KILLARA HIGH SCHOOL

YEAR 12 MATHEMATICS
ASSESSMENT 2 - 2007

TOTAL TIME: 3 hours

TOTAL MARKS: 120

Read each question thoroughly.

Attempt all questions.

Start each question on a new page.

Non-Programmable Calculator may be used.

All necessary working out must be shown to obtain full marks

Total marks 120
Attempt all questions 1 – 10
All questions are approximately of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklet available.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate (round the answer off to 3 significant figures). $\sqrt{\dfrac{5.2^3-10.8}{15.9\times 23.6}}$ **1**

(b) Evaluate $\lim_{x\to 2}\dfrac{x^2-3x+2}{x-2}$ **1**

(c) Solve for x: $|3x-4|\geq 8$ **2**

(d) Prove the identity: $(1-\tan x)^2+(1+\tan x)^2 = 2\sec^2 x$. **2**

(e) Express as a fraction in its simplest term: $\dfrac{5}{x^2-5x+4}-\dfrac{3}{x^2-16}$ **2**

(f) Rationalise the denominator and express in simplest terms $\dfrac{2-\sqrt{3}}{2\sqrt{3}+1}$ **2**

(g) Given that $\log_a 10 = 3.321928$ *and* $\log_a 7 = 2.807355$, calculate $\log_a 49000$ **2**

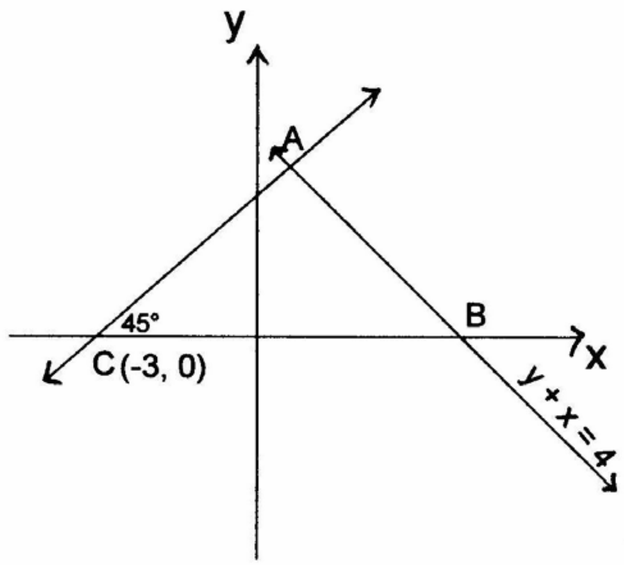
Marks

Question 2 (11 marks) Use a SEPARATE writing booklet.

(a) Show, by shading on a sketch, the region defined by the inequalities **3**

$$2x + 3y < 6,$$
$$x - y \geq 1$$

(b) In the diagram below, the line through C(-3, 0), makes an angle of 45° to the x-axis and intersects the line $y + x = 4$ at the point A. Copy the diagram into your examination booklet.



i) Show that AC is perpendicular to AB. **1**

ii) Find the equation of the line through B which is parallel to AC. **2**

iii) Show that the equation of the line through the point C which is parallel to the line AB is $y + x + 3 = 0$. **1**

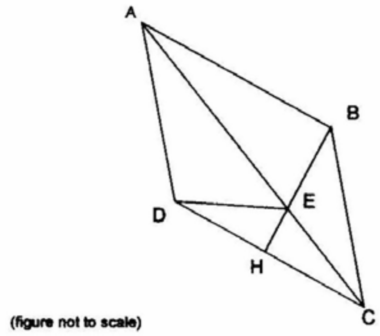
iv) The lines from part (ii) and (iii) intersect at D. Find the coordinates of the point D. **2**

v) What type of quadrilateral is ABDC? Give reasons. **2**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a)



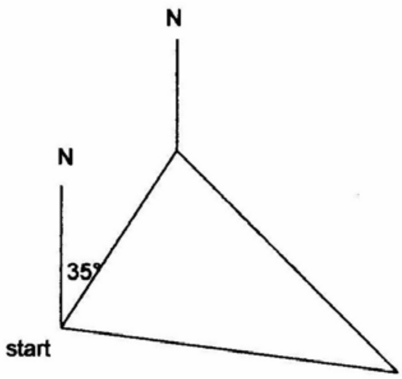
ABCD is a rhombus, BH is perpendicular to DC and intersect AC at E.

