Killara HIGH SCHOOL

STUDENT NUMBER

HSC MATHEMATICS
COURSE

ASSESSMENT TASK NO 3
Date: Friday 27/5/11

TIME ALLOWED: *Reading time: 5 minutes*
Working time: 60 minutes

INSTRUCTIONS: This paper consists of 2 questions
ANSWER all 2 questions
Start each question in a new booklet
Marks may not be given for untidy and poorly arranged work.
All relevant working should be shown.

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

OUTCOMES ASSESSED:

A1 - Appreciation of the scope, usefulness, beauty and elegance of mathematics
H1 - seeks to apply mathematical techniques to problems in a wide range of practical contexts
H2 - constructs arguments to prove and justify results
H3 - manipulates algebraic expressions involving logarithmic and exponential functions
H5 - applies appropriate techniques from the study of calculus & trigonometry to solve problems
H8 - uses techniques of integration to calculate areas and volumes
H9 - communicates using mathematical language, notation, diagrams and graphs

	MARKS
QUESTION 1 Trigonometric Functions	/22
QUESTION 2 Exponentials and logarithm Functions	/18
	/40

This assessment task constitutes 15% of the Higher School Certificate Course Assessment.

QUESTION 1: (22 marks) MARKS

a. Differentiate the following with respect to x:

i) $tan\ 3x$ 1

ii) cos^3x 1

iii) $e^{2x}sinx$ 2

b. Find:

i) $\int_0^{\pi} \sin\frac{x}{2}\, dx$. 2

ii). $\int_0^{\frac{\pi}{4}}(2sinx + sin2x)dx$ 2

c. Consider the function $y = 1 + \sqrt{3}\ sinx + cosx.$

i) Find the equation of the tangent to the graph of the function 3

at $x = \frac{5\pi}{6}.$

ii) Find the maximum and minimum values of $1 + \sqrt{3}sinx + cosx$

in the interval $0 \leq x \leq 2\pi$ 3

d. Evaluate $\int_0^{\frac{\pi}{3}} sec^2xdx$ by:

i) direct integration (give answer as exact value). 2

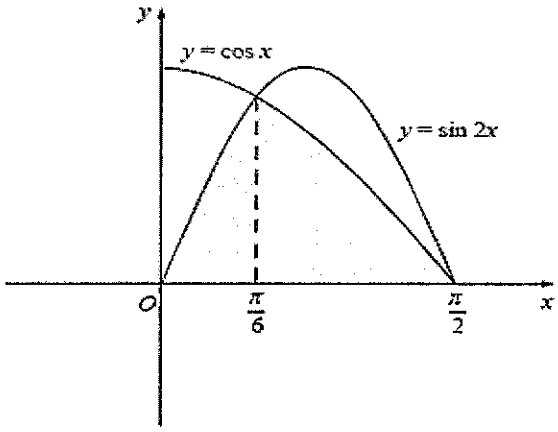
ii) using Simpson's rule using 3 function values(answer correct

to 2 significant values). 2

e. The diagram shows the graphs of the functions $y = cosx$ and $y = sin2x$ 4

between x=0 and $x = \frac{\pi}{2}$. The two graphs intersect at $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$.

Calculate the area of the shaded region.



QUESTION 2: (18 marks) MARKS

a. Differentiate the following with respect to x:

i) e^{5x} 1

ii) $\frac{lnx}{x}$ 2

iii) $log_e(\cos 2x)$ 2

b. Given that $log_a b = 2.75$ and $log_a c = 0.25$, find the value of:

i) $log_a\left(\frac{b}{c}\right)$ 1

ii) $log_a(bc)^3$ 1

c. Evaluate $log_5 12$, correct to 2 decimal places. 2

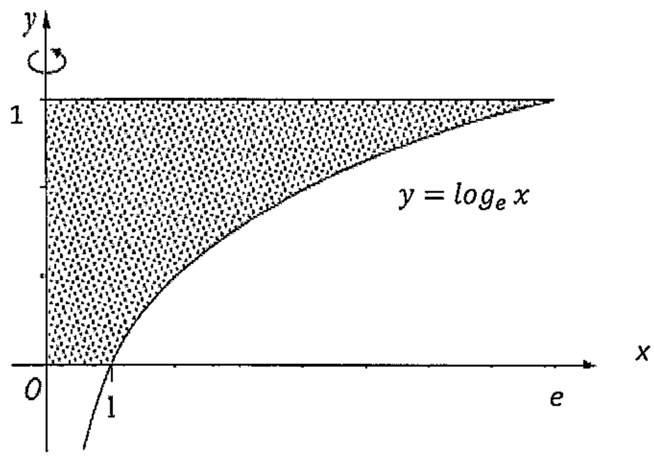
d. Find $\int \frac{x-1}{x^2-2x}dx$. 2

e. Evaluate i) $\int_0^1 e^{4x}dx$. 2

ii) $\int_0^1 (e^{2x} + e^{-x})dx$ 2

f. The diagram shows the graph of $y = log_e x$ between $x = 1$ and $x = e$. 3

The shaded region, bounded by $y = log_e x$, the line $y = 1$, and the x and y axes, is rotated about the y axis to form a solid. Find the volume of this solid.



STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

2011 MATHEMATICS TASK 3 ANSWERS

QUESTION 1

a i $\frac{d\tan 3x}{dx} = 3\sec^2 3x$

1 mark correct answer

ii $\frac{d}{dx}\cos^3 x = -3\sin x\cos^2 x$

1mark correct answer

iii $\frac{d}{dx} e^{2x}.\sin x = 2e^{2x}\sin x + e^{2x}\cos x$

$= e^{2x}(2\sin x + \cos x)$

u'v + v'u 1mark for each correct part.

b i) $\int_0^{\pi} \sin\frac{x}{2}dx = \left[-2\cos\frac{x}{2}\right]_0^{\pi}$

$= -2\left(\cos\frac{\pi}{2} - \cos 0\right)$

$= -2(0-1)$

$= 2$

1mark correct integ ration.

1mark correct answer

ii) $\int_0^{\pi/4} (2\sin x + \sin 2x)\,dx = \left[-2\cos x - \frac{\cos 2x}{2}\right]_0^{\pi/4}$

$= -\left[\left(2\cos\frac{\pi}{4} + \frac{1}{2}\cos\frac{\pi}{2}\right) - \left(2\cos 0 + \frac{\cos 0}{2}\right)\right]$

$= -\left[2\frac{1}{\sqrt{2}} - 0 - \left(2 + \frac{1}{2}\right)\right]$

$= -\sqrt{2} + \frac{5}{2}$

1mark correct integration

1mark correct substitution and answer

c i $y = 1 + \sqrt{3}\sin x + \cos x$

$\frac{dy}{dx} = \sqrt{3}\cos x - \sin x$

at $x = \frac{5\pi}{6}$ $\quad m = \sqrt{3}\cos\frac{5\pi}{6} - \sin\frac{5\pi}{6}$

$= \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{2}$

$= -2$

1mark differentiantion and finding m or y

2marks gradient, y and differentiation

at $x = \frac{5\pi}{6}$ $\quad y = 1 + \sqrt{3}\sin x + \cos x$

$= 1 + \sqrt{3}\sin\frac{5\pi}{6} + \cos\frac{5\pi}{6}$

$= 1 + \sqrt{3}\times\frac{1}{2} - \frac{\sqrt{3}}{2}$

$= 1$

$\therefore y - 1 = -2\left(x - \frac{5\pi}{6}\right) \Rightarrow y = -2x + \frac{5\pi}{3} + 1$

eqn of tangent at $\frac{5\pi}{6} = x$

+

3marks equation of tangent

Question 1 continue

c ii $y' = \sqrt{3}\cos x - \sin x$ | 1 mark $\tan x = \sqrt{3}$

$y' = 0 \quad \sqrt{3}\cos x = \sin x$

$\tan x = \sqrt{3}$, $x = \frac{\pi}{3}, \frac{4\pi}{3}$, turning pts:

$x = \frac{\pi}{3} \quad y = 1 + \sqrt{3}\sin x + \cos x$

$= 1 + \frac{3}{2} + \frac{1}{2}$ | 1 mark turning points

$= 3 \quad \therefore \left(\frac{\pi}{3}, 3\right)$

$x = \frac{4\pi}{3} \quad y = 1 + \sqrt{3}\sin\frac{4\pi}{3} + \cos\frac{4\pi}{3}$

$= 1 + \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)$

$= -1 \quad \therefore \left(\frac{4\pi}{3}, -1\right)$

Nature of turning points

$y'' = -\sqrt{3}\sin x - \cos x$ | 1 mark correctly finding nature of turning points

at $x = \frac{\pi}{3}$

$y'' = -\sqrt{3}\sin\frac{\pi}{3} - \cos\frac{\pi}{3}$

$= -\sqrt{3}\,\frac{\sqrt{3}}{2} - \frac{1}{2}$

$= -2 < 0$ concave down, max point.

at $x = \frac{4\pi}{3} \quad y'' = -\sqrt{3}\sin\frac{4\pi}{3} - \cos\frac{4\pi}{3}$

$y'' = -\sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) - \frac{-1}{2}$

$= 2 > 0$ conc. up. min pt.

