7e 10ciii

Killara High School

2010

TERM 1 EXAMINATION **Monday, 3rd March**

Mathematics

General Instructions

Total marks – 120

- Reading time – 5 minutes
- Working time – 3 hours
- Start each question in a new booklet
- Write using black or blue pen
- Board-approved calculators maybe used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Attempt Questions 1–10
- All questions are of equal value

Question 1 **(12 Marks)** Start a new booklet **Marks**

(a) Evaluate correct to three significant figures, **2**

$$\sqrt{\frac{3^2+12^2}{231-12^2}}$$

(b) Factorise: $2x^3+16$ **2**

(c) Solve: $\dfrac{a-5}{3}-\dfrac{a+1}{4}=5$ **2**

(d) Find integers a and b such that $\dfrac{2}{3-\sqrt{5}}=a+b\sqrt{5}$ **2**

(e) Differentiate x^5+3x^{-1} **2**

(f) Find the values of x for which $|x+1|\le 5$ **2**

Question 2 **(12 Marks)** Start a new booklet **Marks**

(a) Sketch the graph of y = 2x + 3 on the cartesian plane, showing clearly the x and y intercepts. **2**

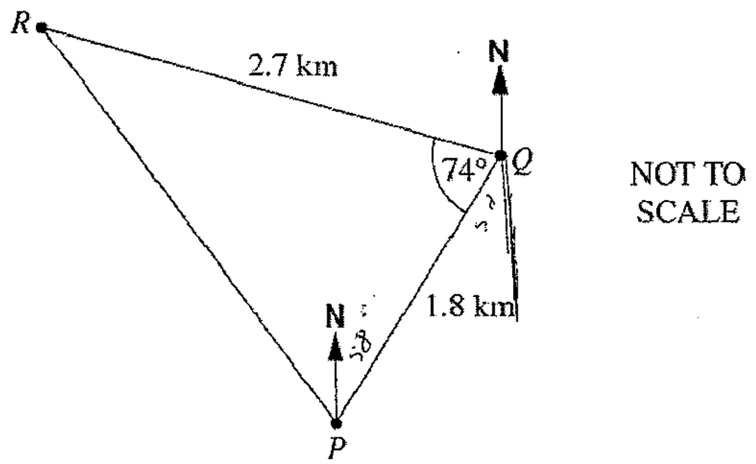
(b) Differentiate:

(i) $y = \dfrac{3x}{5-x}$ **2**

(ii) $y = x^2\sqrt{3x-7}$ **2**

(iii) $y = \dfrac{5}{(3x-2)^8}$ **2**

(c) A yacht race follows the triangular course as shown in the diagram. The course from P to Q is 1.8 km on a bearing of 058°. At Q the course changes direction. The course from Q to R is 2.7 km and $\angle PQR = 74°$



(i) What is the bearing of R from Q? **1**

(ii) What is the distance from R to P? **2**

(iii) The area inside this course is set as a 'no-go' zone for other boats while the race is on. **1**

What is the area of this 'no go' zone?

Question 3 **(12 Marks)** Start a new booklet **Marks**

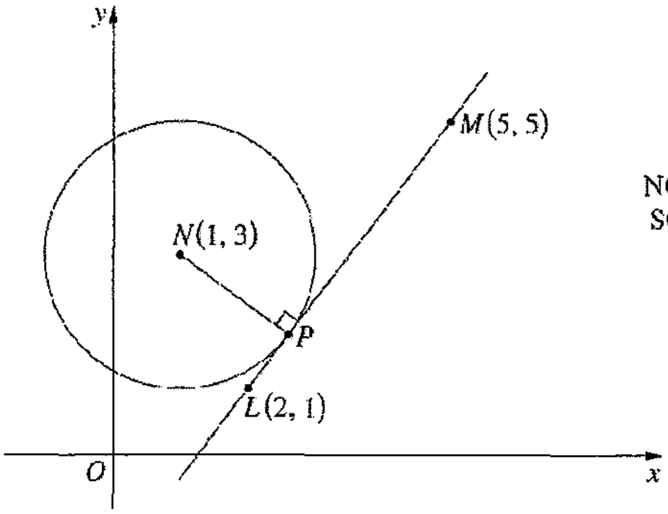
(a) Find the coordinates of the midpoint of the interval joining the points (2, 1) and (4, -3). **1**

(b) Find the equation of the tangent to the curve $y = x^2 - 2x$ at the point (-1, 3). **2**

(c) Find the exact value of such that $\sqrt{3}\cos\theta = 1$ for $0^\circ \le \theta \le 360^\circ$ **2**

(d) Shade the region in the number plane defined by $y \ge 0$ and $y \le 9 - x^2$ **2**

(e)



NOT TO SCALE

The circle in the diagram has centre N. The line LM is a tangent to the circle at P.

(i) Find the equation of LM and leave your answer in the form $ax + by + c = 0$ **2**

(ii) Find the distance NP. **2**

(iii) Find the equation of the circle **1**

Question 4. (12 Marks) Start a new booklet **Marks**

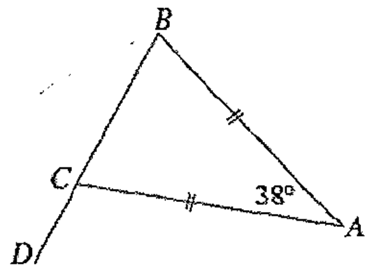
(a) Differentiate, from first principles, $f(x) = 2 - x^2$ **2**

(b) The equation of a parabola is $x^2 = 8(y+3)$

(i) Find the coordinates of the vertex of the parabola **1**

(ii) Find the equation of the directrix of the parabola **2**

(c)



NOT TO SCALE

In the diagram, ABC is an isosceles triangle with AB = AC and $\angle BAC = 38°$.

