Basic Arithmetic, Algebra, Surds & Equations, Geometry, Functions & Graphs, Trigonometry, Linear Functions (Straight line graphs), Quadratic polynomials, Calculus & its applications, And Locus.

Killara High School

2009

TERM 1 EXAMINATION **Thursday, 19th March**

Mathematics

General Instructions **Total marks – 120**

- Reading time – 5 minutes
- Working time – 3 hours
- Start each question in a new booklet
- Write using black or blue pen
- Board-approved calculators maybe used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Attempt Questions 1–10
- All questions are of equal value

Question 1 **(12 Marks)** Start a new booklet **Marks**

(a) Evaluate $\dfrac{2}{\pi - 2}$, correct to two significant figures. **2**

(b) Factorise: $64 - x^3$ **2**

(c) 2

(d) Solve: $3x^2 = -5x + 2$ **2**

(d) (d

(b) Solve $\dfrac{a-5}{3} - \dfrac{a+1}{4} = 5$

(c) Find integers a and b such that $\dfrac{2}{3-\sqrt{5}} = a + b\sqrt{5}$ 2

(d) Differentiate $x^5 + 3x^{-1}$ 2

(e) Find the values of x for which $|x+1| \le 5$ 2

Question 2 **(12 Marks)** Start a new booklet **Marks**

(a) Evaluate: $\sum_{r=3}^{5} 3-r^2$ **1**

(b) A and B have coordinates (3, 4) and (-4, 3) respectively. The point O is the origin and the point M is the midpoint of AB.

(i) Draw a sketch of these points, and mark M on your diagram. **1**

(ii) Show the coordinates of M are (-½, 3½) **1**

(iii) Show the equation of the line passing through A and parallel to OB is $3x + 4y - 25 = 0$. Draw this line on your diagram and label it L_1. **2**

(iv) The equation of the line which passes through O and M is $y = -7x$. Draw this line on your diagram and label it L_2.
C is the point of intersection of L_1 and L_2.
Show that C has coordinates (-1, 7). **2**

(v) Show BC is parallel to OA. **2**

(c) Solve for x: $2\cos 2\theta - 1 = 0$ for $0 \le \theta \le 360^\circ$ **3**

Question 3. **(12 Marks)** Start a new booklet **Marks**

(a) Differentiate with respect to x:

(i) $\sqrt{4-x}$ **2**

(ii) $\dfrac{5-x}{x^2-5}$ **2**

(iii) $x^2(x-2)^9$ **2**

(b) Find the perpendicular distance from the point (2, -3) and the line $3x - 2y + 5 = 0$. **2**

(c) Differentiate, from first principles, $f(x) = 5 - x^2$ **2**

(d) If $\sin\theta = -\dfrac{2}{7}$ and $\cos\theta$ is positive, find the exact value of $\tan\theta$ **2**

Question 4. **(12 Marks)** Start a new booklet **Marks**

(a) Find the following indefinite integrals :

(i) $\int \sqrt{3t-6}\, dt$ **2**

(ii) $\int \frac{x^4+3x^3-x^2-6}{x^2}\, dx$ **2**

(b) (i) Sketch the curve $y=\sqrt{9-x^2}$ and state the domain. **1**

(ii) Mark on your diagram, the area between the curve $y=\sqrt{9-x^2}$ and the x axis between $x=1$ and $x=3$. Use the trapezoidal rule with 2 strips to approximate $\int_1^3 \sqrt{9-x^2}\,dx$ **2**

(c) Use Simpson's rule with 5 function values to find an approximation to the area under the curve $y=\frac{1}{4-x}$ from $x=1$ to $x=3$. **3**

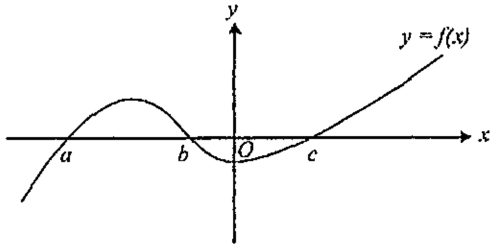
(d) Evaluate the following definite integral **2**

$$\int_1^2 \frac{dx}{(3x+2)^2}$$

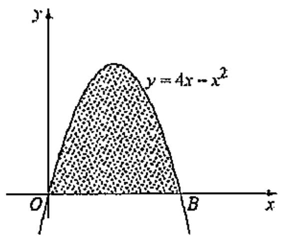
Question 5. **(12 Marks)** Start a new booklet **Marks**

(a) If $\frac{dy}{dt}=6t^2-2$, and y = 1 when t = 1, find the value of y when t = 2 **2**

(b) Part of the function f is shown below. Write an **expression** for the total area bounded by the curve $y=f(x)$ and the x axis from $x=a$ to $x=c$. **2**



(c)



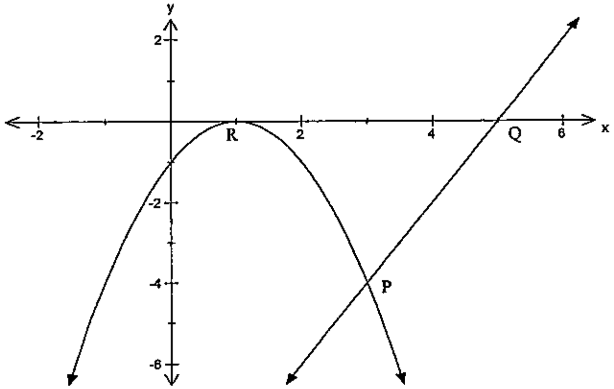
The diagram shows the graph of the function $y=4x-x^2$.