$\therefore \left(\frac{\pi}{3}, 3\right)$ max point, $\left(\frac{4\pi}{3}, -1\right)$ min. point.

d i $\int_0^{\pi/3} \sec^2 x\,dx = \left[\tan x\right]_0^{\pi/3}$ | 1 mark correct integration

$= \tan\frac{\pi}{3} - \tan 0$ | 1 mark for $\sqrt{3}$

$= \sqrt{3}$

ii $h = \frac{\pi/3 - 0}{2} = \frac{\pi}{6} \quad \sec^2 x = \frac{1}{\cos^2 x}$ | 1 mark finding h & $\frac{h}{3} = \frac{\pi}{18}$

x	0	$\pi/6$	$\pi/3$
$\sec^2 x$	1	$\frac{4}{3}$	4

d ii continue

$$\int_0^{\pi/3} \sec^2 x\,dx = \frac{\pi/6}{3}\left[\sec^2 0 + 4\sec^2\frac{\pi}{6} + \sec^2\frac{\pi}{3}\right]$$

$$= \frac{\pi}{18}\left[1 + 4\times\frac{4}{3} + 4\right]$$

$$= 1.8$$

1 mark correct substitution and ~~1 mark~~ correct answer

e $$A = \int_0^{\pi/6} \sin 2x\,dx + \int_{\pi/6}^{\pi/2} \cos x\,dx$$

$$= \left[-\frac{\cos 2x}{2}\right]_0^{\pi/6} + \left[\sin x\right]_{\pi/6}^{\pi/2}$$

$$= -\frac{1}{2}\left[\cos\frac{\pi}{3} - \cos 0\right] + \left[\sin\frac{\pi}{2} - \sin\frac{\pi}{6}\right]$$

$$= -\frac{1}{2}\left[\frac{1}{2} - 1\right] + \left[1 - \frac{1}{2}\right] \Rightarrow -\frac{1}{2}\times -\frac{1}{2} + \frac{1}{2}$$

$$= \frac{3}{4}\ \text{unit}^2$$

2 mark correct boundaries and functions

1 mark correct integration

1 mark correct working out and answer

QUESTION 2

a i) $\frac{d}{dx} e^{5x} = 5e^{5x}$

ii) $\frac{d}{dx}\frac{\ln x}{x} = \frac{\frac{1}{x}\cdot x - \ln x}{x^2}$

$$= \frac{1 - \ln x}{x^2}$$

iii) $\frac{d}{dx}\log_e \cos 2x = \frac{-\sin 2x \cdot 2}{\cos 2x}$

$$= -2\tan 2x$$

b) i $\log_a \frac{b}{c} = \log_a b - \log_a c$

$$= 2.75 - 0.25$$

$$= 2.50$$

ii) $\log_a (bc)^3 = 3\log_a bc$

$$= 3[\log_a b + \log_a c)$$

$$= 3[2.75 + 0.25)$$

$$= 9$$

c) $\log_5 12 = \frac{\log_e 12}{\log_e 5}$

$$= 1.54$$

Question 2 continue

d) $\int \frac{x-1}{x^2-2x}dx$

$f(x)=x^2-2x \quad f'(x)=2x-2$

$\int \frac{2(x-1)}{2(x^2-2x)}dx = \frac{1}{2}\int \frac{2x-2}{x^2-2x}dx$ ✓

$=\frac{1}{2}\ln(x^2-2x) + c$ ✓

(you can ignore c)

e) i $\int_0^1 e^{4x}dx = \left[\frac{e^{4x}}{4}\right]_0^1$

$=\frac{1}{4}\left[e^4-1\right]$

ii) $\int_0^1 (e^{2x}+e^{-x})dx =$

$=\left[e^{2x}\frac{1}{2}+-e^{-x}\right]_0^1$

$=\left[\frac{e^2}{2}-e^{-1}\right]-\left[\frac{e^0}{2}-e^0\right]$

$=\frac{e^2}{2}-\frac{1}{e}-\frac{1}{2}+1$

$=\frac{e^3+e-2}{2e}$

f) $y=\log_e x$

$e^y = e^{\log_e x}$

$x=e^y$

$V=\pi\int_0^1 x^2dy$

$=\pi\int_0^1 e^{2y}dy$

$=\pi\left[\frac{e^{2y}}{2}\right]_0^1$

$=\frac{\pi}{2}\left[e^2-e^0\right]$

$=\frac{\pi}{2}\left[e^2-1\right]$ unit^3