Killara HIGH SCHOOL

YEAR 12

TERM 2
MATHEMATICS EXAMINATION

MAY 2010

Total Time:	**70 minutes**
Total Marks:	**60 marks**

Directions to Candidates:

- **Attempt ALL questions.**
- **All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.**
- **Board approved calculators may be used.**
- **Answer each question in a SEPARATE writing booklet.**
- **You may ask for extra writing booklets if you need them.**

SECTION I – Integration (24 marks) **Marks:**

(a) (i) Find $\int 5.dx$ **1**

(ii) Find $\int \frac{3}{(x-6)^2}\,dx$ **2**

(iii) Find $\int \frac{5x^3 - 8}{x^2}\,dx$ **2**

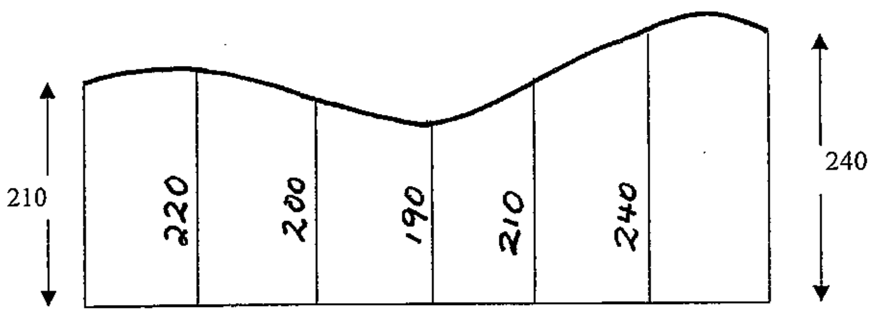
(iv) Evaluate $\int_1^4 x^2 + \sqrt{x}.dx$ **3**

(b) (i) On a number plane, sketch the curve $y = 4x - x^2$ showing its important features. **2**

(ii) On the same diagram shade the region R, which is bounded by the curve $y = 4x - x^2$ and the line $y = 5$, between $x = 0$ and $x = 4$ **1**

(iii) Find the area of the shaded region R. **4**

(c) The diagram shows a block of land and its dimensions, in metres. The block of land is bounded, on one side, by a river. Measurements are taken perpendicular to the line AB, from AB to the river, at equal intervals of 50m.



4

Use Simpson's Rule with six subintervals to find an approximation to the area of the block of land.

(d) (i) The area under the curve $y = \sqrt{9 - x^2}$, $-3 \le x \le 3$, is rotated about the X axis. Find the volume of the solid of revolution leaving answer in terms of π. **4**

(ii) Describe the shape of this solid. **1**

Section II over page ....

SECTION II – Exponentials and Logarithmic Functions (36 marks) **Marks:**

(a) (i) Evaluate $\log_2 10$, correct to 3 decimal places. **2**

(ii) Solve for x : $\log_9 x = \frac{-3}{2}$, giving your answer in exact form. **2**

(b) Differentiate each of the following:

(i) $4e^{-2x}$ **1**

(ii) $x^3 e^{3x}$ **2**

(iii) $\ell n\,(3x^2 - 7x + 2)$ **2**

(iv) $[\ell n\,(2x - 1)]^4$ **2**

(v) $\ell n \left(\frac{3-4x}{2x+3}\right)$ **2**

(c) Find the following integrals :

(i) $\int \frac{8dx}{2x-1}$ **1**

(ii) $\int 1 + \frac{3}{x-2}\ dx$ **2**

(iii) $\int e^{5-2x}\ dx$ **1**

(iv) $\int \frac{4x+2}{x^2+x}\ dx$ **2**

(d) (i) Show that $\frac{d}{dx} e^{2x^4} = 8x^3\ e^{2x^4}$ **1**

(ii) Hence, or otherwise, find $\int x^3 . e^{2x^4}\ dx$ **1**

(e) Find the value of 'k' , such that $\int_1^k \frac{4}{x^2}\ dx = 3$ **3**

(f) Find the equation of the tangent to the curve $y = e^{\frac{x}{2}}$ at the point where

where $(1, e^{\frac{1}{2}})$ **3**

(g) Continued over page . . . .