i) Copy the diagram into your examination booklet and prove that $\Delta ADE \equiv \Delta ABE$. **3**

ii) Show that $\angle ABE$ is a right angle. **1**

(b) Solve $\cos\theta = 2\cos^2\theta$ for the domain $0 \leq \theta \leq 360°$ **3**

(c) A helicopter flies 54 km on a bearing of 035˚. It then flies 67 km on a bearing of 140˚.



i) How far is it from its starting point (correct to one decimal place)? **2**

ii) What bearing needs to be taken to return to its starting point? Give your answer correct to the nearest degree. **3**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) i) Find the locus of a point P which moves so that PA = $\sqrt{2}$ PB where P is the point (x, y), A is (-1, 0) and B is (1, 1). **2**

ii) Show that the locus is a circle **1**

iii) Find the length of the radius and the coordinates of the centre of the circle. **2**

(b) A parabola has its vertex at the point (4, -3) and focus at the point (1, -3).

i) What is the focal length? **1**

ii) What is the equation of the directrix? **1**

iii) What is the equation of the parabola? **2**

(c) Solve for x: $9^x - 12(3^x) + 27 = 0$ **3**

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) The roots of the equation $2x^2 + bx + c = 0$ are $-\frac{1}{2}$ and -4. Find the values of b and c. **2**

(b) For what values of m does the quadratic equation $x^2 - 2mx + 8m - 15 = 0$ have two unequal roots? **2**

(c) Find the constants a, b, and c such that $x^2 + 2x - 2 \equiv ax(x+1) + bx^2 + c(x+1)$. **3**

(d) A function is defined by the rule $f(x) = \begin{cases} \sqrt{x-1}, & x \geq 1 \\ x-1, & x < 1 \end{cases}$

i) Find $f(-5) - f(10)$ **1**

ii) If $f(x) = 8$, find x. **2**

(e) State the largest possible domain and range for the function defined by

$$f(x) = |x| - 2$$ **2**

Marks

Question 6 (13 marks) Use a SEPARATE writing booklet.

(a) Differentiate with respect to x:

i) $5x^3 - \frac{2}{x} + 5$ **1**

ii) $\frac{8x-1}{3x+1}$ **2**

iii) $3x^2(2x+1)^4$ **2**

iv) $\frac{10}{\sqrt{(12-5x)^3}}$ **2**

(b) For the curve y = $x^3 + x^2 - x$

i) find the stationary points and determine their nature. **3**

ii) find any points of inflexion. **1**

iii) sketch the curve. **2**

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) Given the series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+........$

i) find the seventh term correct to 3 decimal places. **1**

ii) find the sum to infinity. **1**

(b) The first term of an arithmetic series is 5, and the fifteenth term is three times the fourth term. Find the common difference. **3**

(c) Jane invests \$800 at the beginning of each year in a superannuation fund. Compound interest is paid at 12% per annum on the investment. The first \$800 is to be invested at the beginning of 2008 and the last is to be invested at the beginning of 2022.

i) Calculate the amount to which the 2008 investment will have grown by the beginning of 2023. **2**

ii) Show that the amount (A) to which the total investment will have grown by the beginning of 2023 is: **2**

$$A = 800\left[\frac{1.12(1.12^{15}-1)}{0.12}\right]$$

iii) Find the total value of investment at the beginning of 2023. **1**

(d) Simplify $\log(x^2-2x+1)-\log(x-1)$ **2**

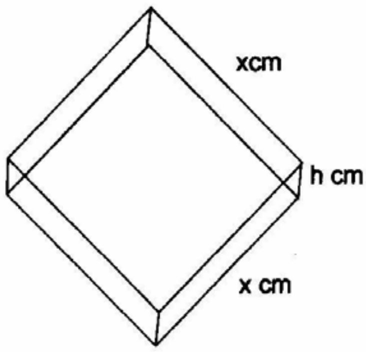
Marks

Question 8 (13 marks) Use a SEPARATE writing booklet.

