KILLARA HIGH SCHOOL

YEAR 12 TERM 2 MATHEMATICS EXAMINATION

JUNE 2007

TOTAL TIME: 70 MINUTES

TOTAL MARKS: 64

Read each question thoroughly.

Attempt all questions.

Start each part on a new page.

Marks may be deducted for untidy or carelessly arranged working.

All necessary working must be shown for full marks.

Marks will be deducted if the answer is not written as directed by the question.

Non-Programmable Calculators may be used.

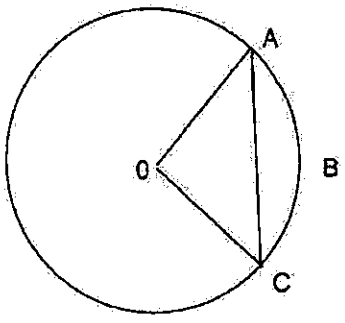
PART A TRIGONOMETRIC FUNCTIONS Total marks: 35

QUESTION 1:

a) (i) Convert 120° to radians. (1)

(ii) Convert $\frac{5\pi}{12}$ radians to degrees. (1)

b) OABC is a sector subtending an angle of 82° 20' at the centre 0 of a circle of radius 8 cm.



Write the answers to the following questions correct to two decimal places.

(i) Convert 82° 20' to a measure in radians. (1)

(ii) Find the length of the arc ABC. (1)

(iii) Find the area of the sector OABC. (1)

(iv) Find the area of the triangle OAC. (2)

(v) Find the area of the minor segment ABC. (1)

The sector OABC is rolled to form a cone so that the two sides 0A and 0C are joined together.

Find the radius and height of this cone and hence find the volume and the curved surface area of the cone, given that $V = \frac{1}{3}\pi r^2 h$ and $A = \pi r^2 + \pi r l$. (4)

QUESTION 2:

Draw the curve $y = 2 + 2\sin 2x;\ 0 \le x \le 2\pi$. (3)

State the period and amplitude of this function. (2)

QUESTION 3:

Differentiate the following : (i) $y = \cos(3x+1)$ (1)

(ii) $y = \sin(\cos x)$ (2)

(iii) $y = \log_e(\sin x)$ (2)

(iv) $y = \sin x \cos x$ (2)

QUESTION 4:

Integrate the following (i) $\int \sin 3x\, dx$ (1)

(ii) $\int \cos x . e^{\sin x}\, dx$ (2)

(iii) Express tanx in terms of sinx and cosx and hence find $\int \tan 3x\, dx$ (2)

(iv) $\int_0^{\frac{\pi}{8}} \sec^2(2x+\pi)dx$ (3)

QUESTION 5:

If $y = e^x \cos x$, find the value of k from which $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + ky = 0$ is true for all x. (3)

PART B **PROBABILITY** **Total marks: 16**

QUESTION 6:

The probability of relief from the flu with antibiotic X is 0.85 whilst with antibiotic Y is 0.7.

One flu sufferer A takes antibiotic X and another flu sufferer B takes antibiotic Y.

What is the probability that (i) Both are cured. (1)

(ii) At least one is cured. (1)

QUESTION 7:

The table below gives the classification of 60 students in a class by hair and eye colour.

	Eye Colour			
Hair Colour	**Blue**	**Green**	**Brown**	
Black	3	1	18	22
Brown	15	2	12	19
Blond	11	3	5	19
	19	6	35	60

Find the probability that two students chosen at random will have the same hair colour if it is known that they

both have green eyes? (4)

QUESTION 8:

In a sporting club of 30, 18 play Tennis and 20 play Netball.

If a member is chosen at random to be their captain, what is the probability that this member

(a) plays Tennis only (2)

(b) plays both Tennis and Netball (2)

QUESTION 9:

A factory produces light bulbs. When tested, if a light bulb does not last 1000 hrs it is called a failure. It is known that the probability of a failure is 0.002.

An inspector selects 3 light bulbs from their daily production for testing.

