

KILLARA HIGH SCHOOL



YEAR 11 MATHEMATICS

TERM 4 EXAMINATION - DECEMBER 2004

Time Allowed: 1 hour

Total Marks: 65 marks

Instructions:

- * Attempt all questions
- * Approved calculators may be used
- * Show all necessary working clearly. Answers without appropriate working may not be awarded marks.
- * Marks may be deducted for untidy or poorly set out work
- * Write solutions on one side of your paper only using black or blue pen
- * Begin each question on a new page

SECTION A: Calculus and Applications

- Find
 - $\lim_{h \rightarrow 0} \frac{h^2 + 7h}{h}$
 - $\lim_{x \rightarrow 4} \frac{x^2 - 3x - 4}{x^2 - 7x + 12}$
- Differentiate
 - $y = 3x^4$
 - $y = 1 - \frac{x}{2}$
 - $y = \frac{x}{1 + x}$
 - $y = (5x + 6)(2x - 1)^3$
 - $y = \frac{x^3 - 3x^2 + 2}{x}$
 - $y = \sqrt[3]{x^3 - 6}$
- What is the gradient of the curve $y = x(x^2 - 3)^4$ at the point (2, 2)
- Find the equation of the tangent to the parabola $y = 2x^2 - 5x + 1$ at the point on the parabola where the tangent is parallel to the line $y = 3x - 4$

SECTION B. Sequences & Series

1. What is the 7th term of the following series:
 - i) 3, 7, 11
 - ii) 12, -6, 3
 - iii) $3\sqrt{3}$, 9, $9\sqrt{3}$
 - iv) $3A - 4B$, $4A - 3B$, $5A - 2B$
2. If $A + 6$, $2A + 1$, and $A + 10$ are the first 3 terms of an arithmetic series find
 - i) A
 - ii) T_{10}
3. If the first term of a geometric series is 12 and the 4th term is 27, find the 2nd and 3rd term.
4. If $T_N = N^2 + 1$, find the value of T_7 . Is the series arithmetic, geometric or neither? Give a reason for your answer.
5. Find the sum of the first 25 terms of an A.P., given that $T_{15} = 34$ and $S_8 = 20$

Soln Yr 11 2004 Term 4

t. Q1: $\lim_{h \rightarrow 0} \frac{h(h+7)}{h} = 7$ ✓

ii) $\lim_{x \rightarrow 4} \frac{(x-4)(x+1)}{(x-3)(x-4)}$
 $= \frac{5}{1} = 5$ ✓

Q2 i) $y = 3x^4$
 $y' = 12x^3$ ✓

ii) $y = 1 - \frac{1}{2}x$
 $y' = -\frac{1}{2}$ ✓

iii) $u = x$ $v = 1+x$
 $u' = 1$ $v' = 1$ ✓

$y' = \frac{(1+x) \cdot 1 - x \cdot 1}{(1+x)^2}$
 $= \frac{1}{(1+x)^2}$ ✓

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iv) $u = 5x+6$ $v = (2x-1)^3$
 $u' = 5$ $v' = 3(2x-1)^2 \cdot 2$

$y' = (5x+6) \cdot 6(2x-1)^2 + (2x-1)^3 \cdot 5$
 $= (2x-1)^2 [6(5x+6) + 5(2x-1)]$ ✓

v) $y = x^2 - 3x + 2x^{-1}$
 $y' = 2x - 3 - 2x^{-2}$ ✓

vi) $y = (x^3 - 6)^{\frac{1}{3}}$
 $y' = \frac{1}{3} (x^3 - 6)^{-\frac{2}{3}} \cdot 3x^2$
 $= \frac{x^2}{\sqrt[3]{(x^3 - 6)^2}}$ ✓

Q3 $u = x$ $v = (x^2 - 3)^4$
 $u' = 1$ $v' = 4(x^2 - 3)^3 \cdot 2x$
 $y' = x \cdot 8x(x^2 - 3)^3 + (x^2 - 3)^4$

at $x = 2$, $y' = m = 8 \cdot 4 \cdot 1 + 1 = 33$
 at $(2, 2)$ grad = 33 ✓

Q4 $y = 2x^2 - 5x + 1$
 $y' = 4x - 5$
 at $y' = m = 3$
 $3 = 4x - 5$
 $x = 2$
 $y = 2 \cdot 4 - 10 + 1 = -1$
 equ $y + 1 = 3(x - 2)$
 $0 = 3x - y - 7$ ✓