Marks:

(g) For the curve $y = x \log_e x,\ x > 0$,

(i)	find its X intercept(s)	1
(ii)	find any stationary point(s), and determine their nature	3
(iii)	determine if there are any points of inflexion on the curve	1
(iv)	find y when $x = e$	1
(vi)	draw a neat sketch of the curve $y = x \log_e x$ showing its important features	3

END OF ASSESSMENT

SOLUTIONS – YR. 12 MATHEMATICS EXAM

TERM II '10.

SOLUTIONS + SUGGESTED MARKING SCALE

SECTION I – Integration (24 marks)

a) i) $\int 5\,.dx = 5x + c$ (1)

ii) $\int \frac{3}{(x-6)^2}dx = \int 3(x-6)^{-2}dx$

$= 3\frac{(x-6)^{-1}}{-1} + c$

$= \frac{-3}{x-6} + c$ (2)

iii) $\int \frac{5x^3-8}{x^2}dx = \int \frac{5x^3}{x^2} - \frac{8}{x^2}dx$

$= \int 5x - 8x^{-2}.dx$

$= \frac{5x^2}{2} - \frac{8x^{-1}}{-1} + c$

$= \frac{5x^2}{2} + \frac{8}{x} + c$ (2)

iv) $\int_1^4 x^2 + \sqrt{x}.dx = \int_1^4 x^2 + x^{1/2}.dx$

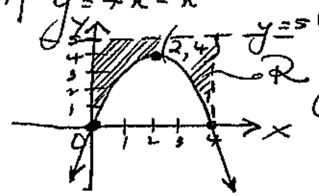
$= \left[\frac{x^3}{3} + \frac{2x^{3/2}}{3}\right]_1^4$

$= \frac{4^3}{3} + \frac{2\times 4^{3/2}}{3} - \left(\frac{1^3}{3} + \frac{2\times 1^{3/2}}{3}\right)$

$= \frac{64}{3} + \frac{16}{3} - \left(\frac{1}{3} + \frac{2}{3}\right)$

$= \frac{77}{3}$ (3)

b) i) $y = 4x - x^2$ (1)



$x = 2, y = 4$

(2)

ii) R on diagram

iii) R = rectangle – area under curve

$= 5\times 4 - \int_0^4 4x - x^2 dx$ (4)

$= 20 - \left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= 20 - \left[2\times 4^2 - \frac{4^3}{3} - 0\right]$

$= 20 - \left[32 - \frac{64}{3}\right] = 20 - 10\frac{1}{3} = 9\frac{2}{3}u^2$

Section 1 cont...

c) $A = \frac{h}{3}[y_1 + 4y_2 + 2y_3 + \dots + y_n]$

$= \frac{50}{3}[210 + 4\times 220 + 2\times 200 + 4\times 190 + 2\times 210 + 4\times 240 + 240]$

$= 64500\ m^2$ (4)

d) i) $y = \sqrt{9-x^2}$ $\quad -3 \le x \le 3$

$V = \pi \int y^2.dx$

$= \pi \int_{-3}^{3} 9 - x^2.dx.$

$= 2\pi \int_0^3 9 - x^2.dx$

$= 2\pi \left[9x - \frac{x^3}{3}\right]_0^3$

$= 2\pi \left[9\times 3 - \frac{3^3}{3} - 0\right]$

$= 2\pi [27 - 9]$ (4)

$= 36\pi$ units^3

ii) a sphere, radius 3, centre = origin (1)

SECTION II /36

a) i) $\log_2 10 = \dfrac{\log_e 10}{\log_e 2} = 3.32192..$
$\doteqdot 3.322$ (3 d.p) (2)

ii) $\log_9 x = -3/2$
$x = 9^{-3/2} = \dfrac{1}{(\sqrt{9})^3} = \dfrac{1}{27} \doteqdot 0.\dot{0}3\dot{7}$ (2)

b) i) $\dfrac{d}{dx} 4e^{-2x} = -2.4e^{-2x} = -8e^{-2x}$ (1)

ii) $\dfrac{d}{dx} x^3 e^{3x} = VU' + UV'$
$= e^{3x}.3x^2 + x^3.3.e^{3x}$
$= 3x^2 e^{3x}(1+x)$ (2)

iii) $\dfrac{d}{dx} \ln(3x^2 - 7x + 2) = \dfrac{6x-7}{3x^2-7x+2}$ (2)

iv) $\dfrac{d}{dx}[\ln(2x-1)]^4 = 4[\ln(2x-1)]^3 . \dfrac{2}{2x-1}$
$= \dfrac{8[\ln(2x-1)]^3}{2x-1}$ (2)