Copy or trace the diagram into your answer booklet.

Find the size of $\angle ACD$. You must give reasons for your answer. **2**

(d) Evaluate : $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 2}$ **2**

(e) Find the values of k for which the quadratic equation **3**

$$x^2 - (k+4)x + (k+7) = 0)$$

has equal roots

Question 5. (12 Marks) Start a new booklet **Marks**

(a) Solve the equation $m^2 - m - 1 = 0$, correct to three decimal places **2**

(b) Find the exact value of sec $210°$ **2**

(c) Find the equation of the normal to the curve $x^2 = 8y$ at the point where $x = 4$. **4**

(d) Let α and β be the roots of the equation $x^2 - 5x + 2 = 0$. Find the values of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha + 1)(\beta + 1)$ **2**

Question 6. **(12 Marks)** Start a new booklet **Marks**

(a) Show that $1 - x + x^2 - \dfrac{1}{1+x} = \dfrac{x^3}{1+x}$ for $x \neq -1$. **2**

(b) The roots of the quadratic equation $px^2 - x + q = 0$ are -1 and 3. **2**
Find p and q.

(c) Solve: $4^x - 5\left(2^{x+1}\right) + 16 = 0$ **3**

(d) (i) Given $f(x) = \dfrac{x}{x-1}$, evaluate $f\left(\dfrac{1}{2}\right) - f(-2)$ **1**

(ii) Show that $f(x) = \dfrac{x}{x-1}$ is neither an odd function nor an even function. **2**

(e) Prove $(1 - \cos\theta)(1 + \sec\theta) = \sin\theta \tan\theta$ **2**

Question 7 **(12 Marks)** Start a new booklet **Marks**

(a) For what values of x is $2x^2 \geq x$? **2**

(b) Find a primitive function of $6 - x^{-3}$ **2**

(c) (i) Find the sum of the interior angles of a regular 11 sided polygon. **1**

(ii) How large is each exterior angle to the nearest minute? **1**

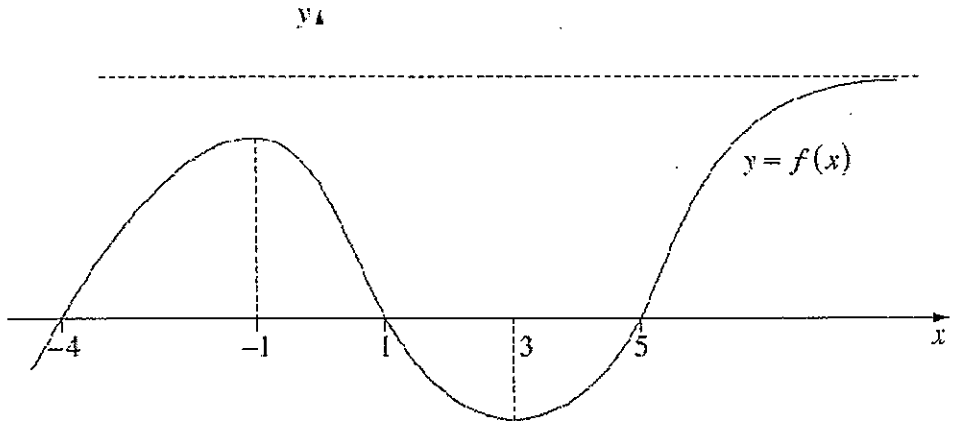
(d) Express $5x^2 + 2x - 3$ in the form $A(x+1)^2 + B(x+1) + C$ **2**

(e) For the parabola defined by, $x^2 + 4x - 8y + 12 = 0$ find the coordinates of the focus. **2**

(f) For all x in the domain $-2 \leq x \leq 2$, a function, $f(x)$ is such that $f(x) < 0$, $f'(x) > 0$ and $f''(x) < 0$. Sketch a possible $y = f(x)$ in this domain. **2**

Question 8. **(12 Marks)** Start a new booklet **Marks**

(a)



The diagram shows the graph of a function $y = f(x)$

(i) Sketch the graph $y = f'(x)$. **2**

(ii) For which values of x is the derivative, $f'(x)$, negative? **1**

(iii) What happens to $f'(x)$ for large values of x? **1**

(b) Consider the curve given by $y = 7 + 4x^3 - 3x^4$

(i) Find the coordinates of the two stationary points. **2**

(ii) Determine the nature of the stationary points **2**

(iii) Find the point of inflexion. **1**

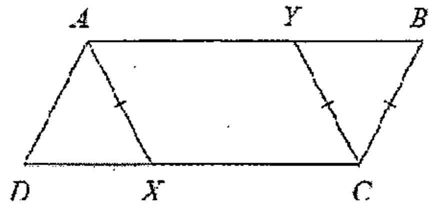
(iv) Sketch the curve for the domain $-1 \le x \le 2$ **2**

(v) State the greatest value of y in the domain. **1**

Question 9. (12 Marks) Start a new booklet **Marks**

(a) If $\tan\theta = -\frac{4}{9}$ and $\cos\theta$ is positive, find the exact value of $\sin\theta$ **2**

(b)



ABCD is a parallelogram. The point *X* lies on *CD*, the point *Y* lies on *AB*, and $AX = CY = BC$, as shown in the diagram.
Copy the diagram into your writing booklet.

