x + 2))\,dx$

$= \frac{1}{4}x^4 + \cos(x + 2) + C$ — 1, 1

(f) $\int (3\cos x + 2\sin 2x)\,dx = 3\sin x - \cos 2x + C$ — 1, 1

Omit $+C$, 1 mk once only

QUESTION 4

(a) (i) $\int 5x^2\,dx = \frac{5}{3}x^3 + C$ 1 1

(ii) $\int 5x^{-2}\,dx = -5x^{-1} + C$ or $\frac{-5}{x} + C$ 1 1

(b) (i) $\int_{-2}^{1} x^3\,dx = \left[\frac{x^4}{4}\right]_{-2}^{1}$ 1

$= \frac{1}{4} - \frac{16}{4}$

$= -3\frac{3}{4}$ 1

(ii) $\int_1^3 (x^2 - 2x + 3)\,dx = \left[\frac{1}{3}x^3 - x^2 + 3x\right]_1^3$ 1

$= (9 - 9 + 9) - (\frac{1}{3} - 1 + 3)$

$= 6\frac{2}{3}.$ 1

(c) (i) $\frac{d}{dx}\left(x + \frac{1}{x}\right)^3 = 3\left(x + \frac{1}{x}\right)^2 \times (1 - x^{-2})$

$= 3\left(1 - \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)^2$

(ii) $\frac{d}{dx}\left(\frac{\cos x}{3 - \sin x}\right)$ Let $u = \cos x$ $v = 3 - \sin x$

$u' = -\sin x$ $v' = -\cos x$ 1

$\therefore \frac{dy}{dx} = \frac{vu' - uv'}{v^2} = \frac{(3 - \sin x)(-\sin x) + \cos x \cos x}{(3 - \sin x)^2}$

$= \frac{-3\sin x + \sin^2 x + \cos^2 x}{(3 - \sin x)^2}$

$= \frac{1 - 3\sin x}{(3 - \sin x)^2}$ 1 1

(iii) $y = (5x + 3)\tan x$ $u = 5x + 3$ $v = \tan x$

$u' = 5$ $v' = \sec^2 x$

$\therefore y' = uv' + vu' = (5x + 3)\sec^2 x + \tan x \times 5$

or $5\tan x + (5x + 3)\sec^2 x$

(iv) $y = \frac{\tan(x^2 - 5)}{x^2 - 5}$ $u = \tan(x^2 - 5)$ $v = x^2 - 5$ 1

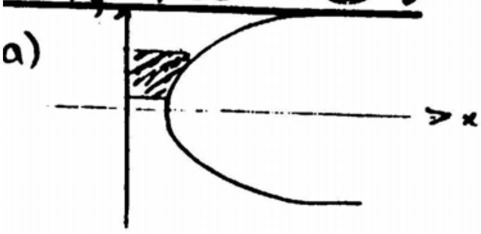
$u' = (2x).\sec^2(x^2 - 5)$ $v' = 2x$ 1

$y' = \frac{vu' - uv'}{v^2} = \frac{(x^2 - 5)2x\sec^2(x^2 - 5) - 2x\tan(x^2 - 5)}{(x^2 - 5)^2}$

$= \frac{2x\left[(x^2 - 5)\sec^2(x^2 - 5) - \tan(x^2 - 5)\right]}{(x^2 - 5)^2}$

QUESTION 5 :

a)



$A = \int_1^3 (x)\, dy$

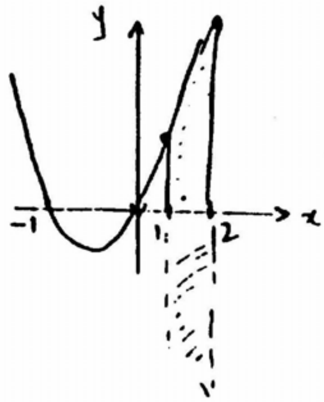
$= \int_1^3 (y^2+1)\, dy$ 1 or diagram

$= \left[\frac{1}{3}y^3 + y\right]_1^3$ 1

$= (9+3) - (\frac{1}{3}+1)$

$= 10\frac{2}{3}$ SQ UNITS 1

b)