By drawing a probability tree neatly to illustrate, find the probability choosing

(2)

(a) 1 failure (b) 2 failures (c) no failures (d) at least 1 failure (4)

PART C APPLICATIONS OF CALCULUS TO THE PHYSICAL WORLD Total marks: 13

QUESTION 10:

Gas escapes a spherical bubble at the constant rate of $70\ mm^3\ s^{-1}$ so that the bubble always remains a sphere.

(a) Write down the value of $\frac{dV}{dt}$. (1)

(b) (i) If the volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$,

then, by the chain rule, $\frac{dV}{dt} = 4\pi r^2 . \frac{dr}{dt}$.

Hence find the rate of decrease of the bubble's radius when the radius is 10 mm. (2)

Write your answer to two decimal places.

(ii) If the surface area of the bubble is given by $S = 4\pi r^2$, then $\frac{dS}{dt} = 8\pi r . \frac{dr}{dt}$,

What is the rate of decrease of its surface area when the radius is 10 mm.

Write your answer to two decimal places. (2)

(iii) Having initially been given the value of $\frac{dV}{dt}$, find an equation for volume in terms of t. (2)

Hence calculate the volume of gas in the bubble after 5 seconds assuming (1)

initially r = 30 mm. Write your answer to 2 decimal places. (1)

QUESTION 11:

If it takes just 2 days for the number of insects in a garden to increase from one million to two million. Assume the growth rate is proportional to the population present, hence the model $\frac{dP}{dt} = kP$

can be used.

(a) Show that the value of k = 0.3466 (4dp) (2)

(b) What is the daily growth rate as a percentage. (1)

(c) Find the number of insects after 1 week.. (1)

YEAR 12 TERM 2 2007 SOLUTIONS.

Question 1

a) (i) $120 \times \frac{\pi}{180} = \frac{2\pi}{3}$ ✓

(ii) $\frac{5\pi}{12} \times \frac{180}{\pi} = 75°$ ✓/2

b) (i) $82°20' \times \frac{\pi}{180} \div 1.44$ ✓
[1.436987751..]

(ii) $l = r\theta$
$= 8 \times 1.43698....$ ✓
[11.49590201..] $\div 11.50$ cm

(iii) $A = \frac{1}{2} r^2 \theta$
$= \frac{1}{2} \times 64 \times 1.43....$
$\div 45.98 \text{ cm}^2$ ✓

(iv) $A = \frac{1}{2} ab \sin C$
$= \frac{1}{2} . 64 . \sin 82°20'$
$\div 31.71 \text{ cm}^2$

(v) $A = 45.98 - 31.71.$
$= 14.27 \text{ cm}^2$

CONE

Base: $C = 2\pi r = 11.4959...$

$\frac{11.4959...}{2\pi} = r$

$r = 1.8296...$ ✓
$\div 1.83$ cm ✓.

height

$h = \sqrt{64 - r^2}$
$= 7.87968...$ ✓
$\div 7.88$ cm ✓.

$V = \frac{1}{3} \pi r^2 h$
$= \frac{1}{3} \pi \times (1.83)^2 \times 7.87968.$
$= 27.62 \text{ cm}^3$

(1)

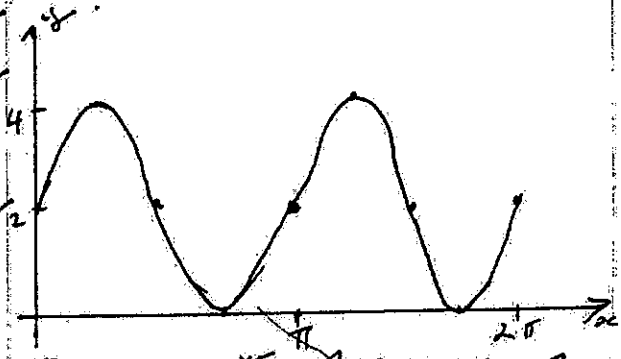
$SA = \pi r l$. [area of sector]
$= \pi \times 1.83 \times 8$ ($l = 8$cm) ✓
$= 45.99$ or
[45.98 Area of Sector] ✓/2

Using answers in Q1 (2dp) etc.