(i) Explain why $\angle ADX = \angle CBY$ **1**

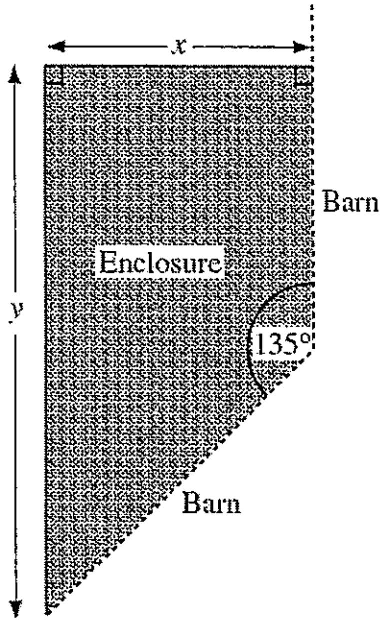
(ii) Show that $AD = AX$ **1**

(iii) Show that triangles *ADX* and *CBY* are congruent. **2**

(iv) Hence prove that *AYCX* is a parallelogram. **1**

QUESTION 9 IS CONTINUED ON THE NEXT PAGE

(c) An enclosure is to be built adjoining a barn, as in the diagram. The walls of the barn meet at 135°, and 117m of fencing is available for the enclosure, so that $x + y = 117$ where x and y are as shown in the diagram.



(i) Show that the shaded area of the enclosure in square metres is given by **2**

$$A = 117x - \frac{3}{2}x^2$$

(ii) Show that the largest area of the enclosure occurs when $y = 2x$. **3**

Question 10. (12 Marks) Start a new booklet **Marks**

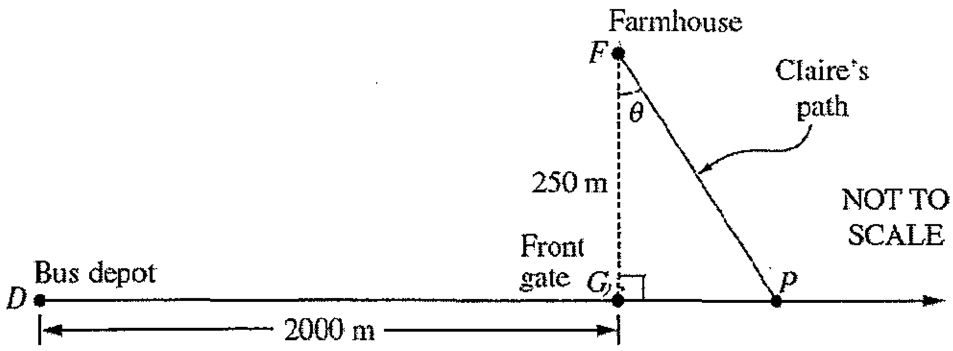
(a) Simplify: $\dfrac{1-x^2}{x-1}$ **1**

(b) Plot the points $A(0, 1)$, $B(4, 0)$ and $C(0, 2)$ on a number plane in your answer booklet.

(i) Show that the length CB is twice that of CA **2**

(ii) ~~(iii)~~ A point $P(X, Y)$ moves such that the length of PB is twice that of PA. Show that the locus of P is a circle, and determine its centre and radius. **3**

(c)



The diagram shows a farmhouse F that is located 250 m from a straight section of road. The road begins at the bus depot D, which is situated 2000 m from the front gate G of the farmhouse. The school bus leaves the depot at 8 am and travels along the road at a speed of 15 m s^{-1}. Claire lives in the farmhouse, and she can run across the open paddock between the house and the road at a speed of 4 m s^{-1}. The bus will stop for Claire anywhere on the road, but will not wait for her.

Assume that Claire catches the bus at the point P on the road where $\angle GFP = \theta$.

(i) Find two expressions in terms of θ, one expression for the time taken for the bus to travel from D to P and the other expression for the time taken by Claire to run from F to P. **2**

(ii) What is the latest time that Claire can leave home in order to catch the bus? **4**

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

KHS.

$\frac{1}{2}$ yrly. Mathematics 2010.

SOLUTIONS.

Ques 1.

a) $\sqrt{\dfrac{3^2+12^2}{231-12^2}} = \sqrt{\dfrac{9+144}{231-144}}$

$= \sqrt{\dfrac{153}{87}}$

$= \sqrt{1.758620690}$

$= 1.35$ 3 s.f.

b) $2x^3+16 = 2(x^3+8)$

$= 2(x+2)(x^2-2x+4)$

c) $\dfrac{a-5}{3} - \dfrac{a+1}{4} = 5$

$4(a-5) - 3(a+1) = 60$

$4a - 20 - 3a - 3 = 60$

$a - 23 = 60$

$a = 83$

d) $\dfrac{2}{3-\sqrt{5}} \times \dfrac{3+\sqrt{5}}{3+\sqrt{5}} = \dfrac{6+2\sqrt{5}}{9-5}$