(a) A sealed tin rectangular box is to have a square base and a volume of 64 cm^3.
If the length of the edge of the base is x cm,

i) express the height h in term of x, **1**

ii) show that the total surface area $y\ cm^2$ is given by $y = \dfrac{256}{x} + 2x^2$, **2**



iii) find the minimum surface area of this box. **3**

(b) If $\dfrac{dy}{dx} = 2x + 3$ and y = 5 when x = - 2, express y as a function of x, **2**

(c) Find the indefinite integrals:

i) $\int (x^4 - 3x^2 + 3)\, dx$ **1**

ii) $\int \dfrac{4x^3 - x^2 + 5}{x^2}\, dx$ **2**

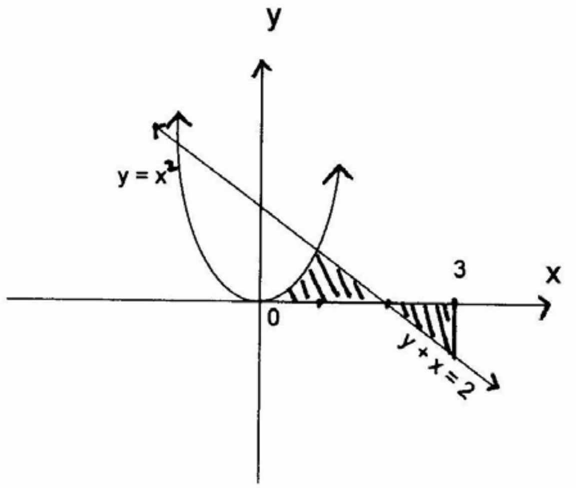
iii) $\int \dfrac{5}{\sqrt{(2-3x)^3}}\, dx$ **2**

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the area of the shaded region bounded by $y + x = 2$, $y = x^2$ and the x-axis as shown.

i) Find the x-coordinates of the points of intersection of the curve $y = x^2$ and the line $y + x = 2$. **2**

ii) Find the area of the shaded region. **3**



(b) Consider the function given by $y = x\log_e x$.

i) Copy and complete the following table in your examination booklet. **1**

x	1	2	3	4	5
y					

ii) Apply Simpson's rule with five function values to find an approximation to **2**

$$\int_1^5 x\log_e x \; dx$$

(c) A cone is formed by rotating the segment of the line $x + 2y = 4$ cut off by the axes about the Y-axis.

i) Draw a diagram to illustrate this information. **1**

ii) Find the volume of the cone. **3**

Marks

Question 10 (11 marks) Use a SEPARATE writing booklet.

(a) Differentiate with respect to x:

i) $\log_e 5x + e^{-3x} - 2x$ **1**

ii) $\log_e(x^2 - 3x + 1)$ **1**

iii) $e^{2x} \log_e x$ **2**

(b) Find the indefinite integral:

i) $\int \frac{x}{x^2 + 1}\, dx$ **1**

ii) $\int \frac{x-1}{x+4}\, dx$ **2**

(c) Evaluate $\int_1^2 (e^{2x+3} + 1)\, dx$ **2**

(d) Find the equation of the tangent to the curve $y = e^{2x}$ at the point where x = 1. **2**

Q1 | ... Solution

a) $= 0.58816$

$= 0.588$ (1)

c) $3x - 4 \geq 8$ & $3x - 4 \leq -8$

$3x \geq 12$ $\quad$ $3x \leq -4$

$x \geq 4$ $\quad$ $x \leq -\frac{4}{3}$ (2)

d)

$L.H.S = 1 - 2\tan x + \tan^2 x + 1 + 2\tan x + \tan^2 x$

$= 2 + 2\tan^2 x$

$= 2(1 + \tan^2 x)$

$= 2\sec^2 x \quad = R.H.S$ (2)

e)

$$\frac{5}{(x-4)(x-1)} - \frac{3}{(x+4)(x-4)}$$ (1)

$$= \frac{5(x+4) - 3(x-1)}{(x^2-16)(x-1)} = \frac{5x+20 - 3x + 3}{(x^2-16)(x-1)}$$

$$= \frac{2x + 23}{(x^2-16)(x-1)}$$ (1)

f) $$= \frac{(2-\sqrt{3})(2\sqrt{3} - 1)}{12 - 1} = \frac{4\sqrt{3} - 2 - 6 + \sqrt{3}}{11}$$ (1)

$$= \frac{5\sqrt{3} - 8}{11}$$ (1)

b) $$= \lim_{x \to 2} \frac{(x-1)\cancel{(x-2)}}{\cancel{(x-2)}} = 1$$ (1)

g) $\log_a 49000 = \log_a (49 \times 1000)$

$= \log_a 49 + \log_a 1000$ (1)