$y = x^2 + x = x(x+1)$

$Vol = \int_1^2 \pi y^2\, dx$

$= \pi \int_1^2 (x^2+x)^2\, dx$ 1

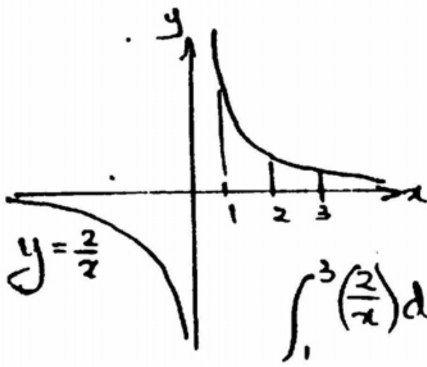
$= \pi \int_1^2 (x^4 + 2x^3 + x^2)\, dx$

$= \pi \left[\frac{1}{5}x^5 + \frac{1}{2}x^4 + \frac{1}{3}x^3\right]_1^2$ 1

$= \pi \left[\left(\frac{32}{5} + 8 + \frac{8}{3}\right) - \left(\frac{1}{5} + \frac{1}{2} + \frac{1}{3}\right)\right]$

$= \frac{481\pi}{30}$ CUB UNITS

$\doteqdot 50 \cdot 37\ u^3$ 1



(c) $y = \frac{2}{x}$

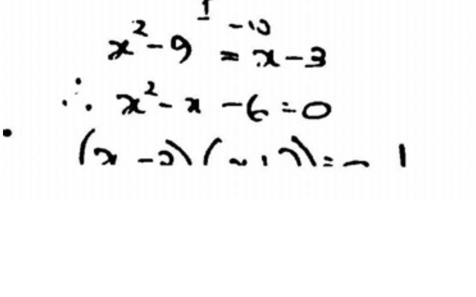
$\int_1^3 \left(\frac{2}{x}\right) dx \doteqdot \frac{b-a}{2}\left[f(a) + f(b)\right]$ twice

$= \frac{2-1}{2}\left[f(1) + f(2)\right] + \frac{3-2}{2}\left[f(2) + f(3)\right]$

$= \frac{1}{2}(2+1) + \frac{1}{2}\left(1 + \frac{2}{3}\right)$ 1

$= 2\frac{1}{3}$ SQ UNITS 1

(d) Using $y = x^2 - 9$ and $y = x - 3$



$x^2 - 9 = x - 3$

$\therefore x^2 - x - 6 = 0$

$A = \left| \text{Area}(x^2-9) - \text{Area}(x-3) \right|$ 1

$= \left| \int_{-2}^{3} (x^2-9)\, dx \right| - \left| \int_{-2}^{3} (x-3)\, dx \right|$

$= \left| \left[\frac{1}{3}x^3 - 9x\right]_{-2}^{3} \right| - \left| \left[\frac{1}{2}x^2 - 3x\right]_{-2}^{3} \right|$

$= \left| (9-27) - \left(-\frac{8}{3} + 18\right) \right| - \left| \left(\frac{9}{2} - 9\right) - \left(\frac{4}{2} + 6\right) \right|$

QUESTION 6.

(a) x^2+6x+m Pos. D.f. If $a>0$ $\Delta<0$ 1

$a=1 \therefore >0$ 1 $\Delta = 36-4\times1\times m < 0$

If $36 > 4m$

i.e. $m > 9$ 1

(b) $3x^2+2x+5 = A(x-1)^2 + B(x-1) + C$

RHS $= A(x^2-2x+1) + B(x-1) + C$

$= Ax^2 + x(-2A+B) + (A-B+C)$

$\therefore A = 3$ 1

also $-2A+B = 2$

$\therefore -6+B = 2$

$\therefore B = 8$ 1

also $A-B+C = 5$

$\therefore 3-8+C = 5$

$+C = 10$ 1

Sub:

$\therefore 3x^2+2x+5 = 3(x-1)^2 + 8(x-1) + 10$

(c) $2x^2-3x+4=0$

$\therefore$ (i) $\alpha+\beta = -b/a$

$= 3/2$ 1

(ii) $\alpha\beta = \frac{c}{a}$

$= 2$. 1

(iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{3}{2}\right)^2 - 2\times2$

$= -1\frac{3}{4}$ 1

(iv) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$\frac{1\frac{3}{4}}{2}$

$-\frac{7}{8}$ 1

(d) $2\tan^2\theta - \tan\theta - 6 = 0$

$\therefore$ (i) $2x^2 - x - 6 = 0$ 1

(ii) $(2x+3)(x-2) = 0$

$\therefore x = -3/2$ or $x = 2$ 1

(iii) $\therefore \tan\theta = -3/2$ or $\tan\theta = 2$.

