Killara
HIGH SCHOOL

STUDENT NUMBER

MATHEMATICS

COURSE

ASSESSMENT TASK NO 2
2011

TIME ALLOWED: *Reading time: 5 minutes*
Working time: 3 hours

INSTRUCTIONS: This paper consists of 10 questions
ANSWER all 10 questions
Start each question in a new booklet
Marks may not be given for untidy and poorly arranged work.
All relevant working should be shown.

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

OUTCOMES ASSESSED:

H1 - Seeks to apply mathematical techniques to problems in a wide range of practical contexts
H2 - Constructs arguments to prove and justify results
H4 - Expresses practical problems in mathematical terms based on simple given models
H5 - Applies appropriate techniques from the study of calculus and trigonometry to solve problems
H6 - Uses the derivative to determine the features of the graph of a function
H7 - Uses the features of a graph to deduce information about the derivative
H8 - Uses techniques of integration to calculate areas and volumes
H9 - Communicates using mathematical language, notation, diagrams and graphs

This assessment task constitutes 25 % of the Higher School Certificate Course Assessment.

Total marks – 120
Attempt Questions 1 – 10
All questions are of equal value

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing Booklet.

(a) Evaluate $\sqrt{\frac{3.37^3-2.13}{14.67\times10^3}}$ write your answer in scientific correct to 3 significant figures. **2**

(b) Solve for x $1-3x\le 7$ **2**

(c) Solve $x+\frac{2}{x}=\frac{9}{2}$ **2**

(d) Evaluate $3^{2x}\times 9^{1-x}$ **2**

(e) Rationalise the denominator $\frac{2}{2\sqrt{5}+3}$ **2**

(f) Simplify, write your answer with positive index $\left(\frac{1}{3a^{-2}}\right)^2$ **2**

Marks

Question 2 (12 marks) Use a SEPARATE writing Booklet.

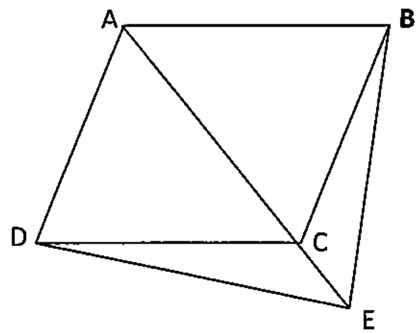
(a) Completely factorise $a^2 - b^2 + a + b$ **2**

(b) Solve $|2x - 3| = 1$ **2**

(c) Evaluate $\lim_{x \to 2} \frac{x^2+x-6}{x-2}$ **1**

(d) Solve $a = a^3$ **2**

(e) ABCD is a rhombus



NOT TO SCALE

(i) State why $\angle ACD = \angle ACB$ **1**

(ii) Prove that $\triangle DCE \equiv \triangle BCE$ **3**

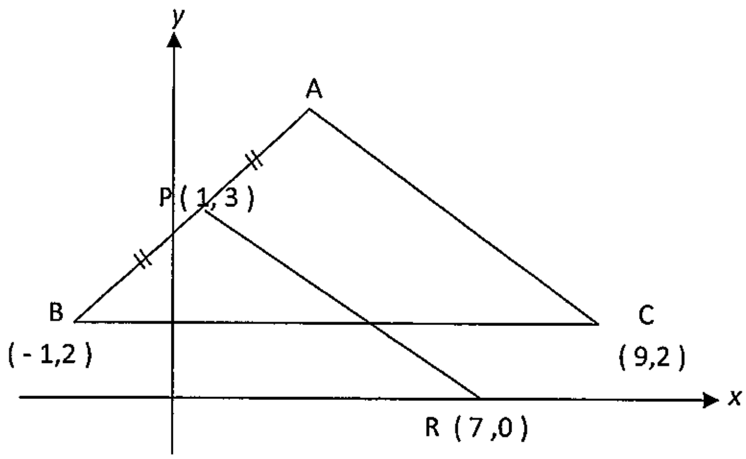
(iii) State why DE = EB **1**

Marks

Question 3 (12 marks) Use a SEPARATE writing Booklet.