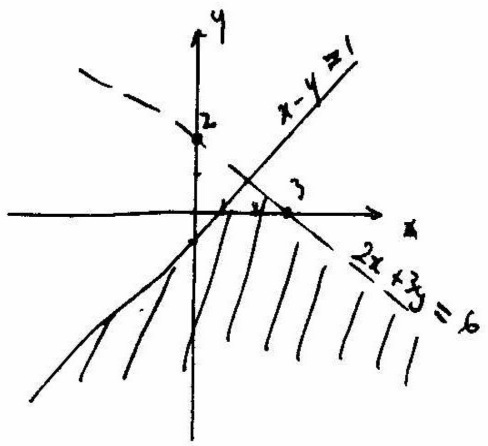
$= 2\log_a 7 + 3\log_a 10$

$= 2 \times 2.807355 + 3 \times 3.3219 = 15.58$ (1)

12

Q2

a)



① correct { dark lines / broken

① correct shaded region

① labelling

(−1) for any mistake.

,) (i) $m_{AC} = \tan 45^{\circ}$ $\qquad m_{AB} = -1$

$= 1$

$m_{AC} \times m_{AB} = -1 \quad \therefore AC \perp AB$ ①

i)

$\frac{y-0}{x-4} = 1 \Rightarrow x - 4 = y$ ② Co-ords of B = (4, 0)

i)

$\frac{y-0}{x+3} = -1 \Rightarrow x + 3 + y = 0$ ②

v)

$$\begin{array}{rl} & x - y = +4 \quad (1) \\ + & x + y = -3 \\ \hline & 2x \quad = +1 \end{array}$$

$x = +\frac{1}{2}$ sub. into (1) ①

$\Rightarrow y = -3\frac{1}{2}$ ①

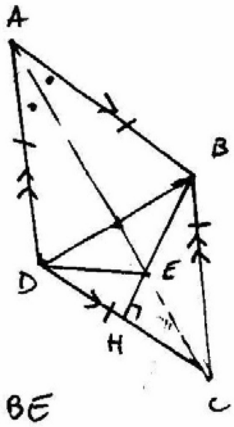
$\therefore D\left(+\frac{1}{2}, -3\frac{1}{2}\right)$

v) ABDC has opp. sides // with 1 right angle ①

$\therefore$ ABCD is a rectangle. ①

Q3

a)



i) in $\triangle ADE$ and $\triangle ABE$

$AB = AD$ (sides of a rhombus) 1

AE is common 1

$\angle DAE = \angle EAB$ (Diag. of rhombus bisects the angle).

$\therefore \triangle ADE \equiv \triangle ABE$ (SAS) 1

ii) $AB \parallel DC$ (opp. sides of rhombus)

$\therefore \angle EHC = \angle ABH$ (alt. $\angle s'$ in $\parallel$ lines). 1

b)

$\cos\theta = 2\cos^2\theta$

$2\cos^2\theta - \cos\theta = 0$

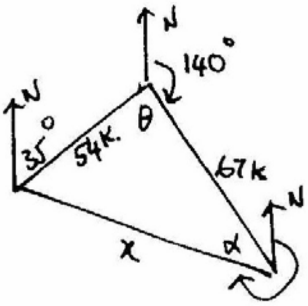
$\cos\theta(2\cos\theta - 1) = 0$ 1

$\cos\theta = 0 \Rightarrow \theta = 90^\circ, 270^\circ$ 1

$2\cos\theta - 1 = 0 \Rightarrow \theta = 60^\circ, 300^\circ$ 1

$\therefore \theta = 90^\circ, 60^\circ, 270^\circ, 300^\circ$

c)



i) $\theta = 360^\circ - 140^\circ - (180^\circ - 35^\circ)$

$= 75^\circ$ 1

$x^2 = 54^2 + 67^2 - 2\times54\times67\times\cos75$

$\doteqdot 5532.1854$

$x = 74.4$ Km 1

ii) $\dfrac{74.4}{\sin 75} = \dfrac{54}{\sin\alpha} \Rightarrow \sin\alpha = \dfrac{54\times\sin75}{74.4} = 0.701$ 1