(i) Find the coordinates of point B. **1**

(ii) Find the area of the region bounded by the curve $y=4x-x^2$ and the x axis. **2**

(iii) Write down a pair of inequalities that specify the shaded region. **2**

(d) The diagram shows the curves $y=-(x-1)^2$ and $y=2x-10$.
$y=-(x-1)^2$ meets the x axis at R
$y=2x-10$ meets the x axis at Q
The curves intersect in the 4th Quadrant at P.



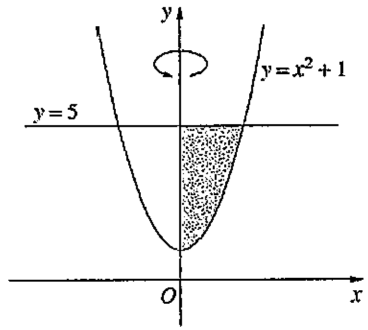
The coordinates of the point P are (3,-4)

Find the area PQR bounded by the curves $y=-(x-1)^2$, $y=2x-10$ and the x-axis. **3**

Question 6. **(12 Marks)** Start a new booklet **Marks**

(a) If $\int_0^a (4-2x)dx = 4$, find the value of a **3**

(b) **3**



In the diagram, the shaded region is bounded by the parabola $y=x^2+1$, the y-axis and the line $y=5$.
Find the volume of the solid formed when the shaded region is rotated about the y-axis.

(c) Solve: $2^x - \frac{10}{2^x} = -3$ **3**

(d) (i) For the function $y=f(x)$: $f'(x)=0$ at $x=0$ and $x=2$
$f'(x)<0$ for $0<x<2$ and $x>2$
$f'(x)>0$ for $x<0$

Draw a possible sketch of the graph of $y=f(x)$ **2**

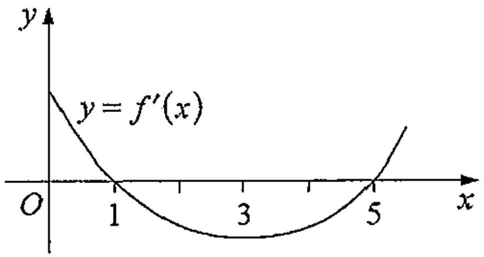
(ii) The coordinates of the turning points of the graph of $y=f(x)$ **1** are $(a, 1)$ and $\left(b, \frac{23}{27}\right)$. Find the values of a and b.

Question 7. (12 Marks) Start a new booklet **Marks**

(a) Prove: $(1-\cos\theta)(1+\sec\theta)=\sin\theta\tan\theta$ **2**

(b) Find the equation of the normal to the curve $y=\sqrt{x+2}$ at the point where $x = 7$. 4

(c) The diagram shows the gradient function of the curve $y = f(x)$ **2**



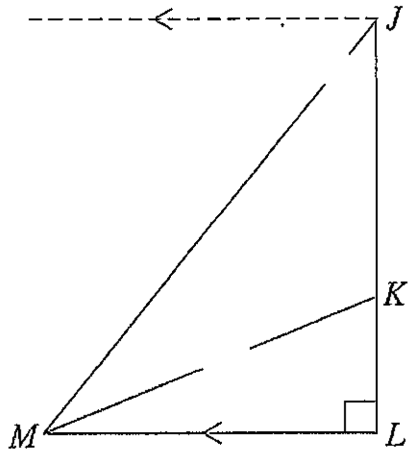
Sketch the curve $y = f(x)$ and indicate for what value of x the curve has a local minimum

(d) The angle of depression of J to M is 75°. The length of JK is 20m and the length of MK is 18m.

Copy the diagram below into your answer booklet and mark on your diagram all relevant information.

(i) Calculate the angle of elevation from M to K. Answer to the nearest degree. **3**

(ii) Calculate the area of triangle *JMK*. Answer correct to 1 decimal place. **1**



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Question 8 : (12 Marks) Start a new booklet **Marks**

(a) Find the exact value of tan (-150°) 1

(b) Simplify: $\dfrac{\sin\left(90^\circ-\theta\right)}{\cos\left(180^\circ+\theta\right)}$ 1

(c) Consider the graph of $y=x^3-mx^2+3x$

(i) Find $\dfrac{dy}{dx}$ 1

(ii) For what values of m will the graph have two distinct stationary points. 2

(iii) Sketch the graph for $m = 3$, by determining the stationary point and its nature. 3

(d)

A can of chick peas is in the shape of a closed cylinder with height h cm and radius r cm, as shown in the diagram.

(i) The volume of the can is 500 cm^3. 1
Find an expression for h in terms of r.

(ii) Show that the surface area, S cm^2, of the can is given by
$S=2\pi r^2+\dfrac{1000}{r}$. 1

(iii) If the area of metal used to make the can is to be minimized, find the radius of the can. 2

Question 9. **(12 Marks)** Start a new booklet **Marks**

(a) The first three terms of an arithmetic series are -2 + 3 + 8 +.......

(i) Find the 65th term. 2

(ii) Hence, or otherwise, find the sum of the first 65 terms of the series. 2

(b) The first term of a geometric series is $\dfrac{1}{4}$ and the fourth term is $\dfrac{27}{32}$. 2

(i) Find the common ratio

(ii) Find the sum to 6 terms.