$= \dfrac{6+2\sqrt{5}}{4}$

$= \dfrac{3}{2} + \dfrac{\sqrt{5}}{2}$

$\therefore a = 3/2,\ b = \frac{1}{2}$

e) $\dfrac{d}{dx}\ x^5 + 3x^{-1} = 5x^4 - 3x^{-2}$

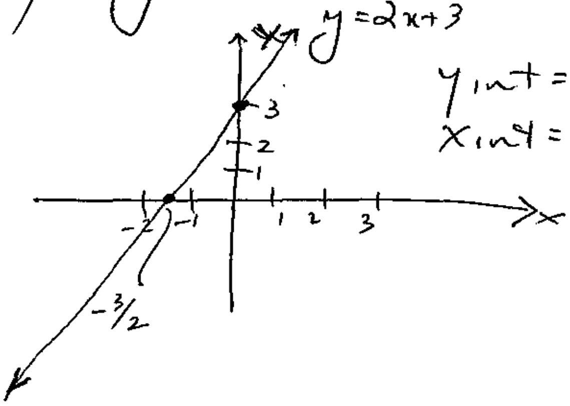
f) $|x+1| \le 5$

$-5 \le x+1 \le 5$

$-6 \le x \le 4.$

Ques 2

a) $y = 2x+3$



y int $=3$
x int $= -\frac{3}{2}$

b) i) $\frac{d}{dx}\frac{3x}{5-x} = \frac{(5-x).3-(3x).-1}{(5-x)^2}$

$= \frac{15-3x+3x}{(5-x)^2}$

$= \frac{15}{(5-x)^2}$

ii) $y = x^2\sqrt{3x-7}$

$\frac{d}{dx} = \sqrt{3x-7}\,.\,2x + x^2.\frac{1}{2}(3x-7)^{-\frac{1}{2}}.3$

$= 2x\sqrt{3x-7} + \frac{3x^2}{2\sqrt{3x-7}}$

iii) $y = \frac{5}{(3x-2)^8} = 5(3x-2)^{-8}$

$y' = -8.5(3x-2)^{-9}.3$

$= \frac{-120}{(3x-2)^9}$

c) i) R from Q $= 180+58+74$
$= 312°T$

ii) $RP^2 = 1.8^2 + 2.7^2 - 2\times1.8\times2.7\times\cos 74°$
$= 7.850804902$
$\therefore RP = 2.8$ km (1dp)

c) iii) $A = \frac{1}{2}\times2.7\times1.8\times\sin 74°$

$= 2.335865924$
$\doteqdot 2.34\ km^2$

Ques 3

a) $(2,1)$ and $(4,-3)$

$$m.p = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{2+4}{2}, \frac{1+-3}{2}\right)$$

$$= \left(\frac{6}{2}, \frac{-2}{2}\right)$$

$$= (3,-1)$$

b) $y = x^2 - 2x \quad (-1,3)$

$$y' = 2x - 2$$

$$m = 2\times -1 - 2$$

$$= -4$$

$\therefore$ tangent,

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -4(x - -1)$$

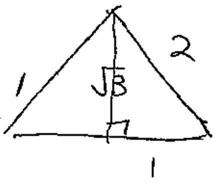
$$y - 3 = -4x - 4$$

$$4x + y + 1 = 0$$

c) $\sqrt{3}\cos\theta = 1 \quad 0 \leq \theta \leq 360$

$$\cos\theta = \frac{1}{\sqrt{3}}$$

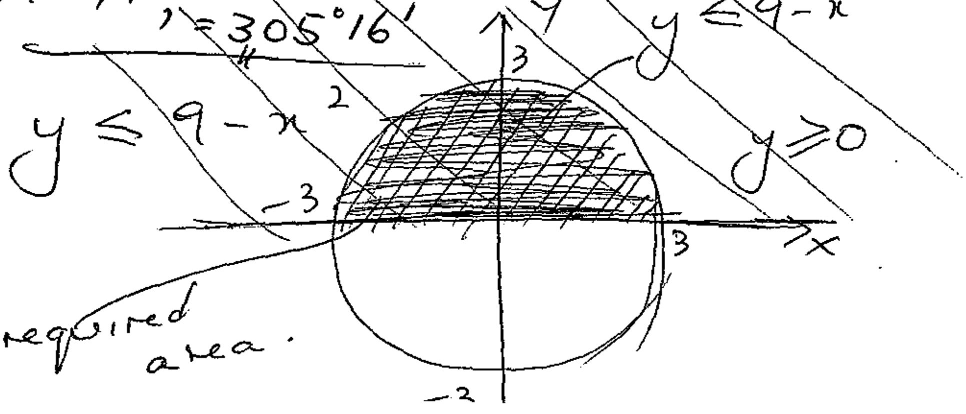
basic $\theta = 54°44'$



S	A
T	C

$*\ \therefore \theta = 54°44',\ 360 - 54°44'$
$= 305°16'$

d) $y \geq 0 \quad y \leq 9 - x^2$



e) i) $L(2,1) \quad M(5,5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{5-2}$$

$$= \frac{4}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{4}{3}(x - 2)$$

$$3y - 3 = 4x - 6$$

$$0 = 4x - 3y - 3$$

ii) perp dist.

$$= \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right|$$

$$= \frac{|4\times 1 - 3\times 3 - 3|}{\sqrt{4^2 + (-3)^2}}$$

$$= \frac{|4 - 9 - 3|}{\sqrt{25}}$$

$$= \frac{8}{5} \text{ units.}$$

iii) $(x-1)^2 + (y-3)^2 = \left(\frac{8}{5}\right)^2$

$$(x-1)^2 + (y-3)^2 = \frac{64}{25}.$$

Ques 4

a) $y = 2 - x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2 - (x+h)^2 - (2 - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2} - (\cancel{x^2} + 2xh + h^2) - \cancel{2} + \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{-2xh - h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(-2x - h)}{h}$$

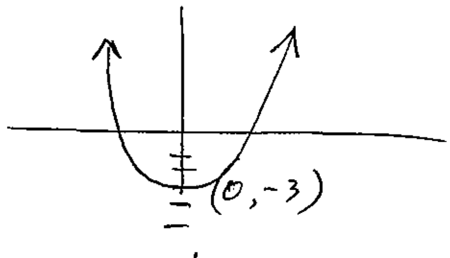
$$= -2x.$$

b) $x^2 = 8(y+3)$

like $(x-h)^2 = 4a(y-k)$

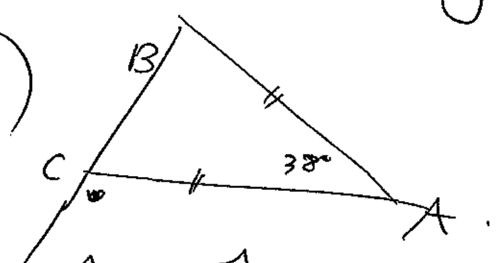
i) $\therefore V = (0, -3)$

ii) $4a = 8 \quad \therefore a = 2$



$\therefore$ directrix is $y = -5$.

c)



