KILLARA HIGH SCHOOL

YEAR 12 TERM 2 MATHEMATICS EXAMINATION

JUNE 2009

TOTAL TIME: 70 MINUTES

TOTAL MARKS: 66

DIRECTIONS TO CANDIDATES

- **Attempt all questions**
- **All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.**
- **Board-approved Calculators may be used.**
- **Answer each question in a SEPARATE Writing Booklet.**
- **You may ask for extra Writing Booklets if you need them.**

QUESTION 1 **Total marks: 30**

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

(a) Evaluate $e^{\log_e(2x+3)}$ when $x = 3$, correct to 2 significant figures. (2)

(b) If $\log_m 2 = 0.623$ and $\log_m 3 = 0.847$, evaluate $\log_m 12$. (2)

(c) Find $f''(5)$ if $f(x) = \log_e 2x$. (2)

(d) Differentiate the following,.

(i) $\frac{d}{dx}(\log_e 6x)$ (1)

(ii) $\frac{d}{dx}(2x^3 - 4e^{3x})$ (1)

(iii) $\frac{d}{dx}\log_e\left(\frac{x+1}{x-1}\right)$ (2)

(iv) $\frac{d}{dx}e^x \log_e(x^2 - 2)$ (2)

(v) $\frac{d}{dx}\log\sqrt[3]{5x}$ (2)

(e) Find the Indefinite Integrate of the following with respect to x.

(i) $\int 4e^x dx$ (1)

(ii) $\int \frac{3}{2x} dx$ (1)

(iii) $\int \frac{6}{e^{2x}} dx$ (1)

(iv) $\int \frac{x}{3x^2 - 2} dx$ (1)

(f) (i) Using the Trapezoidal Rule with three function values, calculate the area between the curve $y = \log_e x$, the x axis and the lines x = 1 and x = 3. (2)

(ii) Find the exact volume of revolution for the region bounded by the curve $y = e^{2x}$, the x-axis and the lines x = 2 and x = 3 when it is rotated about the x-axis. (2)

(g) (i) Show there is a minimum when $f(x) = e^{2x} - 2e^x - 12x$. (2)

(ii) What is this minimum value, correct to 3 decimal places (2)

(h) The gradient of the tangent at any point on a curve is given by $\frac{dy}{dx} = e^{-2x}$.

(i) If the curve passes through the point (0 , 1) , find the equation of the curve. (2)

(ii) Find the point on the curve that has an abscissa of $x = -\frac{1}{2}\log_e 3$. (2)

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1a) $e^{\log_e(2x+3)} = e^{\log_e 9}$

$= 9$

b) $\log_m 12 = \log_m 3 \times \log_m 4$

$= \log_m 3 \times \log_m 2^2$

$= 0.847 \times 2 \times 0.623$

$= 1.06$

c) $f(x) = \log_e 2x$

$f'(x) = \frac{\cancel{2}}{\cancel{2}x} = \frac{1}{x} = x^{-1}$

$f''(x) = -1 \times x^{-2}$

$= -\frac{1}{x^2}$

$f''(5) = -\frac{1}{25}$

d i) $\frac{d}{dx}\log_e 6x = \frac{6}{6x}$

$= \frac{1}{x}$

ii) $\frac{d}{dx}(2x^3 - 4e^{3x}) = 6x^2 - 4e^{3x}\cdot 3$

$= 6x^2 - 12e^{3x}$

$= 6(x^2 - 2e^{3x})$

iii) $\frac{d}{dx}\log_e \frac{x+1}{x-1}$

$u = \frac{x+1}{x-1}$ $\quad u' = \frac{x-1-(x+1)}{(x-1)^2}$

d iii continue $u' = -\frac{2}{(x-1)^2}$

$\frac{d}{dx}\log_e u = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{\frac{(x+1)}{(x-1)}} \times \frac{-2}{(x-1)^2}$