(c) Nathan, a slightly obsessive toddler, decide to attempt to empty the sandpit at pre-school, one bucket load at a time along a path that runs from the sandpit to the back fence.. 6
After depositing each bucketful, he returns to the sandpit to collect the next deposit. The first deposit is emptied 20 cm from the sandpit, the second 35cm from the sandpit, the third 50cm from the sandpit. He empties each subsequent bucket 15cm from the previous one.

(i) How far is the 25th deposit from the sandpit?

(ii) How many loads can he deposit along the total length of the 18m path.

(iii) How far has Nathan walked in order to make all sand deposits and return to the sandpit in time for storytime?

Question 10. (12 Marks) Start a new booklet **Marks**

(a) The sum of the first n terms of a certain arithmetic series is given by $S_n = \dfrac{n(3n+1)}{2}$.

(i) Calculate S_1 and S_2. **1**

(ii) Find the first three terms of this series. **1**

(b) A ball is dropped from a height of 4 metres and bounces onto a hard surface. After each bounce, the ball rebounds to a height that is 75% of the previous maximum height.

(i) What is the height reached by the ball on the third bounce. **1**

(ii) What kind of sequence is formed by the successive heights. (explain your answer) **1**

(iii) What is the total distance travelled by the ball from the time it was first dropped until it eventually comes to rest on the floor. **2**

(c) Elvis, a fairy penguin enthusiast, bequeaths an amount of \$10000 in his will to support upkeep of the penguin enclosure at the zoo. The money is to be invested as a single amount, accruing interest at a rate of 6% per annum compounding annually. An amount of \$720 is to be paid on each anniversary of his death; the first \$720 is to be paid to the zoo on the first anniversary of his death

(i) Calculate the balance in the fund at the beginning of the second year. **1**

(ii) Let \$P be the amount of the investment remaining after n years, and after the nth donation has been made. Show that $P = 12000 - 2000 \times 1{\cdot}06^n$ **2**

(iii) On the tenth anniversary of Elvis' death, following the donation for that year, the trustee of his estate decides to increase the amount of each donation to \$900. Find the amount still remaining in the fund following the 12[th] anniversary of his death. **3**

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x > 0$

athematics Half Yearly Exam 2009.

stion 1

1.751938...

= 1.8 (2 sig figs)

(2) Award 2 for correct answer.
Award 1 for correct rounding from incorrect answer.

$64 - x^3 = 4^3 - x^3$

$= (4-x)(16 + 4x + x^2)$

(2) Award 2 for correct answer
Award 1 for $4^3 - x^3$.

$5 - \frac{2}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 5 - \frac{4-2\sqrt{3}}{1}$

$= 1 + 2\sqrt{3}$

(2) Award 2 for correct ans.
Award 1 for rationalizing the denominator

$3x^2 = -5x + 2$

$3x^2 + 5x - 2 = 0$

$(3x-1)(x+2)$

$x = \frac{1}{3}$ or $x = -2$

(2) Award 1 for each correct sol'n from correct working.

$|2x - 3| \leq 5$

$-5 \leq 2x - 3 \leq 5$

$-2 \leq 2x \leq 8$

$-1 \leq x \leq 4$

(2) Award 1 for each correct sol'n from correct working.
Award 0 for evidence of $2x + 3 \leq 5$

$2x - 3 \leq 5$ or $2x - 3 \geq -5$

$2x \leq 8$ $\quad 2x \geq -2$

$x \leq 4$ $\quad x \geq -1$

$\therefore -1 \leq x \leq 4$.

$110\% = \$5.94$

$10\% = \$5.94 \div 11$

$100\% = \$5.94 \div 11 \times 10$

$= \$5.40$

$\therefore$ Pre GST price $= \$5.40$

(2) Award 2 for correct ans.
Award 1 for $\frac{1}{11}$ of 5.94 or equivalent
Award 0 for 10% of 5.94.

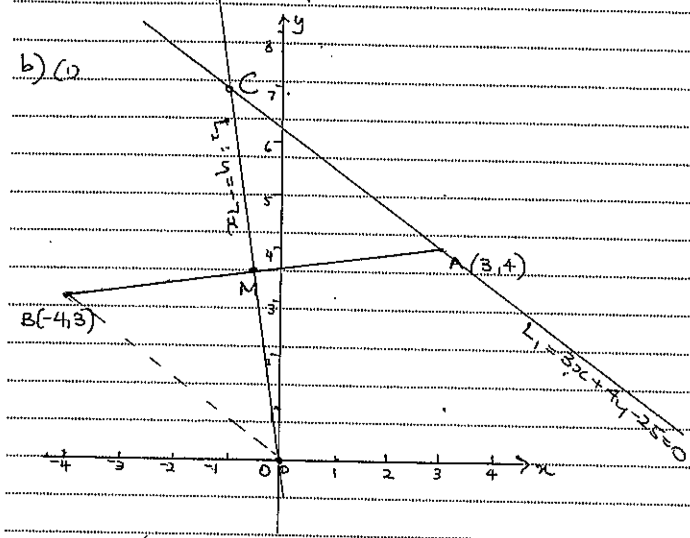
Question 2.

a) $\sum_{r=3}^{5} 3 - r^2 = (3-3^2) + (3-4^2) + (3-5^2)$

$= -41$

(1) Award 1 for correct sol'n.

b) (i)



(i) Award 0 for no ruler and/or inconsistent scale on either axis, if this OK,
Award 1 for evidence of A, B, M correctly located

(ii) $M: \left(\frac{-4+3}{2}, \frac{3+4}{2}\right) = \left(-\frac{1}{2}, 3\frac{1}{2}\right)$

(1) Award 1 for correct substitution into formula.