$A\hat{B}C = A\hat{C}B = \frac{180 - 38}{2}$ (base $\angle$s of isos. $\triangle$)

$= 71°$

$\therefore A\hat{C}D = 180 - 71°$ (st. $\angle = 180$) $= 109°$

d) $$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$$

$$= 4$$

e) $x^2 - (k+4)x + (k+7) =$

equal roots

$\Delta = 0, \quad b^2 - 4ac = 0$

$-(k+4)^2 - 4 \cdot 1 \cdot (k+7) = 0$

$k^2 + 8k + 16 - 4k - 28 = 0$

$k^2 + 4k - 12 = 0$

$(k+6)(k-2) = 0$

$\therefore k = -6$ or 2

Ques 5

a) $m^2 - m - 1 = 0$

$m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{--1 \pm \sqrt{(-1)^2 - 4.1.-1}}{2.1}$

$= \dfrac{1 \pm \sqrt{1+4}}{3}$

$= \dfrac{1 \pm \sqrt{5}}{3}$

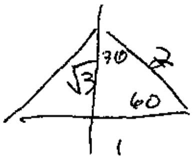
$= 1.079, \ -0.412$ (3 d.p.)

b) $\sec 210° = \dfrac{1}{\cos 210°}$

S	A
T	C

$= \dfrac{1}{-\cos 30}$

$= \dfrac{1}{-\sqrt{3}/2}$



$= -\dfrac{2}{\sqrt{3}}.$

c) $x^2 = 8y \qquad x = 4$

$y = \dfrac{x^2}{8}$

$y' = \dfrac{2x}{8} = \dfrac{x}{4}$

at $x = 4$, $y' = 4/4 = 1$, ∴ normal has $m = -1$

$x = 4, \ y = 2$

equation of normal,

$y - y_1 = m(x - x_1)$

$y - 2 = -1(x - 4)$

$y - 2 = -x + 4$

d) $x^2 - 5x + 2 = 0$

i) $\alpha + \beta = -b/a$

$= --5/1$

$= 5$

ii) $\alpha\beta = \dfrac{c}{a}$

$= 2/1$

$= 2$

iii) $(\alpha+1)(\beta+1)$

$= \alpha\beta + \alpha + \beta + 1$

$= 2 + 5 + 1$

$= 8$

e) $(1 - \cos\theta)(1 + \sec\theta) = \sin\theta \tan$

LHS $= 1 + \sec\theta - \cos\theta - \cos\theta\sec$

$= 1 + \sec\theta - \cos\theta - \cos\theta\left(\dfrac{1}{\cos}\right.$

$= \cancel{1} + \sec\theta - \cos\theta - \cancel{1}$

$= \dfrac{1}{\cos\theta} - \cos\theta$

$= \dfrac{1 - \cos^2\theta}{\cos\theta}$

Ques 6(e)

$= \dfrac{\sin^2\theta}{\cos\theta}.$

$= \dfrac{\sin\theta . \sin\theta}{\cos\theta}$

$= \sin\theta . \tan\theta$

$=$ RHS.

Ques 6

a) $1 - x + x^2 - \frac{1}{1+x} = \frac{x^3}{1+x}$

$$LHS = \frac{(1-x+x^2)(1+x) - 1}{1+x}$$

$$= \frac{\cancel{1} + \cancel{x} - \cancel{x} - \cancel{x^2} + \cancel{x^2} + x^3 - \cancel{1}}{1+x}$$

$$= \frac{x^3}{1+x}$$ ✓

b) $p^2 - x + q = 0 \quad -1, 3.$

$\therefore$ sum, $-1+3 = \frac{-b}{a}$

$2 = \frac{1}{p}$

$\therefore p = \frac{1}{2}$

product $-3 = \frac{c}{a}$

$-3 = \frac{q}{p}$

$-3p = q$

$-3 \cdot \frac{1}{2} = q$

$\therefore q = -3/2$

c) $4^x - 5(2^{x+1}) + 16 = 0$

$(2^x)^2 - 5 \cdot 2^x \cdot 2^1 + 16 = 0$

let $u = 2^x \rightarrow u^2 - 10u + 16 = 0$

$(u-8)(u-2) = 0$

$\therefore u = 8$ or 2.

$\therefore 2^x = 8$ or 2

$\therefore x = 3$ or 1

d) i) $f(x) = \frac{x}{x-1}$

$f(\frac{1}{2}) - f(-2)$

$\frac{\frac{1}{2}}{\frac{1}{2}-1} - \frac{-2}{-2-1}$

$\frac{\frac{1}{2}}{-\frac{1}{2}} - \frac{-2}{-3}$

$-1 \cancel{+} -2/3$

$= -1\frac{2}{3}.$

Ques 6 cont...

ii) $f(x) = \frac{x}{x-1}$

even; $f(x) = f(-x)$
odd; $f(x = -f(-x)$

$$\frac{x}{x-1} = \frac{-x}{-x-1}$$

$$\cancel{\frac{x}{x-1}} = \cancel{\quad}$$

$$\frac{x}{x-1} = \frac{-x}{-(x+1)}$$

$$= \frac{x}{x+1}$$

LHS = $f(x)$ RHS = $f(-x)$

now $f(x) \neq f(-x)$
$f(x) \neq -f(-x)$

$\therefore$ neither odd nor even

e) $(1-\cos\theta)(1+\sec\theta) = \sin\theta . \tan\theta$

$$1 + \sec\theta - \cos\theta - \cos\theta . \sec\theta$$

$$= 1 + \frac{1}{\cos\theta} - \cos\theta - \cancel{\cos\theta} . \frac{1}{\cancel{\cos\theta}}$$

$$= \frac{1}{\cos\theta} - \cos\theta$$

$$\frac{1-\cos^2\theta}{\cos\theta}$$

$$= +\frac{\sin^2\theta}{\cos\theta} = +\frac{\sin\theta . \sin\theta}{\cos\theta}$$

$$= +\tan\theta . \sin\theta$$

= RHS

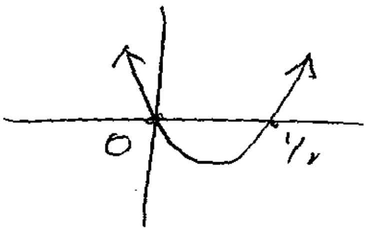
Ques 7.

a) $2x^2 \geq x$

$2x^2 - x \geq 0$

$x(2x-1) \geq 0$

$x = 0, \frac{1}{2}$.