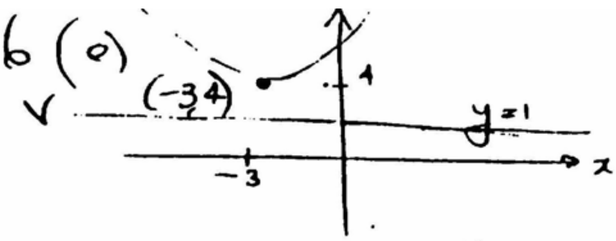
$\theta = 56°19'$ in 2nd/4th or $63°26'$ in 1st/3rd

$= 123°41'$ or $303°41'$ $63°26'$ $243°26'$

$= 2.158$ or 5.300 $= 1.107$ or 4.248

4

6 (a)



Focal length = 3

Parabola is of form

$$4ay = x^2$$

$$4a(y-k) = (x-h)^2$$

$$\therefore 12(y-4) = (x+3)^2 \quad 1$$ or sketch complete

i.e. $$12y - 48 = x^2+6x+9$$

$$x^2 + 6x - 12y + 57 = 0 \quad 1$$

QUESTION 7.

a) (i) $$\frac{3}{\sec^2\theta} + \frac{3}{\operatorname{cosec}^2\theta}$$

$$= 3\cos^2\theta + 3\sin^2\theta$$

$$= 3. \qquad (1)$$

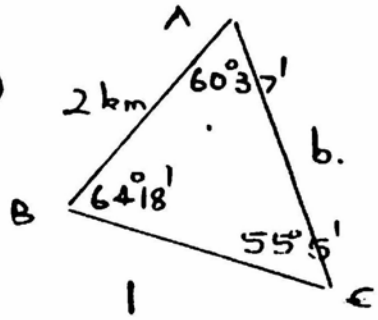
(ii) $$\frac{\sec\theta}{\cos\theta} + \frac{\tan\theta}{\cot\theta}$$

$$= \sec^2\theta + \tan^2\theta$$

$$= (1+\tan^2\theta) + \tan^2\theta$$

$$= 2\tan^2\theta + 1 \quad (1)$$

b)



1

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

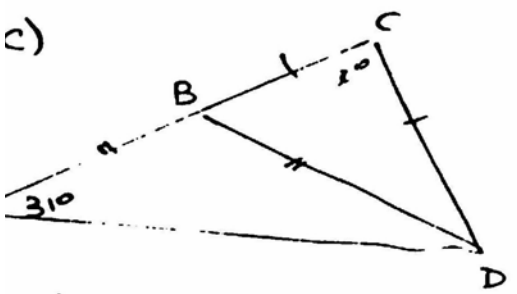
$$\frac{b}{\sin 64°18'} = \frac{2000}{\sin 55°5'} \quad 1$$

$$b = \frac{2000 \times \sin 64°18'}{\sin 55°5'}$$

$$= 2197{\cdot}7879\ldots \quad 1$$

The target is 2198m from C 1
any sensible answer

c)



$\angle BDC = 31°$ ... $\angle$'s opp = sides 1

$\angle CBD = 62°$ ... ext $\angle$ Δ = sum 2 int opp $\angle$'s. 1

$\angle CDB = 62°$ ... $\angle$'s opp = sides

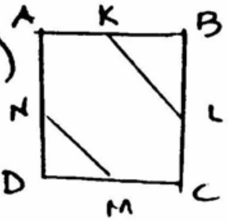
$\therefore \angle BCD = 180° - 124°$ ... $\angle$ sum Δ 1

$$x° = 56°$$

-1 any ommission

Question 7 (cont)

7(d)



$AB = BC = CD = AD$ ... ABCD a square

$\therefore KB = BL = DM = DN$ .... each $\frac{1}{2}$ of above 1

In $\triangle DNM$ and $\triangle BKL$

$DN = KB$ ... above

$DM = LB$ ... above

$\angle NDM = 90° = \angle KBL$ ... ABCD square

$\therefore \triangle DNM \equiv \triangle KBL$ ... SAS

$\therefore NM = KL$ ... corresp sides congruent $\triangle$'s. 1

Question 8.