v) $\dfrac{d}{dx} \ln\left(\dfrac{3-4x}{2x+3}\right) = \dfrac{d}{dx} \ln(3-4x) - \ln(2x+3)$
$= \dfrac{-4}{3-4x} - \dfrac{2}{2x+3}$
$= \dfrac{-8x-12-6+8x}{(3-4x)(2x+3)} = \dfrac{-18}{(3-4x)(2x+3)}$ (2)

c) i) $\displaystyle\int \frac{8\,dx}{2x-1} = 4\int \frac{2\,dx}{2x-1}$
$= 4\ln(2x-1) + C$ (1)

ii) $\displaystyle\int 1 + \frac{3}{x-2}\,dx = x + 3\ln(x-2) + C$ (2)

iii) $\displaystyle\int e^{5-2x}.dx = \frac{e^{5-2x}}{-2} + C$ (1)

SECTION II cont'd ..

iv) $\displaystyle\int \frac{4x+2}{x^2+x}.dx = 2\int \frac{2x+1}{x^2+x}\,dx$
$= 2\ln(x^2+x) + C$ (2)

d) i) $\dfrac{d}{dx} e^{2x^4} = e^{2x^4} . \dfrac{d}{dx}(2x^4)$
$= e^{2x^4} . 8x^3$
$= 8x^3 . e^{2x^4}$ (1)

ii) $\displaystyle\int x^3 . e^{2x^4} dx = \frac{1}{8} e^{2x^4} + C$ (1)

e) $\displaystyle\int_1^k \frac{4}{x^2} = 3$
LHS $= \displaystyle\int_1^k 4x^{-2}.dx$
$= \left[\dfrac{4x^{-1}}{-1}\right]_1^k$
$= \left[-\dfrac{4}{x}\right]_1^k$
$\therefore \dfrac{-4}{k} - -\dfrac{4}{1} = 3$
$\dfrac{-4}{k} + 4 = 3$
$1 = \dfrac{4}{k}$
$\therefore k = 4.$ (3)

f) $y = e^{x/2}$ $(1, e^{1/2})$
$y' = \dfrac{1}{2} e^{x/2}$
$f'(1) = \dfrac{1}{2} e^{1/2}$

$\therefore$ tangent is
$y - y_1 = m(x - x_1)$
$y - \sqrt{e} = \dfrac{\sqrt{e}}{2}(x-1)$
$2y - 2\sqrt{e} = \sqrt{e}x - \sqrt{e}$
$\sqrt{e}x - 2y + \sqrt{e} = 0.$ (3)

3) $y = x \log_e x \quad , \quad x > 0$

i) X intercept $\Rightarrow y = 0$

$0 = x \log_e x$

$x \neq 0 \quad \therefore \log_e x = 0$

$\therefore x = 1$ (1)

ii) st. pts when $y' = 0$

$y' = x \cdot \frac{1}{x} + \log_e x \cdot 1$

$= 1 + \ln x$

$1 + \ln x = 0$

$\ln x = -1$

or $\log_e x = -1$

$\therefore x = e^{-1} = \frac{1}{e}$

$x = \frac{1}{e} \quad , y = \frac{1}{e} \log_e {}^{1/e}$

$= \frac{1}{e} \times -1$

$= \frac{-1}{e}$

$\therefore \left(\frac{1}{e}, \frac{-1}{e}\right)$

test for nature y'',

$y'' = \frac{1}{x}$ (3)

$f''\left(\frac{1}{e}\right) > 0 \quad \therefore$ min tpt at $\left(\frac{1}{e}, -\frac{1}{e}\right)$

iii) pts of inflex, $y'' = 0$

$\frac{1}{x} = 0 \quad \therefore$ no. solution

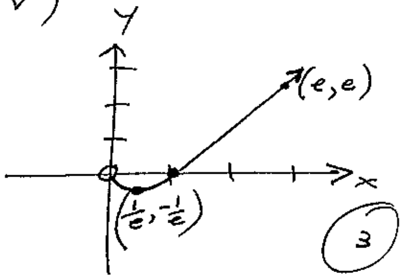
$1 = 0 \; ??$

$\therefore$ no pts. of inflexion. (1)

iv) $x = e \; , \; y = e \log_e e = e \times 1 = e$

$\therefore (e, e)$ (1)

v)



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