$\therefore x \leq 0 \cup x \geq \frac{1}{2}$.

b) $\int 6 - x^{-3}\,dx$

$= 6x - \frac{6x^{-2}}{-2} + C$

$= 6x + \frac{6}{2x^2} + C$

$= 6x + \frac{3}{x^2} + C$

OR $6x + 3x^{-2} + C$

c) i) angle sum $= 180(n-2)$

$= 180(11-2)$

$= 180 \times 9$

$= 1620°$

ii) $\therefore \angle = \frac{1620}{11}$

$= 147\frac{3}{11}$

$\times 60 = \frac{180}{11}$ $\quad = 147° 16'$

d) $5x^2 + 2x - 3$

$= A(x+1)^2 + B(x+1) + C$

RHS $= A(x^2 + 2x + 1) + Bx + B + C$

$= Ax^2 + 2Ax + A + Bx + B + C$

$\therefore A = 5$

$2A + B = 2$

$\therefore 10 + B = 2$

$B = -8$

and $A + B + C = -3$

$\therefore 5 - 8 + C = -3$

$-3 + C = -3$

$\therefore C = 0$

$\therefore A = 5, B = -8, C = 0$.

e) $x^2 + 4x - 8y + 12 = 0$

$x^2 + 4x = 8y - 12$

$x^2 + 4x + 4 = 8y - 12 + 4$

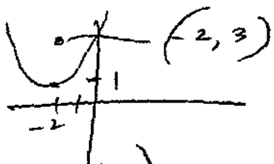
$(x+2)^2 = 8y - 8$.

$(x+2)^2 = 8(y-1)$

$\therefore$ Vertex $= (-2, 1)$

$4a = 8$

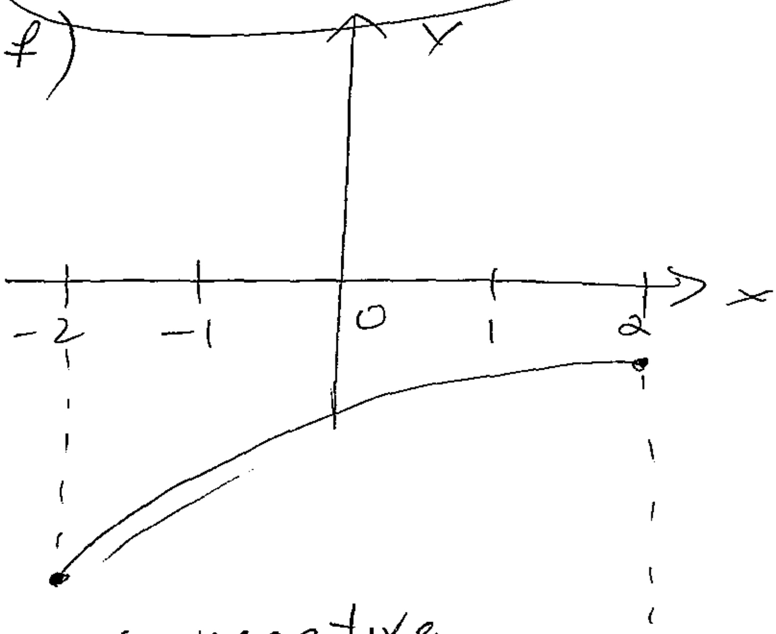
$a = 2$



$\therefore$ focus $= (-2, 3)$

Ques 7 cont...

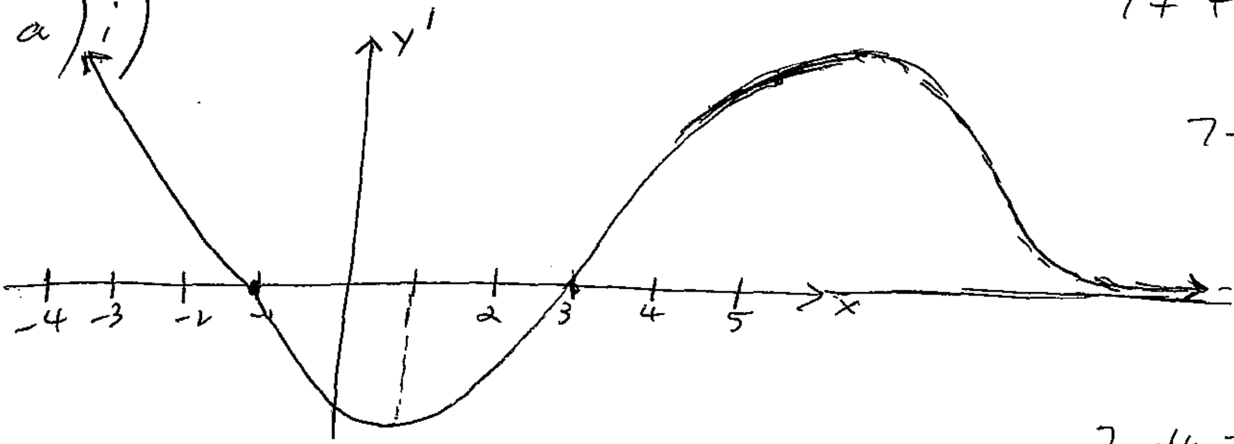
f)



y is negative
gradient is positive
concave down.