(a) $f(x) = 2x^3 - 8x$

(i) $f'(x) = 6x^2 - 8$ 1

(ii) $f''(x) = 12x$ 1

(iii) Let $f'(x) = 0 \quad \therefore 6x^2 = 8$

$x^2 = \frac{8}{6}$

$x = \pm\frac{2}{\sqrt{3}}$ 1

$\therefore$ Stat Points are $\left(\frac{2}{\sqrt{3}}, f\left(\frac{2}{\sqrt{3}}\right)\right)$ $\left(-\frac{2}{\sqrt{3}}, f\left(-\frac{2}{\sqrt{3}}\right)\right)$

or $\left(\frac{2}{\sqrt{3}}, \frac{-32}{3\sqrt{3}}\right)$ $\left(-\frac{2}{\sqrt{3}}, \frac{+32}{3\sqrt{3}}\right)$

$\therefore$ Minimum $\quad \therefore$ Minimum

OR Using $f''(x)$

$f''\left(\frac{2}{\sqrt{3}}\right) = \frac{24}{\sqrt{3}} > 0$ $\quad f''\left(-\frac{2}{\sqrt{3}}\right) = \frac{-24}{\sqrt{3}} < 0$ 1

$\therefore$ Minimum $\quad \therefore$ Maximum

(b) $y = \tan 3x$

$y' = 3\sec^2 3x.$

$f'(\pi/3) = 3\sec^2 \pi$

$= 3 \times (-1)^2$

$= 3$

$\therefore -\frac{1}{m} = -\frac{1}{3}$ 1

$f(\pi/3) = \tan \pi$

$= 0$ 1

Normal $y - y_1 = m(x - x_1)$

$y - 0 = \frac{1}{3}(x - \pi/3)$

$y = -\frac{x}{3} + \frac{\pi}{3}$

or $\frac{x}{3} + y - \frac{\pi}{3}$ 1

or $x + 3y = \frac{\pi}{3}$

(c) $y' = 2\sin 3x$

$\therefore y = -\frac{2}{3}\cos(3x) + c.$ 1

$\therefore \frac{8}{3} = -\frac{2}{3}\cos(\pi) + c.$

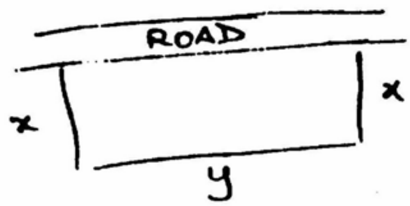
$\therefore \frac{8}{3} = \frac{2}{3} + c$

$\therefore c = 2.$

Curve $y = -\frac{2}{3}\cos(3x) + 2.$ 1

$= 2 - \frac{2}{3}\cos(3x)$

(d)



$2x + y = 700$

$y = 700 - 2x.$ 1

$\therefore$ Road frontage

$= 700 - 2 \times 175$

$= 350\text{m}$ 1

$A = x \times y$

$= x(700 - 2x)$

$= 700x - 2x^2$

$A' = 700 - 4x$ 1

For stat point

$700 = 4x$

$x = 175.$

$A'' = -4 < 0$

$\therefore$ Max.

(e) $y = 3\cos(x+2)$

$A = -\int_{\frac{\pi}{3}}^{\frac{3\pi}{4}} 3\cos(x+2)\,dx.$

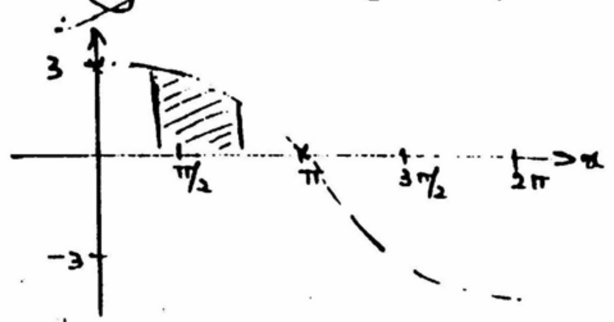
$= -3\int_{\frac{\pi}{3}}^{3\pi/4} \cos(x+2)\,dx$