$\alpha = 44^\circ 30' 48''$ 1

Bearing $= 360^\circ - (180^\circ - 140^\circ) - 44^\circ 30' 48'' = 275^\circ T$ 1

Q4 i) $PA = \sqrt{2}\, PB$
a)
$PA^2 = 2PB^2$

$(x+1)^2 + y^2 = 2\{(x-1)^2 + (y-1)^2\}$ (1)

$x^2 + 2x + 1 + y^2 = 2x^2 - 4x + 2 + 2y^2 - 4y + 2$

$x^2 + y^2 - 6x - 4y = -3$ (1)

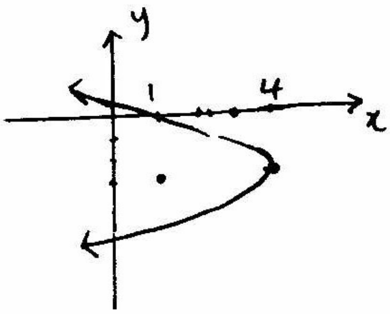
ii) $x^2 - 6x + 9 + y^2 - 4y + 4 = -3 + 9 + 4$

$(x-3)^2 + (y-2)^2 = 10$ (1)

iii) radius $= \sqrt{10}$ (1)

centre $(3, 2)$ (1)

(b)



$(y-k)^2 = -4a(x-h)$

i) $a = 3$ (1)

ii) directrix $x = 7$ (1)

iii) $(y+3)^2 = -12(x-4)$ (2)

(c) let $X = 3^x$

$X^2 - 12X + 27 = 0$ (1)

$(X-9)(X-3) = 0$

$3^x = 9$ & $3^x = 3$

$x = 2$ $x = 1$

(1) (1)

$\boxed{Q5}$

a) $-\frac{1}{2} - 4 = -\frac{b}{2}$ $\qquad$ $-\frac{1}{2} \times -4 = \frac{c}{2}$

$\frac{b}{2} = 4\frac{1}{2}$ $\qquad$ $c = 4$. (1)

$b = 9$ (1)

b) for 2 unequal roots $\Delta > 0$

$4m^2 - 4 \times 1 \times (8m - 15) > 0$ (1)

$4m^2 - 32m + 60 > 0 \Rightarrow m^2 - 8m + 15 > 0$

$(m-5)(m-3) > 0$

$\therefore$ $m > 5$ and $m < 3$ (1)

c) $ax(x+1) + bx^2 + c(x+1) = ax^2 + ax + bx^2 + cx + c$

$= (a+b)x^2 + (a+c)x + C$ (1)

$\therefore$ $C = -2$

$a + c = 2$ $\qquad$ $a + b = 1$

$a = 4$ (1) $\qquad$ $b = -3$ (1)

d) i) $f(-5) - f(10) = -6 - 3 = -9$ (1)

ii) $\sqrt{x-1} = 8$ (1)

$x - 1 = 64$

$x = 65$ (1)

e)

domain : x is all real (1)

range : $y \geq -2$ (1)

5 b) i) $= 15x^2 + \frac{2}{x^2}$ (1)

ii) $\frac{8(3x+1) - 3(8x-1)}{(3x+1)^2} = \frac{24x+8-24x+3}{(3x+1)^2} = \frac{11}{(3x+1)^2}$ (2)

iii) $= 6x(2x+1)^4 + 3x^2 \times 4 \times 2(2x+1)^4$

$= 6x(2x+1)^4 + 24x^2(2x+1)^3$

$= 6x(2x+1)^3(2x+1+4x)$

$= 6x(2x+1)^3(6x+1)$ (2)

iv) $= \frac{10}{\sqrt{(12-5x)^5}} \times -5x - \frac{3}{2} = \frac{75}{\sqrt{(12-5x)^5}}$ (2)

6)

$y = x^3 + x^2 - x$

$y' = 3x^2 + 2x - 1$

i) for stationary point $y' = 0$

$\Rightarrow 3x^2 + 2x - 1 = 0$

$(x+1)(3x-1) = 0 \Rightarrow x = -1$ & $x = \frac{1}{3}$ (1)