Q1 i) A.P. $a = 3$ $d = 4$ ✓
 $T_7 = a + 6d$
 $= 3 + 6 \cdot 4$
 $= 27$ ✓

ii) G.P. $a = 12$ $r = \frac{1}{2}$
 $T_7 = ar^6$
 $= 12 \cdot \left(\frac{1}{2}\right)^6$
 $= \frac{3}{16}$ ✓

iii) G.P. $a = 3\sqrt{3}$ $r = \sqrt{3}$
 $T_7 = ar^6$
 $= 3\sqrt{3} \cdot (\sqrt{3})^6$
 $= 81\sqrt{3}$ ✓

iv) A.P. $a = 3A - 4B$
 $d = A + B$
 $T_7 = a + 6d$
 $= 3A - 4B + 6(A + B)$
 $= 9A + 2B$ ✓

$d = (2A + 1) - (A + 6)$
 $= (A + 10) - (2A + 1)$

$A - 5 = -A + 9$
 $2A = 14$
 $A = 7$ ✓

$a = 13$ $d = 2$
 $T_{10} = a + 9d$
 $= 13 + 9 \cdot 2$
 $= 31$ ✓

Q3 G.P.
 $a = 12$
 $T_4 = ar^3 = 27$ ✓

$12 \cdot r^3 = 27$
 $r^3 = \frac{9}{4}$
 $r = \sqrt[3]{\frac{9}{4}}$ ✓

$T_2 = 12 \cdot 3 \sqrt[3]{\frac{9}{4}}$
 $T_3 = 12 \cdot 3 \sqrt[3]{\frac{81}{16}}$ ✓

Q4 $T_5 = 5^2 + 1$
 $= 26$

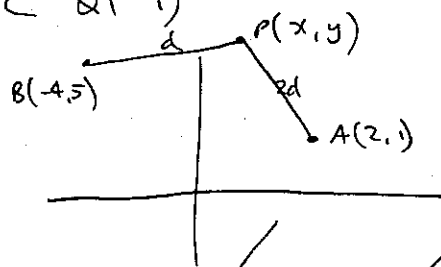
$T_1 = 2$ $T_3 = 10$
 $T_2 = 5$ Neither
 $\therefore$ it does not have a common Ratio or d.f. ✓

Q5 A.P. $T_{15} = a + 14d = 3$
 $S_8 = \frac{8}{2} (2a + 7d) = 20$

$a + 14d = 34$
 $8a + 28d = 20$
 $2a + 28d = 68$
 $6a = -48$
 $a = -8$
 $d = 3$ ✓

$S_{25} = \frac{25}{2} (-16 + 24 \cdot 3)$
 $= 700$ ✓

C Q1 i)



$$AP = 2PB$$

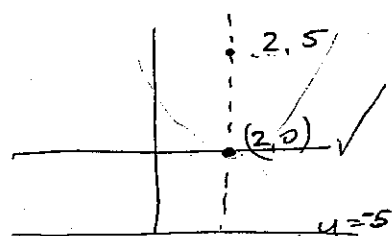
$$\sqrt{(x-2)^2 + (y-1)^2} = 2\sqrt{(x+4)^2 + (y-5)^2}$$

$$x^2 - 4x + 4 + y^2 - 2y + 1 = 4(x^2 + 8x + 16 + y^2 - 10y + 25)$$

$$3x^2 + 36x + 3y^2 - 38y + 159 = 0 \quad (4)$$

ii) A circle (1)

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$$(x-2)^2 = 4 \cdot 5 \cdot y$$

$$(x-2)^2 = 20y \quad (3)$$

Q3

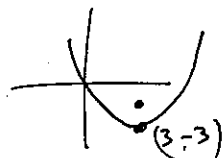
$$x^2 - 6x + 9 = 2y - 3 + 4$$

$$\sqrt{(x-3)^2} = 2\sqrt{y+3}$$

$$= 2(y+3)$$

Vertex (3, -3)

$$4a = 2 \quad a = \frac{1}{2} \checkmark$$



- i) Vertex (3, -3) ✓
- ii) Focus (3, -2½) ✓
- iii) Directrix { y = -3½ } ✓

(7)