$= -3\left[\sin(x+2)\right]_{\pi/3}^{3\pi/4}$

$= -3\left[\sin\left(\frac{3\pi}{4}+2\right) - \sin\left(\frac{\pi}{3}+2\right)\right]$

$= -3\left[(-0.937) - (0.09425)\right]$

$y = 3\cos(x \div 2)$



$A = \int_{\pi/3}^{3\pi/4} 3\cos\left(\frac{x}{2}\right)dx$

$= 3\left[2\sin\frac{x}{2}\right]_{\pi/3}^{3\pi/4}$ 1

$= 6\left[\cdot 9238\ldots - 0\cdot 5\right]$

$= 2.54327\ldots \doteq 2.5u^2$ 1

QUESTION 9

(a) $1\cdot\dot{2}\dot{3}$ $= 1\frac{23}{99}$ — 2

Let $x = 1\cdot\dot{2}\dot{3}$ (Gr)

$\therefore 100x = 123\cdot\dot{2}\dot{3}$ 1

$\therefore 99x = 122$

$x = \frac{122}{99} = 1\frac{23}{99}$ — 1

(b) Borrows

$A = P\left(1+\frac{R}{100}\right)^n$

$= 30000\,(1\cdot016)^{36}$

$= 53124\cdot48597\ldots$ 1

Repays

$P_1 = Q\,(1\cdot016)^{35}$

$P_2 = Q\,(1\cdot016)^{34}$

$\vdots$

$P_{36} = Q$

GP $a = Q$ $n = 36$

Total

$S_n = a\left(\frac{r^n - 1}{r-1}\right)$

$= Q\left(\frac{1\cdot016^{36} - 1}{1\cdot016 - 1}\right)$

$= Q \times 48\cdot176\ldots$

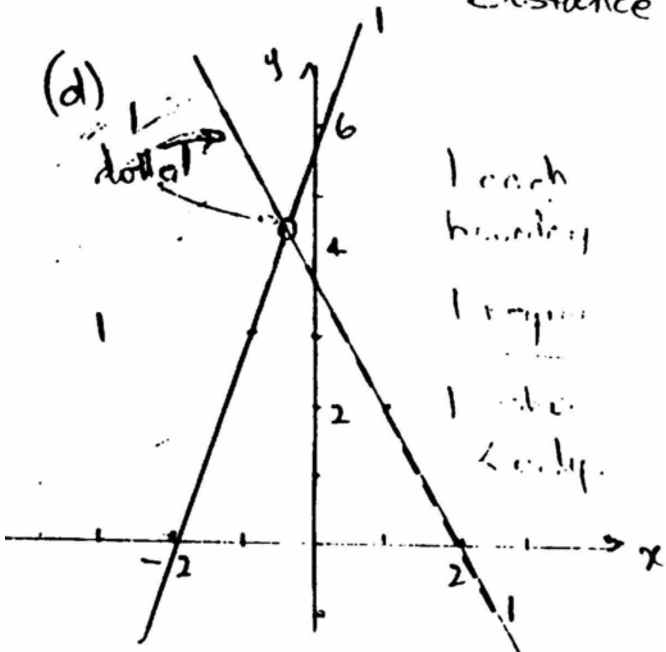
$\therefore Q = \frac{53124\cdot48597\ldots}{48\cdot176012\ldots} = 1102\cdot711555\ldots$ 1

Repayment $\doteqdot$ \$1102·70 or \$1102·72 or \$1103 given

(c) $d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right| = \left|\frac{5\times3 - 3\times4 + 2}{\sqrt{5^2+3^2}}\right|$ 1

Distance $= \frac{5}{\sqrt{34}}$ or $\frac{5\sqrt{34}}{34}$ 1

(d)



$y = 3x+6$ $y = -2x+4$

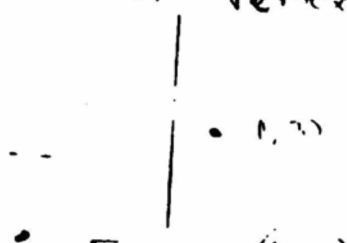
$y \geq 3x+6$ $y < -2x+4$

(e) $x^2 - 2x = 8y - 25$

$x^2 - 2x + 1 = 8y - 24$

$(x-1)^2 = 8(y-3)$

(i) $\therefore a = 2$ Vertex $(1, 3)$ — 1



(ii) $\therefore$ Focus $(1, 5)$ — 1