Ques 8

a) i)



$7 + 4 \cdot \frac{8}{27} - 3 \times \frac{16}{81}$

$7 + \frac{32}{27} - \frac{48}{81}$

$7 + \frac{96 - 48}{81}$

$7 - 4 - 3 \cdot$

$7 + 32 - 48$

ii) $f'(x) < 0$; $-1 < x < 3$

iii) $f'(x) \to 0$ as $x \to \infty$

b) $y = 7 + 4x^3 - 3x^4$

i) $y' = 12x^2 - 12x^3$

$= 12x^2(1 - x)$

at pts, $y' = 0$

$12x^2(1 - x) = 0$

$\therefore x = 0, 0, 1$

ii) $\cancel{x=0}$, $y'' = 24x - 36x^2$

$= 12x(2 - 3x)$

$f''(0)$, $y'' = 0$ $\therefore$ poss. pt. of inflex

$f''(-\frac{1}{2})$, $f''(\frac{1}{2})$

$f''(-\frac{1}{2}) = (-)(+) \to (-)$

$f''(\frac{1}{2}) = (+)(+) \to (+)$

$\therefore$ pt of inflexion at $(0, 7)$

$x = 1$ $y'' = < 0$ $\therefore$ max tpt at $(1, 8)$

iii) pt of inflex

$12x(2 - 3x) = 0.$

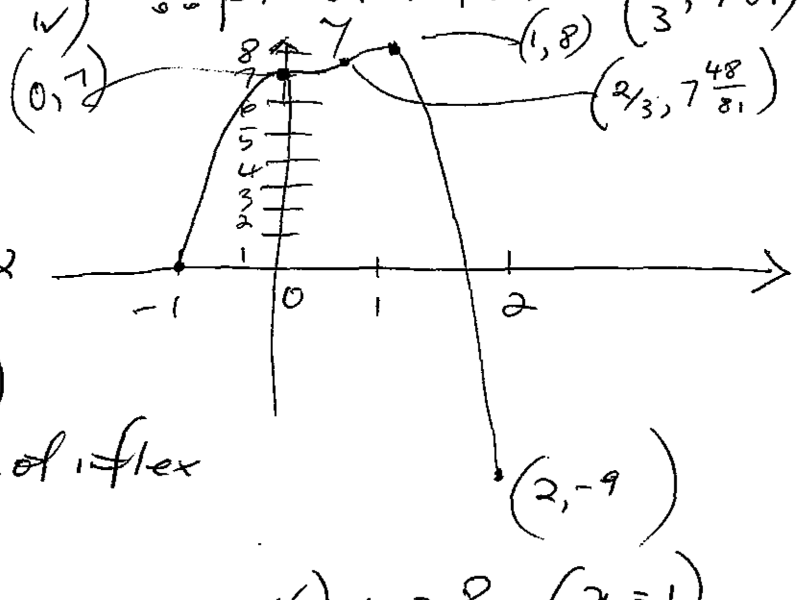
$x = 0$, $x = 2/3$.

$f''(\frac{1}{2}) = (+)$

$f''(1)$ $(-)$

$\therefore$ pt. of inflex at $(\frac{2}{3}, 7\frac{48}{81})$

iv)

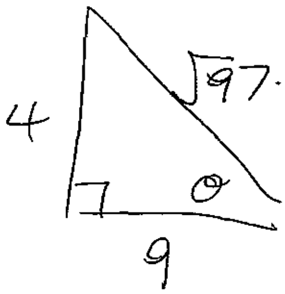


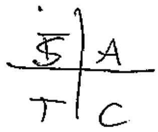
v) $y = 8$ $(x = 1)$.

Ques 9

a) $\tan\theta = \frac{-4}{9}$ $\quad \cos\theta = \cancel{\theta}$

4th quad

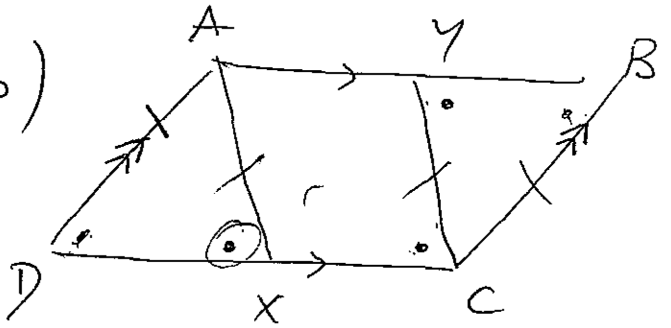




16
81
97

$\therefore \sin\theta = -\frac{4}{\sqrt{97}}$

b)



i) opposite angles of a parallelogram are equal $\therefore A\hat{D}X = C\hat{B}Y$

ii) ~~$C\hat{B}Y = B\hat{Y}C$ (equal $\angle$'s of isos. $\triangle$)~~
~~$Y\hat{C}X = B\hat{Y}C$ (alt. $\angle$'s $AB \parallel CD$)~~
~~$A\hat{X}D =$~~

$BC = AD$ (opp. sides of llogram)
now $BC = YC = AX$ (given)
so $AX = AD$.

