KILLARA HIGH SCHOOL

YEAR 11 – 2010

HSC MATHEMATICS TASK 1

Time allowed: 70 minutes

Total marks: 45

- **NOT ALL** questions are of equal value.
- Start each question on a new page.
- Show all necessary working.

Question 1: (14 marks) **Marks**

a) For what values of k will $3x^2 - kx + 12$ be positive definite **2**

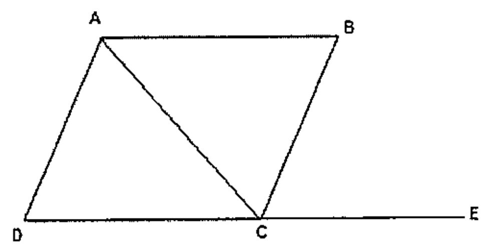
b) Write $8x^2 + 6x + 3$ in the form of $A(2x+1)^2 + B(2x+1) + C$ **3**

c) In the diagram ABCD is a rhombus where angle DAC = 54° and DC is produced to E.

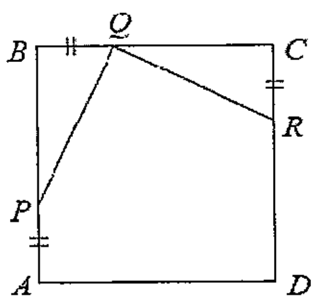
i) What is the value of angle DAB? **1**

ii) What is the value of angle BCE? **2**

Give reasons for your answers.



d) In the diagram, ABCD is a square. The points P, Q and R lie on AB, BC and CD respectively, such that AP = BQ = CR. Prove that:



i) triangles PBQ and QCR are congruent. **4**

ii) $\angle$ PQR is a right angle. **2**

Question 2: (31 marks) **Marks**

Differentiate with respect to x (note: marks will be deducted for incorrect notations)

i) $5x^3 + 6\sqrt{x} - 9$ **2**

ii) $\left(x^2 + \frac{1}{x}\right)(3x^2 - 7x)$ **2**

iii) $\dfrac{3x^4 - 8}{5 - 2x}$ (leave your answer in the simplest form) **3**

iv) $f(x) = (4x^3 - 7)^5$ **2**

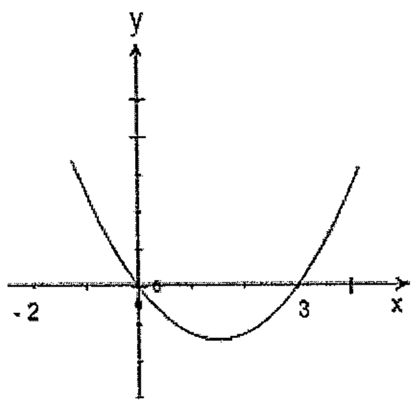
a) Using first principals, differentiate the function $f(x) = x^2 - 3x$ **3**

b) Find the equation of the tangent to the curve $y = 3x^2$ at the point (-1, 3). **2**

c) The diagram shows the graph of the gradient function of the curve y = f(x)..

i) For what values of x is the graph of f(x) increasing? **1**

ii) For what value of x does f(x) have a local maximum? Justify your answer. **2**



Question 2 continue next page

d) A section of wire fencing 18 metres long is erected into the form of a rectangle

Containing 3 sides only, the fourth side is left open.

If the length of the rectangle is x metres,

i) Show that an expression for the area of the rectangle is $y = 9x - \frac{x^2}{2}$ **2**

ii) Find the maximum possible area. **3**

e) The function $f(x) = 6x^2 - x^3$ and is defined in the domain $-1 \leq x \leq 7$

i) Find the coordinates of any stationary points, and determine their nature. **3**

ii) Determine the coordinates of the point/s of inflexion. **2**

iii) Draw a neat sketch of the function, showing all the above features on your sketch. **3**

iv) In the domain $-1 \leq x \leq 7$, find the greatest value of $f(x) = 6x^2 - x^3$ **1**