(iii) $m_{OB} = \frac{3-0}{-4-0}$ $\quad$ A (3,4)

$= -\frac{3}{4}$.

$y - 4 = -\frac{3}{4}(x - 3)$

$4y - 16 = -3x + 9$.

$3x + 4y - 25 = 0$

(2) Award 2 for correct working leading to result ALL WORKING MUST BE SHOWN.
Award 1 for correct calculation of m_{OB}.

(iv) $L_1: 3x + 4y - 25 = 0$ — (1)

$L_2: y = -7x$ — (2)

(2)→(1) $3x + 4 \times -7x - 25 = 0$

$-25x = 25$

$x = -1$

$y = -7x$

$y = -7 \times -1$

$y = 7$ $\therefore$ C $(-1, 7)$

(2) Award 2 for correct sol'n for x and y with FULL WORKING.

(v) $m_{BC} = \frac{7-3}{-1+4} = \frac{4}{3}$ $\quad m_{OA} = \frac{4-0}{3-0} = \frac{4}{3}$

B $(-4, 3)$

C $(-1, 7)$

$m_{BC} = m_{OA}$.

$\therefore$ lines are parallel.

(2) Award 2 for correct cal'n of BOTH gradients & concluding statement
Award 1 for calc. of both gradients

c) $2\cos 2\theta - 1 = 0$ for $0 \le \theta \le 360°$ (3)

$\cos 2\theta = \frac{1}{2}$ for $0 \le 2\theta \le 720$

$2\theta = 60°, 300°, 420°, 660°$

$\theta = 30°, 150°, 210°, 330°$

Award 3 for 4 correct sol'ns

Award 2 for $\theta = 30°, 150°$

Award 1 for $2\theta = 60°, 300°$

Question 3

a (i) $\frac{d}{dx}(4-x)^{\frac{1}{2}} = \frac{1}{2}(4-x)^{-\frac{1}{2}} \cdot -1$ (2)

$= \frac{-1}{2\sqrt{4-x}}$

Award 2 for correct use of ~~chain rule~~.

Award 1 for $\cancel{y = (4-x)^{\frac{1}{2}}}$

(ii) $\frac{d}{dx}\left(\frac{5-x}{x^2-5}\right) = \frac{(x^2-5).-1-(5-x).2x}{(x^2-5)^2}$ (2)

$= \frac{-x^2+5-10x+2x^2}{(x^2-5)^2}$ *

$= \frac{x^2-10x+5}{(x^2-5)^2}$

Award 2 for correct use of quotient rule to *.

Award 1 for evidence of u, v, u', v' (must be correct)

(iii) $\frac{d}{dx}\left(x^2(x-2)^9\right) = x^2 \cdot 9(x-2)^8 + (x-2)^9 \cdot 2x$ (2)

$= 9x^2(x-2)^8 + 2x(x-2)^9$

$= x(x-2)^8(9x+2(x-2))$

$= x(x-2)^8(11x-4)$

Award 2 for correct use of product rule to this point.

Award 1 for correct u, v, u', v'

Award 0 for incorrect formula.

b) $(2,-3)$ $3x-2y+5=0$ (2)

$d = \frac{|3\times 2 - 2\times -3 + 5|}{\sqrt{3^2+(-2)^2}}$

$= \frac{|6+6+5|}{\sqrt{13}}$

$= \frac{17}{\sqrt{13}}$ units or $\frac{17\sqrt{13}}{13}$ units

Award 2 for correct answer

Award 1 for correct substitution into formula

c) $5-x^2 = f(x)$ (2)

$f(x+h) = 5-(x+h)^2$

$= 5-(x^2+2xh+h^2)$

$= 5-x^2-2xh-h^2$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{(\cancel{5}-\cancel{x^2}-2xh-h^2)-(\cancel{5}-\cancel{x^2})}{h}$

$= \lim_{h\to 0} \frac{-2xh-h^2}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{\cancel{h}}$

$= -2x$

Award 2 for correct sol'n from full working

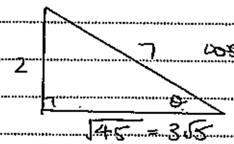
Award 1 for correct substitution

Award 0 for derivative only.

d) $\sin\theta = -\frac{2}{7}$ (2)

$\cos\theta$ +ve $\therefore$ Quadrant 4.

$\therefore \tan\theta < 0$



~~$\cos\theta = \frac{3\sqrt{5}}{7}$~~

$\tan\theta = \frac{-2}{3\sqrt{5}}$ or $\frac{-2\sqrt{5}}{15}$

Award 2 for correct sol'n.

Award 1 for correct $\tan\theta < 0$ or $\tan\theta = \frac{2}{3\sqrt{5}}$

QUESTION 4

a) $\int (3t-6)^{\frac{1}{2}} dt = \frac{2}{3}\frac{(3t-6)^{\frac{3}{2}}}{3} + C$ (2)

$= \frac{2}{9}(3t-6)^{\frac{3}{2}} + C$

Award 1 for integral.