$f''(x) = 6x + 2$

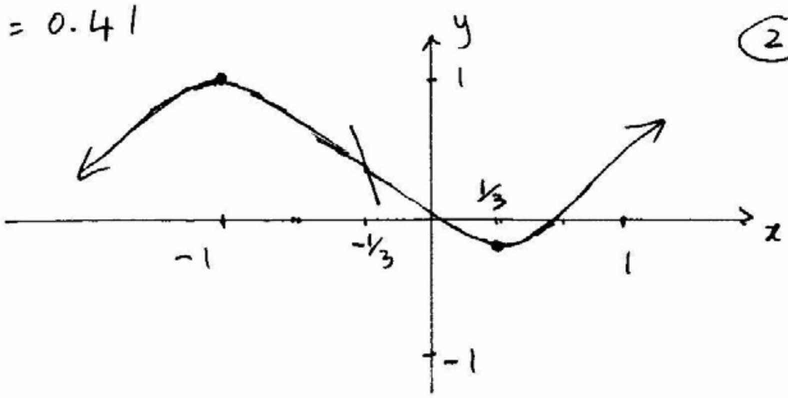
$f''(-1) = -4 < 0 \therefore$ max. point at $(-1, 1)$ (1)

$f''(\frac{1}{3}) = 4 > 0 \therefore$ min. point at $(\frac{1}{3}, -0.185)$ (1)

ii) point of inflexion $y'' = 0$

$6x + 2 = 0$ (1)

$x = -\frac{1}{3}$, $y = 0.41$ (2)



(1) - Correct shape

(1) - labelling.

T3

Q7

a) i) $T_7 = 1 \times \left(-\frac{1}{2}\right)^6$ 1

$= 0.017$

ii) $S_\infty = \dfrac{a}{1-r} = \dfrac{1}{1+\frac{1}{2}} = \underline{\underline{\dfrac{2}{3}}}$ 1

b) $T_4 = 5 + 3d$ 1

$T_{15} = 5 + 14d$

$T_{15} = 3T_4 \Rightarrow 5 + 14d = 3(5 + 3d)$ 1

$5d = 10$

$\underline{\underline{d = 2}}$ 1

c) (i) $A_1 = 800\,(1.12)^{15} = \underline{\underline{\$4378.85}}$ 1

(ii) $A_2 = 800\,(1.12)^{14}$

$A_3 = 800\,(1.12)^{13}$ 1

$\vdots$

$A_{15} = 800\,(1.12)^1$

Total value $A = A_{15} + \dots + A_1$

$= 800 \times 1.12 + 800 \times 1.12^2 + \dots + 800 \times 1.12^{15}$ 1

$= \dfrac{800 \times 1.12\,(1.12^{15} - 1)}{1.12 - 1}$ 1

$= \dfrac{800 \times 1.12\,(1.12^{15} - 1)}{0.12}$

iii) $A = \$33403$ 1

d) $\log(x^2 - 2x + 1) - \log(x - 1) = \log \dfrac{x^2 - 2x + 1}{x - 1}$ 1

$= \log \dfrac{(x-1)\cancel{(x-1)}}{\cancel{x-1}}$

$= \log(x - 1)$ 1

12

28)

a) i) $x^2 \times h = 64 \Rightarrow h = \dfrac{64}{x^2}$ (1)

ii) $S.A = 2x^2 + 4(x \times h)$ (1)

$= 2x^2 + x \times \dfrac{256}{x^2}$

$= 2x^2 + \dfrac{256}{x}$ (1)

iii) $\dfrac{dA}{dx} = 4x - \dfrac{256}{x^2} = 0$

$4x^3 - 256 = 0$ (1)

$x^3 = 64 \Rightarrow x = 4$ (1)

$\therefore S.A = 2 \times 4^2 + \dfrac{256}{4} = 32 + 64 = 96\ cm^2$ (1)

b) $y = \int 2x + 3\, dx = x^2 + 3x + C$ (1)

$5 = (-2)^2 - 3 \times 2 + c$

$c = 7$

$y = x^2 + 3x + 7$ (1)

c)

i) $= \dfrac{x^5}{5} - x^3 + 3x + c$ (1)

ii) $\displaystyle\int \frac{4x^3 - x^2 + 5}{x^2}\, dx = \int 4x - 1 + \frac{5}{x^2}\, dx$ (1)

$= 2x^2 - x - \dfrac{5}{x} + c$ (1)

iii)