Q1 (a) For $3x^2 - kx + 12$ to be positive definite

$\Delta < 0$

$\therefore k^2 - 4 \times 3 \times 12 < 0$

$k^2 < 144$

$-12 < k < 12$

(b) $8x^2 + 6x + 3 \equiv A(2x+1)^2 + B(2x+1) + C$

L.H.S $= 4Ax^2 + 4Ax + A + 2Bx + B + C$

$= 4Ax^2 + (4A + 2B)x + A + B + C$

equate the coefs. of L.HS & R.H.S

$\Rightarrow 4A = 8 \qquad 4A + 2B = 6$

$A = 2, \qquad 2B = 6 - 4 \times 2$

$= -2$

$B = -1$

$C + A + B = 3$

$C = 3 - 2 + 1$

$= 2$

$\therefore 8x^2 + 6x + 3 \equiv 2(2x+1)^2 - (2x+1) + 2$

(c) i)

$\angle DAB = 2 \times 54$

$= 108°$ (Diagonals of rhombus bisect the angles)

ii)

$\angle DAB = \angle DCB$ (opp. angles of rhombus are equal)

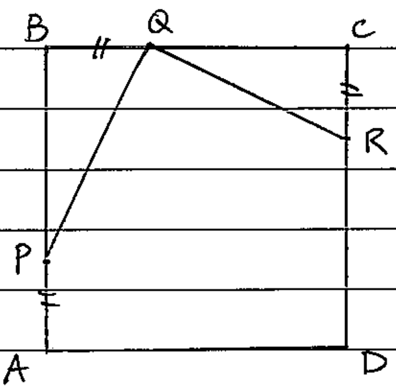
$= 108°$

$\angle BCE = 180° - \angle DCB$ (angles on straight line)

$= 180° - 108°$

$\therefore \angle BCE = 72°$

(d)



i)

In $\triangle PBQ$ & $\triangle QCR$

$BQ = CR$ (S)

$\angle QBP = \angle QCR$

$= 90^\circ$ (property of a square) (A)

$BC = AB$ (sides of square)

$BP = AB - PA$

$QC = BC - BQ$

but $PA = BQ$ (given)

$\therefore BP = QC$ (S)

$\therefore \triangle PBQ \equiv \triangle QCR$ (SAS)

ii)

Hence corresponding angles are equal in congruent $\triangle$s'.

$\Rightarrow \angle BQP = \angle CRQ$ ①

But $\angle CRQ + \angle CQR = 90^\circ$ (angle sum in a right-angled $\triangle$)

$\therefore \angle CRQ + \angle BQP = 90^\circ$ (using ①)

$\therefore \angle PQR = 180^\circ - \angle CRQ - \angle BQB$

$= 90^\circ$ (angles on a straight line)

(a) $f'(x) = 15x^2 + \frac{3}{\sqrt{x}}$ ✓

ii) $f'(x) = \left(2x - \frac{1}{x^2}\right)\left(3x^2 - 7x\right) + \left(x^2 + \frac{1}{x}\right)\left(6x - 7\right)$ ✓ ✓

iii) $f'(x) = \frac{12x^3(5-2x) + 2(3x^4 - 8)}{(5-2x)^2}$ ✓✓

$= \frac{60x^3 - 24x^4 + 6x^4 - 16}{(5-2x)^2}$

$= \frac{60x^3 - 18x^4 - 16}{(5-2x)^2}$ ✓

iv)

$f'(x) = 5 \times 12x^2 (4x^3 - 7)^4$ ✓

$= 60x^2 (4x^3 - 7)^4$ ✓

(b)

$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - x^2 + 3x}{h}$ ✓

$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h - \cancel{x^2} + \cancel{3x}}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 3)}{\cancel{h}}$ ✓

$= 2x - 3$ ✓

(c) $\frac{dy}{dx} = 6x$

at $x = -1$, $m = 6 \times -1$

$= -6$ ✓

equation of the tangent :

$y - 3 = -6(x + 1)$

$y = -6x - 3$ ✓

(d)

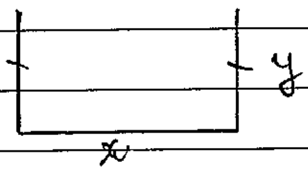
i) $f(x)$ is increasing for $x < 0$ & $x > 3$ ✓

ii) $f(x)$ has a max. turning point at

$x = 0$ ✓

because at $x = 0$, $\begin{cases} f'(x) = 0 \text{ and} \\ f''(x) < 0 \end{cases}$ ✓

(e)



$2y + x = 18$

$\therefore y = \frac{18 - x}{2} = 9 - \frac{x}{2}$ ✓

Area of the rectangle is :

$A = x\left(9 - \frac{x}{2}\right)$

$= 9x - \frac{x^2}{2}$ ✓

ii) $\frac{dA}{dx} = 9 - x$ ✓

for max. point $\frac{dA}{dx} = 0$

$\Rightarrow \quad 9 - x = 0$

$x = 9$

$\frac{d^2A}{dx^2} = -1 \quad < 0 \quad \frown \quad \checkmark$

$\therefore$ at $x = 9$ is a maximum.