Award 1 for +C

b) $\int \frac{x^4+3x^3-x^2-6}{x^2} dx$ (2)

$= \int x^2 + 3x - 1 - 6x^{-2} dx$. *

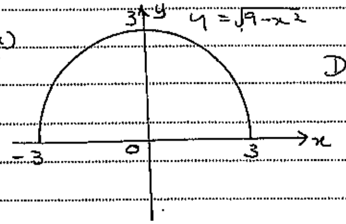
$= \frac{x^3}{3} + \frac{3x^2}{2} - x - \frac{6x^{-1}}{-1} + C$

$= \frac{x^3}{3} + \frac{3x^2}{2} - x + \frac{6}{x} + C$

Award 2 for correct sol'n. (ignore +c)

Award 1 for *

b) (i)

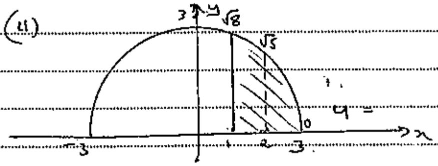


Domain: $-3 \le x \le 3$

(1) Award 1 for sketch + domain.

Award 0 for one or other.

(ii)



$\int_1^3 \sqrt{9-x^2}\,dx \doteqdot \frac{1}{2}\left[\sqrt{8}+2\sqrt{5}+0\right]$

$= \frac{1}{2}\left[\sqrt{8}+2\sqrt{5}\right]$

$= \frac{1}{2}\left[2\sqrt{2}+2\sqrt{5}\right]$

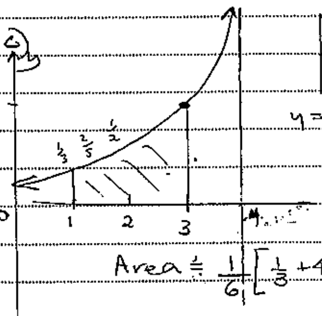
$= \sqrt{2}+\sqrt{5}$

(2) Award 2 for correct sol'n ~~to~~

Award 1 for correct sub. into trap. rule.

Award 0 for Simpson's rule.

c)



$h = \frac{1}{2}$

x	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
$f(x)$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{2}{3}$	1

$y = \frac{1}{4-x}$

(3) Award 3 for correct sol'n.

Award 2 for correct substit^n into Simpson's rule

Area $\doteqdot \frac{1}{6}\left[\frac{1}{3}+4\times\frac{2}{5}+\frac{1}{2}\right]+\frac{1}{6}\left[\frac{1}{2}+4\times\frac{2}{3}+1\right]$

$= \frac{1}{6}\left[\frac{1}{3}+4\times\frac{2}{5}+2\times\frac{1}{2}+4\times\frac{2}{3}+1\right]$

$= \frac{1}{6}\left[\frac{33}{5}\right]$

$= \frac{11}{10}$ units^2.

$\therefore$ Area $\doteqdot \frac{11}{10}$ units^2

Award 1 for correct use of Simpsons rule with 1 strip.

Award 1 for 5 correct y values and correct h.

d) $\int_1^2 (3x+2)^{-2}\,dx = \left[\frac{(3x+2)^{-1}}{-1\times 3}\right]_1^2$

$= \left[\frac{-1}{3(3x+2)}\right]_1^2$

$= \left(\frac{-1}{24}\right)-\left(\frac{-1}{15}\right)$

$= \frac{1}{40}$

(2) Award 2 for correct sol'n.

Award 1 for correct substitution ~~into correct integral~~

Question 5.

a) $\frac{dy}{dt} = 6t^2-2$

$y = \frac{6t^3}{3} - 2t + c.$ $\quad y=1, t=1$

$1 = 2\times 1^3 - 2\times 1 + c$

$c = 1$

$\therefore y = 2t^3-2t+1$

When $t=2$, $y = 2.2^3 - 2.2 + 1$

$y = 13.$

(2) Award 2 for correct sol^n.

Award 1 for $y = f(t)$ [incl. $c=1$]

b) $A = \int_a^b f(x)\,dx + \left|\int_b^c f(x)\,dx\right|$

(2) Award 1 for each part.

c) (i) $0 = 4x - x^2$

$= x(4-x)$ $\therefore x=0$ or $x=4$.

Co-ordinates of B are (4,0)

(1) Award 1 for CO-ORDINATES of B.

(ii) $A = \int_0^4 4x - x^2\,dx.$

$= \left[\frac{4x^2}{2}-\frac{x^3}{3}\right]_0^4$

$= 2.4^2 - \frac{4^3}{3} - 0$

$= 10\frac{2}{3}$ units^2

(2) Award 2 for correct answer.

Award 1 for correct substitution into integral.

(iii) $y \ge 0$, $y \le 4x - x^2$

(2) Award 1 for each correct inequality.

d) Area $= \left|\int_1^3 -(x-1)^2\,dx\right| + \left|\int_3^5 2x-10\,dx\right|$

$= \left|\left[\frac{-(x-1)^3}{3}\right]_1^3\right| + \left|\left[x^2-10x\right]_3^5\right|$

$= \left|-\frac{(3-1)^3}{3} - -\frac{(1-1)^3}{3}\right| + \left|(5^2-10\times 5)-(3^2-10\times 3)\right|$

$= \left|-\frac{8}{3}\right| + \left|-25 - -21\right|$

$= \frac{8}{3} + 4 \quad = \frac{20}{3}$ units^2

(3) Award 3 for correct

Award 2 correct statement + 1 correct evaluation

Award 1 for correct 1st statement.