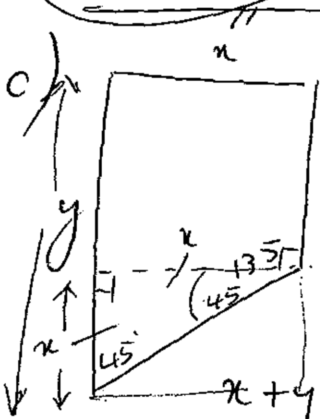
iii) In $\triangle$'s ADC, $\triangle CBY$
$AD = BC$ (opp. sides of llogram)
$AX = YC$ (given)
$D\hat{A}X = Y\hat{C}B$ ($180 - 2$ equal angles)

$\therefore \triangle ADC \equiv \triangle CBY$
(2 sides and included angle test)

iv) $AY \parallel XC$ ($ABCD$ is llogram)
$YC \parallel AX$ ($Y\hat{C}\cancel{A} = A\hat{X}D$ corres. $\angle$'s)

$\therefore AYCX$ is 4 sided figure with 2 pairs of parallel sides
$\therefore$ parallelogram.

Ques 9 cont.

c) 

117m

$x + y = 117$

$\therefore y = 117 - x$

$A = x \times y - \frac{x \times x}{2}$

$= x(117 - x) - \frac{x^2}{2}$

$= 117x - x^2 - \frac{x^2}{2}$

$A = 117x - \frac{3x^2}{2}$

ii) $A' = 117 - \frac{6x}{2}$

st pt. $A' = 0$

$117 - 3x = 0$

$3x = 117$

$x = 39$

$\therefore y = 117 - 39$

$= 78$

$\therefore y = 78 = 2 \times 39$

i.e $y = 2x$

$A'' = -3$

$\therefore$ max area when $x = 39, y = 78$

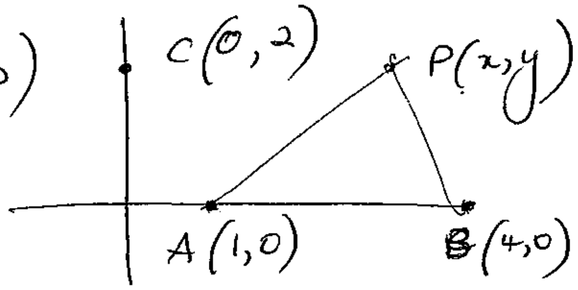
or $y = 2x$.

Ques 10

a) $\dfrac{1-x^2}{x-1} = \dfrac{(1-x)(1+x)}{x-1}$

$= -(1+x)$

b)



Show $CB = 2CA$.

$CB = \sqrt{2^2 + 4^2}$

$= \sqrt{4+16}$

$= \sqrt{20} = 2\sqrt{5}$.

$CA = \sqrt{2^2 + 1^2}$

$= \sqrt{5}$.

$\therefore$ ~~$CA = 2C$~~

$CB = 2CA$.

$(2\sqrt{5} = 2 \times \sqrt{5})$

ii) $PB = \cancel{2}PA$.

$PB = \sqrt{(x-4)^2 + (y-0)^2}$

$PA = \sqrt{(x-1)^2 + (y-0)^2}$

$(x-4)^2 + y^2 = 4((x-1)^2 + y^2$

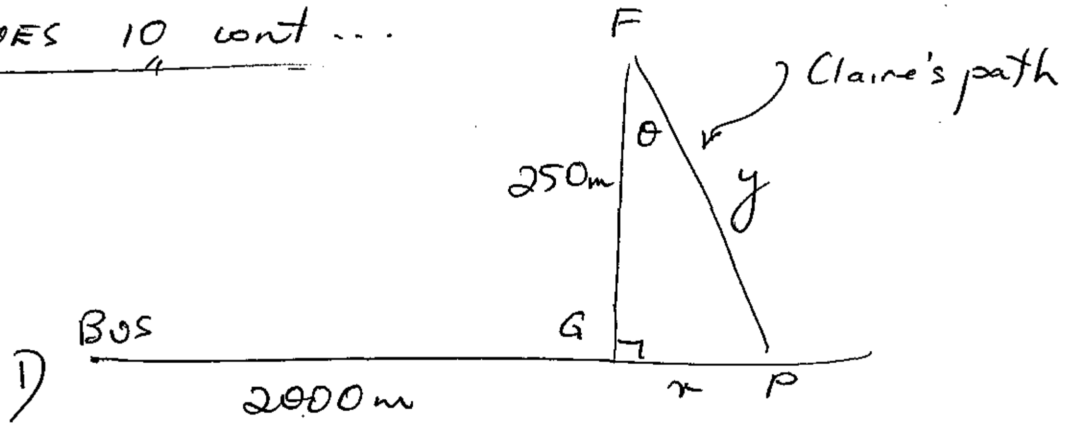
$x^2 - 8x + 16 + y^2 = 4(x^2 - 2x + 1 + y^2)$

$\cancel{x^2} - \cancel{8x} + 16 + y^2 = \cancel{4x^2} - \cancel{8x} + 4 + 4y^2$

$3x^2 + 3y^2 = 12$ circle centre (0,0) -

QUES 10 cont ...

c)



i) $S = \frac{D}{T}$

$T = \frac{D}{S}$

$15\,m/s = 54000\,m/h$

$4\,m/s = 14400\,m/h$

let $GP = x$

now $\tan\theta = \frac{x}{250}$

$x = 250\tan\theta$

also $y^2 = x^2 + 250^2$

$y = \sqrt{x^2 + 62500}$

$= \sqrt{62500\tan^2\theta + 62500}$

$$T_{bus} = \frac{200\cancel{0} + 25\cancel{0}\tan\theta}{5400\cancel{0}}\,h = \frac{4 + \tan\theta}{216} \text{ hours}$$

$$T_{claire} = \frac{\sqrt{62500\tan^2\theta + 62500}}{14400} = \frac{25\cancel{0}\sqrt{\tan^2\theta + 1}}{1440\cancel{0}}$$

$$= \frac{25\sqrt{\sec^2\theta}}{1440}$$

$$= \frac{25\sec\theta}{1440} \text{ hours.}$$

$\therefore T_{bus} = \frac{4 + \tan\theta}{216}$; $T_{claire} = \frac{5\sec\theta}{288}$