$= \frac{\cancel{(x-1)}}{(x+1)} \times \frac{-2}{(x-1)^{\cancel{2}}}$

$= -\frac{2}{x^2-1}$

iv) $\frac{d}{dx} e^x \log_e(x^2-2) =$

$= e^x \cdot \log_e(x^2-2) + e^x \cdot \frac{2x}{x^2-2}$

$= e^x\left[\log_e(x^2-2) + \frac{2x}{x^2-2}\right]$

v) $\frac{d}{dx}\log_e \sqrt[3]{5x} = \frac{d}{dx}\log_e (5x)^{\frac{1}{3}}$

$= \frac{1}{(5x)^{1/3}} \times \frac{1}{3}(5x)^{-2/3} \times 5$

$= \frac{5}{3(5x)^{1/3}(5x)^{2/3}}$

$= \frac{\cancel{5}}{3(\cancel{5}x)}$

$= \frac{1}{3x}$

e i) $\int 4e^x dx = 4e^x + c$

ii) $\int \frac{3}{2x} dx = \frac{3}{2}\ln x + c$

iii) $\int \frac{6}{e^{2x}} dx = 6\int e^{-2x} dx$

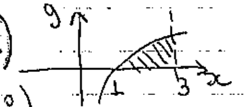
$= 6\frac{e^{-2x}}{-2} + c$

$= -3e^{-2x} + c$ or $-\frac{3}{e^{2x}} + c$

iv) $\int \frac{x}{3x^2-2} dx = \frac{1}{6}\int \frac{6x}{3x^2-2} dx$

$= \frac{1}{6}\ln(3x^2-2) + c$

f) i)



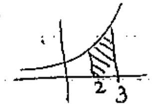
$h = \frac{3-1}{2} = 1$

$A = \frac{h}{2}[f(1) + 2f(2) + f(3)]$

$= \frac{1}{2}[\log_e 1 + 2\log_e 2 + \log_e 3]$

$A = 1.24$ unit^2

ii) $y = e^{2x}$



$V = \pi\int_2^3 y^2 dx$ $\quad y^2 = e^{4x}$

$= \pi\int_2^3 e^{4x} dx$

$= \left[\pi\frac{e^{4x}}{4}\right]_2^3$

$= \frac{\pi}{4}[e^{12} - e^8] = \frac{\pi e^8}{4}(e^4 - 1)$

g) i $y = e^{2x} - 2e^x - 12x$

$\frac{dy}{dx} = e^{2x}\cdot 2 - 2e^x - 12$

$\frac{dy}{dx} = 0$ when there is max/min

$e^{2x}\cdot 2 - 2e^x - 12 = 0$

$e^{2x} - e^x - 6 = 0$ $\quad$ let $u = e^x$

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u - 3 = 0 \qquad u + 2 = 0$

$u = 3 \qquad u = -2$

$e^x = 3 \qquad e^x = -2$

$\ln e^x = \ln 3 \qquad \log_e e^x = \log_e(-2)$

$x = \ln 3 \qquad$ no solution

Stationary point at $x = \ln 3$

Nature of stationary point:

$\frac{d^2y}{dx^2} = 2.e^{2x}.2 - 2e^x$

$= 4e^{2x} - 2e^x$

for $x = \ln 3$ $\quad y'' = 4e^{2\ln 3} - 2e^{\ln 3}$

$= 4e^{\ln 3^2} - 2e^{\ln 3}$

$= 4 \times 9 - 2 \times 3$

$= 30$

h) i) $\frac{dy}{dx} = e^{-2x}$ $\quad (0,1)$

h i continue

$\frac{dy}{dx} = e^{-2x}$

$\int e^{-2x} dx = \frac{e^{-2x}}{-2} + c$

$y = -\frac{e^{-2x}}{2} + c$

At $(0,1)$

$1 = -\frac{e^{0}}{2} + c$

$1 = -\frac{1}{2} + c$

$c = \frac{3}{2}$

$y = -\frac{e^{-2x}}{2} + \frac{3}{2}$ equation of the curve

ii) $x = \frac{1}{2}\log_e 3$

$y = -\frac{e^{-2\left(\frac{1}{2}\log_e 3\right)}}{2} + \frac{3}{2}$

$= -\frac{e^{-\log_e 3}}{2} + \frac{3}{2}$

$= -\frac{e^{\log_e 3^{-1}}}{2} + \frac{3}{2}$

$= -\frac{3^{-1}}{2} + \frac{3}{2}$

$= -\frac{1}{3 \times 2} + \frac{3}{2} \Rightarrow y = -\frac{1}{6} + \frac{3}{2} = \frac{-1+9}{6} = \frac{8}{6} = \frac{4}{3}$

$\therefore \left(\frac{1}{2}\log_e 3, \frac{4}{3}\right)$

0

0

0

0

0

0