Question 6.

a) $\int_0^a 4-2x\,dx = 4$ — ③ Award 3 for correct answer

$[4x - x^2]_0^a = 4$ — Award 2 for $4a - a^2 = 4$

$4a - a^2 - 0 = 4$

$a^2 - 4a + 4 = 0$

$(a-2)^2 = 0$ — Award 1 for $4a - a^2$

$a = 2$.

b) $V = \pi \int_1^5 y - 1\,dy$ — ③ Award 3 for correct sol'n

$= \pi \left[\frac{y^2}{2} - y\right]_1^5$

$= \pi \left[\left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right)\right]$ — Award 2 for correct subst. into integral

$= 8\pi$ units^3

Award 1 for $\pi\int_1^5 y - 1\,dy$

c) $2^x - \frac{10}{2^x} = -3$.

$(2^x)^2 + 3.2^x - 10 = 0$ — ③ 3 for correct sol'n WITH WORKING.

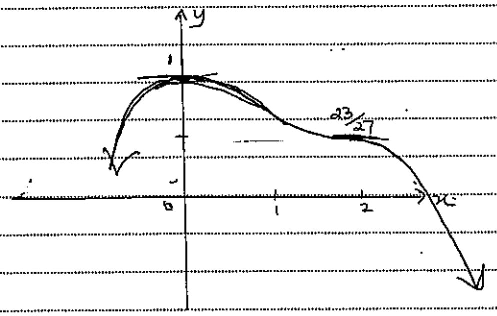
$(2^x + 5)(2^x - 2) = 0$

$2^x = -5$ $2^x = 2^1$ — 2 for $(\ )(\ ) = 0$

no sol'n. $x = 1$ $\therefore x = 1$

1 for quadratic expression.

d) (i)



② Award 1 for each stationary point and 1 for correct graph.

(ii) $a = 0$, $b = 2$ — ① Award 1 for correct answer

Question 1.

a) LHS $= (1 - \cos\theta)(1 + \sec\theta)$ — ② Award 2 for correct working to final result.

$= 1 + \sec\theta - \cos\theta - \cos\theta\sec\theta$.

$= 1 + \sec\theta - \cos\theta - 1$

$= \sec\theta - \cos\theta$ — Award 1 to *

$= \frac{1}{\cos\theta} - \cos\theta$ *

$= \frac{1 - \cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$= \sin\theta \cdot \frac{\sin\theta}{\cos\theta}$

$= \sin\theta\tan\theta$

$=$ RHS

b) $y = \sqrt{x+2}$.   When $x = 7$, $y = \sqrt{7+2} = 3$ — Award 1 for point

$y' = \frac{1}{2}(x+2)^{-\frac{1}{2}}$ $\therefore$ Point $(7, 3)$ — Award 1 for m_{tang}

$y' = \frac{1}{2\sqrt{x+2}}$

at $x = 7$, $y' = \frac{1}{2\sqrt{9}} = \frac{1}{6}$. $\therefore m_{tang} = \frac{1}{6}$ $m_{norm} = -6$ — Award 1 m_{norm}

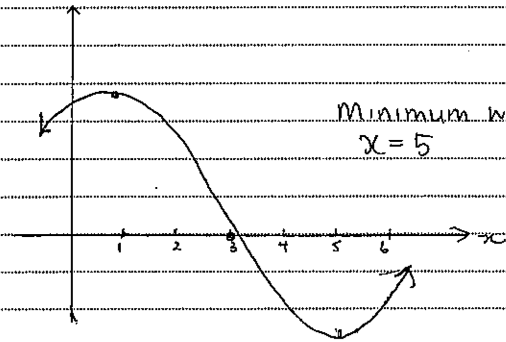
Award 1 for eq'n.

$y - 3 = -6(x - 7)$

$y - 3 = -6x + 42$

$6x + y - 45 = 0$

c)

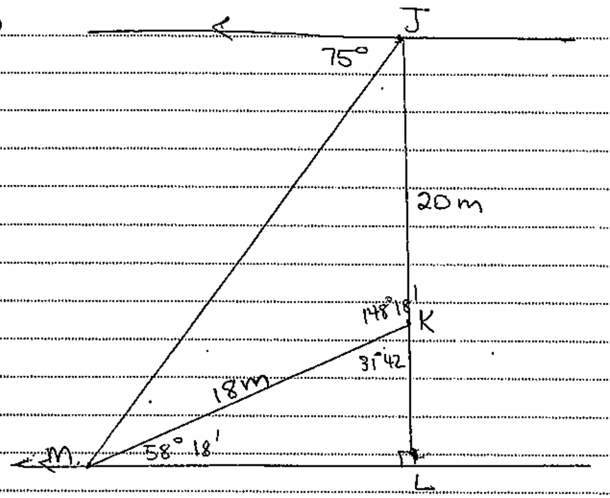


Minimum when $x = 5$

Award 1 for curve

Award 1 for minimum.

d) (i)



(i) $\angle JML = 75°$. (3)

$\angle MJL = 15°$.