$r \div 1.83$ cm ✓
$h \div 7.79$ cm. ✓
$V = 27.32 \text{ cm}^3$. ✓
$SA = \pi \times 1.83 \times 8.$
$\div 45.99 \text{ cm}^2$ ✓ ✓

10

Question 2



PERIOD = π AMPLITUDE = 2 ✓✓/5

Question 3:

i) $\frac{dy}{dx} = -3 \sin(3x+1)$ ✓

(ii) $\frac{dy}{dx} = -\sin x . \cos(\cos x)$ ✓ ✓

(iii) $\frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x.$ ✓ ✓

(iv) $\frac{dy}{dx} = vu' + uv'$, $u = \sin x$, $u' = \cos x$, $v = \cos x$, $v' = -\sin x$

$= \cos x \cos x + \sin x . -\sin x$ ✓

$= \cos^2 x - \sin^2 x$ [$= \cos 2x$]. ✓/4

YEAR 12 TERM 2 2007 SOLUTIONS

Question 4:

i) $\int \sin 3x \, dx$

$= -\frac{1}{3}\cos 3x + C$ ✓

ii) $\int \cos x \cdot e^{\sin x} \, dx$

$= e^{\sin x} + C$ ✓ ✓

iii) $\tan x = \frac{\sin x}{\cos x}$.

$\int \tan 3x \, dx$

$= \int \frac{\sin 3x}{\cos 3x} \, dx$

$= -\frac{1}{3}\log(\cos 3x) + C$ ✓ ✓

iv) $\int_0^{\pi/8} \sec^2(2x+\pi) \, dx$.

$= \frac{1}{2}\left[\tan(2x+\pi)\right]_0^{\pi/8}$ ✓

$= \frac{1}{2}\left[\tan\frac{5\pi}{4} - \tan\pi\right]$. ✓

$= \frac{1}{2}\left[1 - 0\right]$ ✓

$= \frac{1}{2}$.

Question 5

$y = e^x \cos x$

$\frac{dy}{dx} = e^x \cos x + e^x \sin x$. ✓

$\frac{d^2y}{dx^2} = e^x \cos x - e^x \sin x - \sin x e^x - e^x \cos x$

$= -2e^x \sin x$. ✓

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = 0$.

LHS $=$

$-2e^x \sin x - 2(e^x \cos x - e^x \sin x)$

$+ k \cdot e^x \cos x$

$= -2e^x \sin x - 2e^x \cos x + 2e^x \sin x$

$+ ke^x \cos x$.

$= -2e^x \cos x + ke^x \cos x$.

$\therefore k = 2$ for ✓

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = 0$ true ✓

for all values of x.

Question 6

$P(x) = 0.85 \quad P(y) = 0.7$

A		B	
C (0.85)	C	0.7	CC
	$\bar{C}$	0.3	$C\bar{C}$
$\bar{C}$ (0.15)	C	0.7	$\bar{C}C$
	$\bar{C}$	0.3	$\bar{C}\bar{C}$

(i) $P(CC) = 0.85 \times 0.7 = 0.595$ ✓✓

(ii) $P(\text{at least one}) = 1 - P(\bar{C}\bar{C}) = 0.955$ ✓✓

YEAR 12 TEAM 2 2007 SOLUTIONS

Question 7

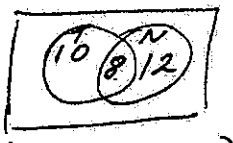
P(same hair colour)

$= P(BB) + P(B_1B_1) + P(B_2B_2)$

$= \left(\frac{1}{6} \times 0\right) + \left(\frac{2}{6} \times \frac{1}{5}\right) + \left(\frac{3}{6} \times \frac{2}{5}\right)$ ✓

$= \frac{2+6}{6 \times 5}$ ✓ $= \frac{8}{30} = \frac{4}{15}$ ✓

Question 8



(a) Tennis only. ✓ for (10)

