



KILLARA HIGH SCHOOL

HSC MATHEMATICS

ASSESSMENT – DECEMBER 2007

Time Allowed: 70 minutes

Maximum marks: 60

Instructions:

- * Attempt all questions
- * Approved calculators may be used
- * Start your answer to each question on a NEW page.
- * Marks may not be awarded for careless or badly arranged

work.

CONTENT: Calculus (30 marks) – Questions 1 to 6

Sequence and Series (30 marks) – Questions 7 to 12

Question 1. (6 marks)

Find the second derivative for each of the following functions:

(a) $y = 5x^3 - 9x$ (2 marks)

(b) $y = (x^2 + 7x)^2$ (2 marks)

(c) $y = \frac{x+1}{x}$ (2 marks)

Question 2. (4 marks)

Find the equation of

(a) the tangent and (2 marks)

(b) the normal to the curve $y = x^2 + 3x - 2$ at the point $(-3, -2)$ on the curve.
(2 marks)

Question 3. (6 marks)

For the function $y = x^3 - 3x$

(a) Find the turning points (2 marks)

(b) Use the second derivative to determine their nature. (2 marks)

(c) Use this information and any other facts you may determine to draw a neat sketch of the curve. (2 marks)

Question 4. (4 marks)

Find the primitive function for each of the following:

(a) $f(x) = 8x^3 - 6x^2$ (2 marks)

(b) $f(x) = 5 + \frac{4}{x^2}$ (2 marks)

Continued over page . . .

Question 5. (4 marks)

The gradient function of a curve is $x - 4$.

- (a) Find the equation of the family of curves that satisfy this condition. (2 marks)
- (b) Find the specific equation from this family whose curve passes through $(-2, 6)$ (2 marks)

Question 6. (6 marks)

A hospitality company caters for parties and charges according to the formula

$P = 800 + 200n - 2n^2$ where P is the profit in dollars and n is the number of guests at the party.

- (a) Show that a maximum possible profit exists. (2 marks)
- (b) Find the number of guests to generate this profit. (2 marks)
- (c) Calculate the value of this profit. (2 marks)

SEQUENCES AND SERIES

Question 7. (4 marks)

For the series given by $T_n = 10 - n - n^2$ find :

- (a) the first term (1 mark)
- (b) the second term (1 mark)
- (c) prove whether the series is an AP (arithmetic), a GP (geometric) or neither. (2 marks)

Question 8. (5 marks)

If $(x + 3) + (2x) + (5x - 11)$ forms an arithmetic series find:

- (a) the value of x (2 marks)
- (b) the common difference (2 marks)
- (c) the fourth term (1 mark)

Question 9. (5 marks)

A worker is employed at a starting salary of \$15,500. After each year they receive an increment of \$1000.

- (a) What is their salary in their 8th year of service? (2 marks)
- (b) During which year will the total of their earnings exceed \$500 000? (3 marks)

Question 10. (7 marks)

- (a) What is the value of the first term of the series $6 + 18 + 54 + \dots$ which is greater than 900? (3 marks)
- (b) Find the limiting sum of the series $192 + 96 + 48 + \dots$ (2 marks)
- (c) Evaluate $\sum_{k=5}^{15} 3 \times 5^{k-1}$

Question 11. (4 marks)

In a certain geometric series, the value of $T_6 - T_{10}$ is 15 and the value of $T_3 - T_7$ is 120. Find the values of :

- (a) a (4 marks)
- (b) r (1 mark)

Question 12. (5 marks)

Bacteria are multiplying in a culture so that at the end of each hour there is 7% more than at the beginning of the hour. If 200 000 bacteria are present now, how many will be present after 6 hours (a school day)? (5 marks)

END OF EXAM