(a) Given that the distance from A(- 1, b) to B(2, 4) is $\sqrt{18}$ units. Find one possible value of b. **2**

(b) The points B and C have coordinates (- 1, 2) and (7, 2), and P is the midpoint of AB and has coordinates (1, 3).



NOT TO SCALE

(i) Show that coordinates of A are (3, 4). **2**

(ii) Write the equation of line BC. **1**

(iii) Show that equation of PR is $x + 2y - 7 = 0$ **2**

(iv) PR cuts y-axis at E. Find the coordinates of E . **1**

(v) Explain how you know AC is not parallel to PR. **2**

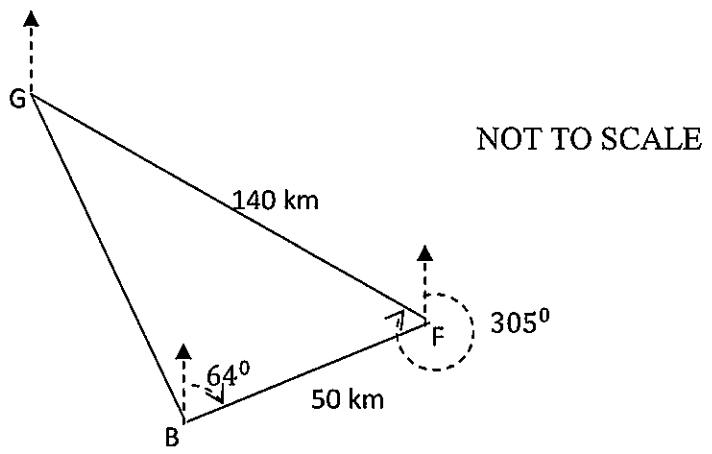
(c) l_1 is the line $2x + y + 2 = 0$ and l_2 is the line $3x + ky - 1 = 0$. Find the value of k when l_1 is perpendicular to l_2 . **2**

Marks

Question 4 (12 marks) Use a SEPARATE writing Booklet.

(a) Write exact value of $sin210^0$ **1**

(b) 'Elwis' the helicopter flies 50 km from Blue Lake 'B' to Forest Hill 'F' on a bearing of 64^0, to dump its water and then flies 140 km from Forest Hill 'F' to Gullatown 'G' on a bearing of 305^0.



(i) Show that $\angle$BFG = 61^0 **2**

(ii) Find, to the nearest km, the distance from B to G. **2**

(iii) Use the sine rule to find $\angle$BGF and hence find the bearing of Blue Lake 'B' from Gullatown 'G', correct to the nearest degree. **3**

(c) Prove the following identity $1 + sec^2\alpha - tan^2\alpha = 2$ **2**

(d) Solve $\sqrt{3}cos\theta = sin\theta$ to find the value of θ, for $0 \leq \theta \leq 2\pi$. *2*

Marks

Question 5 (12 marks) Use a SEPARATE writing Booklet.

(a) Factorise $3a^3 - 24$ **2**

(b) Find the values of k for which quadratic function $f(x) = 3x^2 - 4x + (k + 1)$ have 2 real roots. **2**

(c) The roots of the quadratic equation $2x^2 - 4x + 1$ are α and β . **3**

Find $\frac{1}{\alpha} + \frac{1}{\beta}$.

(d) Express $2x^2 + 9x + 4$ in the form : $A(x + 3)^2 + B(x + 3) + C$ **2**

(e) Find the equation of the normal to the parabola $y = 3 - x - x^2$, where it crosses the y – axis. **3**

Marks

Question 6 (12 marks) Use a SEPARATE writing Booklet.

(a) Differentiate with respect to x.

(i) $4x^2(3-6x)^2$ **2**

(ii) $2\sqrt{x}+\frac{1}{x^2}$ **2**

(iii) $\dfrac{x^2}{3x-1}$ **2**

(b) Show that $f(x)=(x-1)^3$ is always increasing. **1**

(c) For what values of x, is $y=3x-x^3$ increasing ? **2**

(d) Solve $2sin^2x-3sinx+1=0$ for $0\leq x\leq 360^0$. **3**

Marks

Question 7 (12 marks) Use a SEPARATE writing Booklet.