$\therefore$ The max. possible area is

$A = 9 \times 9 - \frac{9^2}{2}$

$= 40.5 \text{ m}^2 \quad \checkmark$

(e)

i) $f'(x) = 12x - 3x^2$

$f''(x) = 12 - 6x$

for stationary points $f'(x) = 0$

$\Rightarrow \quad 3x(4 - x) = 0$

$\begin{cases} x = 0 \\ y = 0 \end{cases}, \quad \begin{cases} x = 4 \\ y = 32 \end{cases} \quad \checkmark$

$f''(0) = 12 \quad > 0 \quad \smile$

$\therefore$ $(0,0)$ is a minimum point $\checkmark$

$f''(4) = 12 - 6 \times 4$

$= -12 \quad < 0 \quad \frown$

$\therefore$ $(4,32)$ is a max. point $\checkmark$

ii) for point of inflexion $f''(x) = 0$

$\Rightarrow \quad 12 - 6x = 0 \quad \Rightarrow \begin{cases} x = 2 \\ y = 16 \end{cases}$

ii) for point of inflexion, $f''(x) = 0$

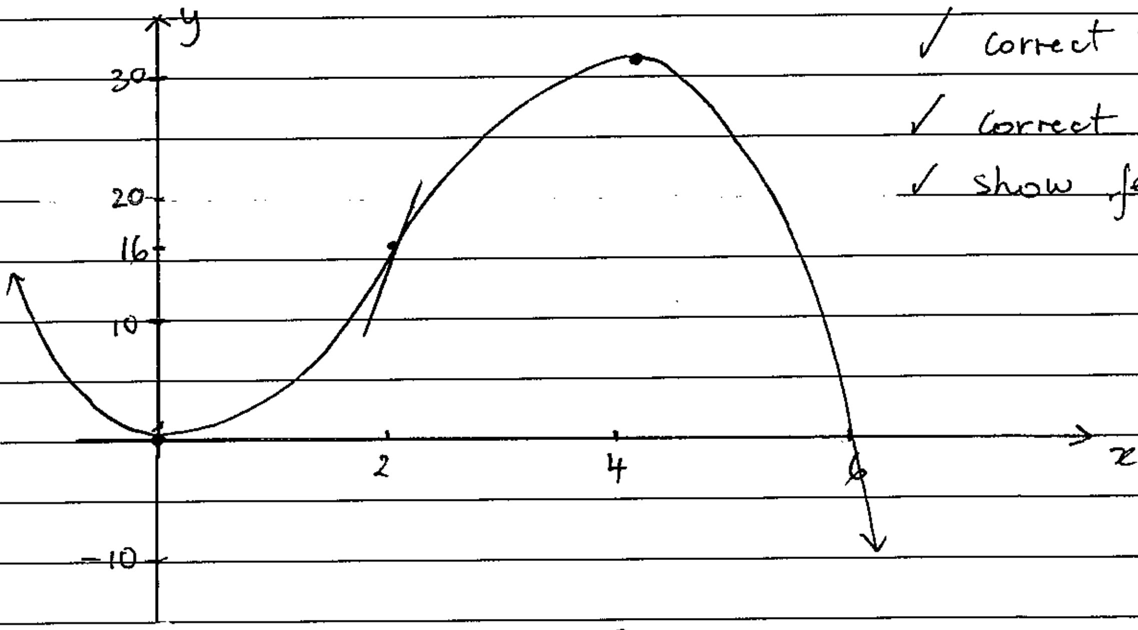
$\Rightarrow \quad 12 - 6x = 0$ ✓

$$\begin{cases} x = 2 \\ y = 16 \end{cases}$$

x	1	2	3
$f''(x)$	6	0	-6

✓

$\therefore$ The point of inflexion is $(2, 16)$



✓ correct shape
✓ correct scale
✓ show features.

x - intercept : $6x^2 - x^3 = 0$

$-x^2(x - 6) = 0$

$x = 0, \quad x = 6$

iv) The least value of $f(x)$ in the domain $-1 \leq x \leq 7$ is

$y = 6 \times 7^2 - 7^3 = -49$ ✓