$\displaystyle\int \frac{5}{\sqrt{(2-3x)^3}}\, dx = \frac{\pm 10}{3\sqrt{2-3x}} + c$ (2)

Q9 1

(a) i) $y = x^2$ ①

$x + y = 2$ ② sub. ① → ②

$\Rightarrow \quad x^2 + x = 2$ (1)

$x^2 + x - 2 = 0$

$(x-1)(x+2) = 0 \quad \Rightarrow \quad x = 1, -2.$ (1)

(ii)

$$A = \int_0^1 x^2 \, dx \quad + \quad 2\int_1^2 2 - x \, dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 + 2\left[2x - \frac{x^2}{2}\right]_1^2 = \frac{1}{3} + 2\left(2 - 1\tfrac{1}{2}\right)$$

(1) (1)

$= 1\tfrac{1}{3}$ sq. units. (1)

(b) i)

x	1	2	3	4	5
y	0	1.386	3.296	5.545	8.047

(1)

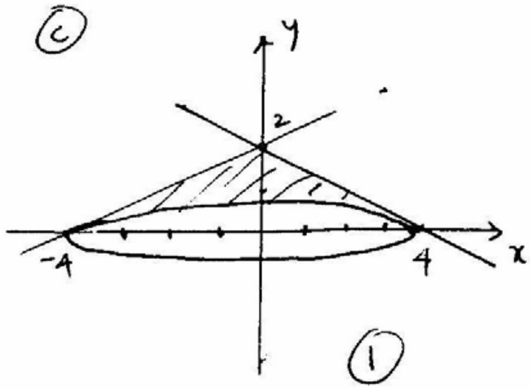
ii) $$A = \int_1^5 x \log_e x \, dx \qquad h = \frac{5-1}{4} = 1$$

$$\doteqdot \frac{1}{3}\left\{f(1) + f(5) + 2\,f(3) + 4\left(f(2) + f(4)\right)\right\}$$ (1)

$$\doteqdot \frac{1}{3}\left(0 + 8.047 + 2 \times 3.296 + 4 \times 1.386 + 4 \times 5.545\right)$$

$\doteqdot$ 14.121 sq units. (1)

(c)



(1)

$x = 4 - 2y$

$x^2 = 16 - 16y + 4y^2$ (1)

$$V = \pi \int_0^2 16 - 16y + 4y^2 \, dy$$

$$= \pi \left[16y - 8y^2 + \frac{4}{3}y^3\right]_0^2$$ (1)

(12)

$$= \frac{32\pi}{3} \quad \text{or} \quad 33.51 \text{ unit}^3$$ (1)

Q10

a) $\log_a 49000 = \log_a (49 \times 1000)$

$= 2\log_a 7 + 3\log_a 10$

$= 2 \times 2.807355 + 3 \times 3.321928$

$\doteqdot 15.58$ 1

a)

i) $\frac{d}{dx} \ln 5x + e^{-3x} - 2x = \frac{1}{x} - 3e^{-3x} - 2$ 1

ii) $\frac{d}{dx} \log_e (x^2 - 3x + 1) = \frac{2x - 3}{x^2 - 3x + 1}$ 1

iii) $\frac{d}{dx} e^{2x} \log_e x = 2e^{2x} \ln x + \frac{e^{2x}}{x}$ 2

b) i) $\int \frac{x}{x^2+1} \, dx = \frac{1}{2} \ln (x^2 + 1) + c$ 1

ii) $\int \frac{x-1}{x+4} \, dx = \int \left(\frac{x+4}{x+4} - \frac{5}{x+4} \right) dx$ 1

$= x - 5\ln(x+4) + c$ 1

c) $\int_1^2 (e^{2x+3} + 1) \, dx = \left[\frac{1}{2} e^{2x+3} + x \right]_1^2$ 1

$= 550.31658 - 75.20658$

$\doteqdot 475.11$ 1

d)

$\frac{d}{dx} e^{2x} = 2e^{2x}$

at $\begin{cases} x = 1 \\ y = e^2 \end{cases}$ $m = 2 \times e^{2 \times 1}$

$= 2e^2$ 1

$\frac{y - e^2}{x - 1} = 2e^2 \Rightarrow y - e^2 = 2e^2 x - 2e^2$

$y - 2e^2 x + e^2 = 0$

or $y - 14.8x + 7.4 = 0$ 1