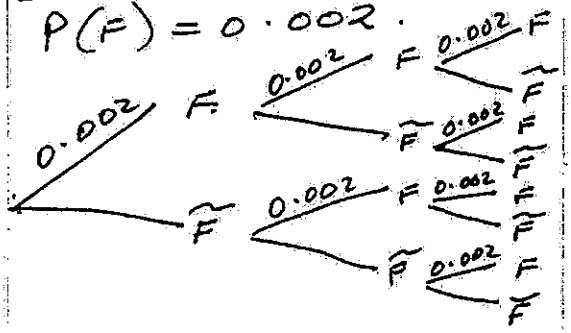
$= \frac{10}{30} = \frac{1}{3}$. ✓ for Ans

(b) $P(T.N) = \frac{8}{30}$ ✓

$= \frac{4}{15}$ ✓

Question 9

$P(F) = 0.002$.



$\{S\} = FFF$ ~~(2) P(1+F)~~

$FF\widetilde{F}$

$F\widetilde{F}F$

$F\widetilde{F}\widetilde{F}$

$\widetilde{F}FF$

$\widetilde{F}F\widetilde{F}$

$\widetilde{F}\widetilde{F}F$

$\widetilde{F}\widetilde{F}\widetilde{F}$

(i) $P(1+F) = P(F\widetilde{F}\widetilde{F}) + P(\widetilde{F}F\widetilde{F}) + P(\widetilde{F}\widetilde{F}F)$

$= 3(0.002 \times 0.998^2)$

$= 0.005976\ldots$ ✓

(ii) $P(2+F) = P(FF\widetilde{F}) + P(F\widetilde{F}F) + P(\widetilde{F}FF)$

$= 3 \times 0.002^2 \times 0.998$

$= 0.000011976$ ✓

(iii) $P(\widetilde{F}\widetilde{F}\widetilde{F}) = (0.998)^3$

$= 0.9940\ldots$ ✓

(iv) $P(\text{at least } 1 F) = 1 - P(\widetilde{F}\widetilde{F}\widetilde{F})$

$= 0.005988\ldots$ ✓

Question 10.

a) $\frac{dV}{dt} = -70\ mm^2 s^{-1}$. ✓

b) $\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$.

(i) $r = 10$

$-70 = 4 \cdot \pi \cdot 10^2 \cdot \frac{dr}{dt}$. ✓

$\frac{dr}{dt} = -0.056\ cm/s$ (3dp)

$= -0.06$. ✓

(ii) $\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$.

$r = 10$; $\frac{dr}{dt} = -0.0557$.

$\frac{dS}{dt} = 8 \cdot \pi \cdot 10 \cdot -0.0557$ ✓

$= -13.9889\ mm^2/s$ ✓

$= -14\ mm^2/s$.

Question 10 (cont)

(iii) $\frac{dV}{dt} = -70 \text{ mm}^3 \text{s}^{-1}$.

$t = 0$

$r = 30$.

$V = \frac{4}{3}\pi r^3$.

$\frac{dV}{dt} = -70$

$V(t) = -70t + C$. ✓

$t = 0$,

$V = \frac{4}{3}\pi r^3$.

$= \frac{4}{3} \cdot \pi \cdot 30^3$

$= 113097.3355...$

$= C$ ✓

$V = -70t + 113097.3355...$

$t = 5$

$V(5) = -350 + 113097.3355$ ✓

$= 112747.34 \text{ mm}^3 \text{s}^{-1}$ ✓

Question 11

a) Since $\frac{dP}{dt} = kP$.

$P = P_0 e^{kt}$.

$t = 0$, $P_0 = 1000000$.

$t = 2$, $P = 2000000$.

$\therefore 2000000 = 1000000 e^{2k}$. ✓

ie $e^{2k} = 2$

$2k = \log_e 2$.

$k = \frac{\log_e 2}{2}$ } ✓

$= 0.3466$ (4dp)

(b) Daily growth rate

$= 0.3466 = 34.66\%$ ✓

(c) when $t = 7$.

$P = 1000000 e^{0.3466 \times 7}$

$= 11,315,800.24$

$= 11,315,800$. ✓

after 1 week.