In ΔJMK, $\dfrac{\sin 15°}{18} = \dfrac{\sin \angle JMK}{20}$

$\sin \angle JMK = \dfrac{20 \sin 15°}{18}$

$\sin \angle JMK = 0 \cdot 2875\ldots$

$\angle JMK = 16°42'$

$\angle KML = 75° - 16°42'$

$= 58°18'$ or $58 \cdot 3°$.

$\therefore$ angle of elevation is 58° (to nearest degree)

Award 3 for correct answer from correct working

Award 2 for $\angle JMK = 16°42'$

Award 1 for correct subst. into sine rule or equivalent

(ii) $\angle JKM = 148°18'$ (1)

Area $= \frac{1}{2} \times 18 \times 20 \times \sin 148°18'$

$= 94 \cdot 58 m^2$.

Award 2 for correct substitution into sine rule.

Question 8.

a) $\tan(-150°) = \tan(-150 + 360°)$ (1)

$= \tan 210°$

$= \dfrac{1}{\sqrt{3}}$

Award 1 for correct exact sol'n

b) $\dfrac{\sin(90° - \theta)}{\cos(180° + \theta)} = \dfrac{\cos\theta}{-\cos\theta}$ (1)

$= -1$

Award 1 for correct answer

c) $y = x^3 - mx^2 + 3x$.

(i) $\dfrac{dy}{dx} = 3x^2 - 2mx + 3$ (1)

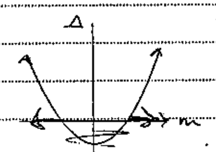
Award 1 for correct answer.

(ii) $\Delta = (-2m)^2 - 4 \times 3 \times 3$ (2)

$= 4m^2 - 36$

$4m^2 - 36 > 0$

$4(m-3)(m+3) > 0$



2 distinct stationary points if $m < -3$ 'or' $m > 3$

Award 2 for correct answer.

Award 1 for $4m^2 - 36 > 0$

(iii) If $m = 3$, $x^3 - 3x^2 + 3x = y$. (3)

$y = x(x^2 - 3x + 3)$

x int is $x = 0$

y int is $y = 0$.

$y = x^3 - 3x^2 + 3x$.

$y' = 3x^2 - 6x + 3$.

$= 3(x^2 - 2x + 1)$

$= 3(x-1)^2$.

$y' = 0 \Rightarrow x = 1$

$\therefore$ Stationary point when $x = 1$, $y = 1$ $(1,1)$.

$y'' = 3.2(x-1)$

at $(1,1)$ $y'' = 0$

$\therefore$ Point of inflexion at $(1,1)$

Check

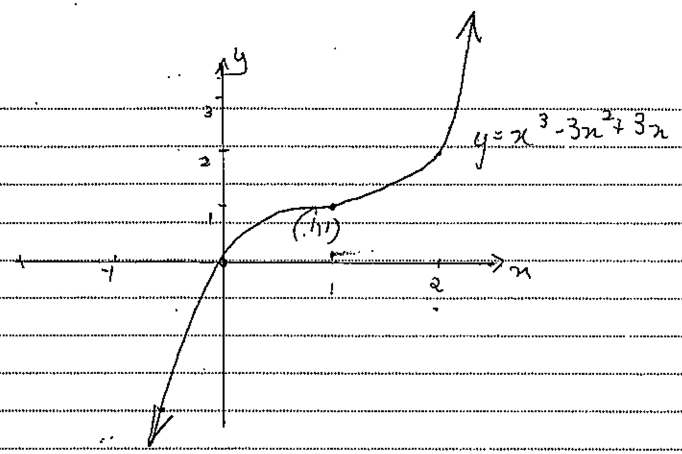
x	1^-	1	1^+
y''	$6(\frac{1}{2}-1)$	0	$6(\frac{3}{2}-1)$

change in concavity $\therefore$ Inflexion point at $(1,1)$

Award 1 for curve

Award 1 for stationary point

Award 1 for inflexion (must show Δ in concavity)



d) (i) $\pi r^2 h = 500 \qquad SA = 2\pi r^2 + 2\pi r h$

$\therefore h = \dfrac{500}{\pi r^2} \longrightarrow = 2\pi r^2 + 2\pi r \times \dfrac{500}{\pi r^2}$

(1) Award 1 for $h = \dfrac{500}{\pi r^2}$

(ii) $SA = 2\pi r^2 + \dfrac{1000}{r}$

(1) Award 1 for all working

(iii) $S = 2\pi r^2 + 1000r^{-1}$

$\dfrac{dS}{dr} = 4\pi r - 1000r^{-2}$

$\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$

$\dfrac{dS}{dr} = 0 \Rightarrow 4\pi r - \dfrac{1000}{r^2} = 0$

$4\pi r^3 = 1000$

$r^3 = \dfrac{250}{\pi}$

$r = \sqrt[3]{\dfrac{250}{\pi}}$ ← Award (1)

$\dfrac{d^2S}{dr^2} = 4\pi + \dfrac{1000}{r^3}$

at $r = \sqrt[3]{\dfrac{250}{\pi}}$, $\dfrac{d^2S}{dr^2} = 4\pi + \dfrac{1000}{\left(\sqrt[3]{\dfrac{250}{\pi}}\right)^3}$

$\therefore > 0$ ← Award (1)

$\therefore$ MINIMUM when radius is $\sqrt[3]{\dfrac{250}{\pi}}$ units

Question 9.

a) (i) $-2, 3, 8, \ldots$ AP $a = -2$, $d = 5$.

Award 2 for correct sol^n

(i) $T_{65} = -2 + 64 \times 5 = 318$ (2)

Award 1 for correct sub^n into $T_n = a + (n-1)d$

(ii) $L = 318$, $a = -2$, $n = 65$ (2)

$S_n = \dfrac{n}{2}(a + L)$

$= \dfrac{65}{2}[-2 + 318]$

$S_{65} = 10270$

Award 2 for correct answer.

Award 1 for correct sub into $S_n = \dfrac{n}{2}(a+L)$

b) (i) $a = \frac{1}{4}$

$T_4 = ar^3 = \dfrac{27}{32}$

$\frac{1}{4} r^3 = \dfrac{27}{32}$

$r^3 = \dfrac{27}{32} \times 4$

$r^3 = \dfrac{27}{8}$

$r = \dfrac{3}{2}$ (1)

(1) Award 1 for $r = \dfrac{3}{2}$

(ii) $S_n = \dfrac{\frac{1}{4}\left(\left(\frac{3}{2}\right)^6 - 1\right)}{\frac{1}{2}} = \dfrac{\frac{1}{4}\left(\frac{729}{64} - 1\right)}{\frac{1}{2}}$

$= \dfrac{665}{128}$

(1) Award 1 for $S_6 = \dfrac{665}{128}$

c) $20 + 35 + 50 + \ldots$

AP $a = 20$, $d = 15$

$T_{25} = 20 + 24 \times 15 = 380$ (2)

$\therefore 25^{th}$ deposit $= 380$m from sandpit

Award 2 for correct answer.

Award 1 for correct sub into AP

(ii) $1800 = 20 + (n-1)15$

$1780 = (n-1)15$

$118.6 = (n-1)$

$119.6\ldots = n$ $\therefore$ Nathan can deposit 119 loads

Award 2 for correct answer of 119.

Award 1 for $119.6\ldots$ or 120

(iii) Nathan walks there and back for each load

$\therefore 2 \times \dfrac{119}{2}(2 \times 20 + 118 \times 15) = 215\ 390$ cm.

$= 2.1539$ km (2)

Award 2 for correct answer

Award 1 for not $\times 2$

Question 10

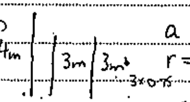
a) $S_n = \frac{n(3n+1)}{2}$ — Award 1 for correct answer

(i) $S_1 = \frac{1(4)}{2} = 2$. $S_2 = \frac{2(7)}{2} = 7$ ①

(ii) $S_3 = \frac{3.10}{2} = 15$. $T_1 = 2$ $T_2 = 7-2 = 5$

$T_3 = 15-7 = 8$ — Award 1 for

∴ First 3 terms are 2, 5, 8 ① — correct answer

b)(i) 

$a = 4$ $r = 0.75$ or 3×0.75

$4 \times 0.75^3 = \frac{27}{16}$ m $= 1.6875$ m ① — 1 for correct answer.

(ii) GP because each term is form by preceding term × by 0.75 ① — must have reason.

(iii) $d = 4 + 2 \times \frac{a}{1-r} = 4 + 2 \times \frac{3}{1-0.75}$

$= 4 + 24$

$= 28$m. ② — Award 2 for 28m.

Award 1 for 16m (fgot to × by 2).

c)

(i) Amount in fund at end of 1^{ST} year is (= amount at beginning of 2^{nd} year)

$\$10000 \times 1.06 - \$720 = \$9880$ ① — Award 1 for correct answer.

(ii) Amount in fund at end of 2^{nd} year is

$(10000 \times 1.06 - 720)1.06 - 720$

$= 10000 \times 1.06^2 - 720 \times 1.06 - 720$

Amount in fund at end of 3^{rd} yr

$= (10000 \times 1.06^2 - 720 \times 1.06 - 720) \cdot 1.06 - 720$

$= 10000 \times 1.06^3 - 720 \times 1.06^2 - 720 \times 1.06 - 720$

Amount in fund after n years (P) ② — Award 2 for correct answer with ALL working.

$P = 10000 \times 1.06^n - 720(1 + 1.06 + 1.06^2 + \dots + 1.06^{n-1})$

GP $a = 1$, $r = 1.06$, $S_n = \frac{1.06^n - 1}{0.06}$

$= 10000 \times 1.06^n - 720\left(\frac{1.06^n - 1}{0.06}\right)$ — Award 1 for $10000 \times 1.06^n - 720\frac{(1.06^n - 1)}{0.06}$.

$= 10000 \times 1.06^n - 12000(1.06^n) + 12000$

$P = 12000 - 2000 \times 1.06^n$

(iii) from (ii) $P = 12000 - 2000 \times 1.06^n$. ③

∴ Balance at end of 10^{th} year (after payment)

$P = 12000 - 2000 \times 1.06^{10}$

$P = \$8418.30$. — Award 1 for correct answer.

Balance at end of 11^{th} year

$= (8418.30) \times 1.06 - \$900$

Balance at end of 12^{th} year

$[(8418.30) \times 1.06 - \$900] \times 1.06 - \$900$ — Award 1 for this or equivalent

$= \$8418.30 \times 1.06^2 - 900 \times 1.06 - 900$

$= \$8418.30 \times 1.06^2 - 900(2.06)$

$= \$7604.80$.

∴ Amount still remaining is \$7604.80 — Award 1 for correct answer.