(a) (i) Sketch the function $f(x)$ defined by the rule **2**

$$f(x) = \begin{cases} x - 1, & x \leq 1 \\ (x-1)^2, & x > 1 \end{cases}$$

(ii) Evaluate $f(2) + f(0)$ **1**

(b) Write the domain of the function $y = \frac{1}{\sqrt{2-x}}$ **1**

(c) For the function $f(x) = x^3 - 3x^2 + 5$,

(i) Find the coordinates of the stationary points and determine their nature **3**

(ii) Explain why there is a point of inflexion at $x = 1$. **2**

(iii) Sketch a neat graph of the function, indicating all important features. **2**

(iv) What is the minimum value of the function in the domain $1 < x < 3$ **1**

Marks

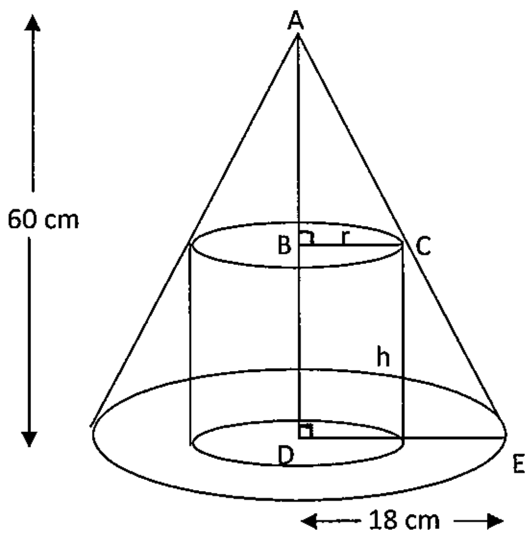
Question 8 (12 marks) Use a SEPARATE writing Booklet.

(a) Find the primitive function of

(i) $(x+2)(x^2-3)$ **2**

(ii) $\dfrac{x^3-5x^2+x}{x}$ **2**

(b) A cylinder of height h cm and radius r cm is enclosed in a cone of height 60cm and radius 18 cm.



(i) Prove $\triangle$ ABC $|||$ $\triangle$ ADE **2**

(ii) By using the ratios of the corresponding sides, show that $h = 60 - \dfrac{10}{3}r$. **2**

(iii) Show that the volume of the cylinder is given by $V = 60\pi r^2 - \dfrac{10}{3}\pi r^3$. **2**

(iv) Find the value of *r* for which the volume of the cylinder is maximum. **2**

You must give reasons why your value of *r* gives the maximum volume.

Marks

Question 9 (12 marks) Use a SEPARATE writing Booklet.

(a) Evaluate $\int_0^1 (\sqrt{x} - 3x^2)dx$ **2**

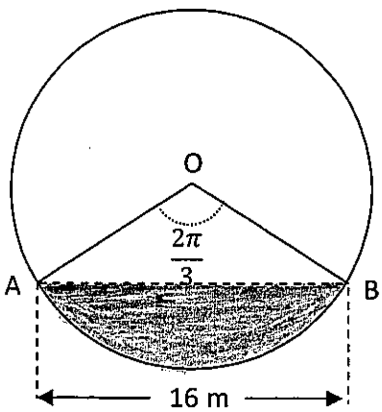
(b) For a certain curve, $\frac{d^2y}{dx^2} = 2x$. The point (3,6) lies on to the curve and its tangent at this point is inclined at 45^0 to the positive direction of x – axis. Find the equation of the curve. **3**

(c) Consider the function $y = \frac{1}{x^2}$. Complete the table below:

x	1	1.5	2	2.5	3	3.5	4
$\frac{1}{x^2}$							

Use Simpson's Rule to approximate $\int_1^4 f(x)dx$. **2**

(d) The surface of the water in a horizontal pipe is 16 m wide and subtends an angle of $\frac{2\pi}{3}$ at the centre of the pipe.



Find (i) the distance from the centre of the pipe to the water surface. **2**

(ii) the value of the cross sectional area above the water. **3**

Marks

Question 10 (12 marks) Use a SEPARATE writing Booklet.

(a) Sketch $y = |x| - 1$ **2**

(b) Find the points of intersection of $y = x^2 + 1$ and $y = x + 3$. **2**

(i) Hence find the area between the two curves. **2**

(ii) Find the volume of the solid generated, when the region in the first quadrant bounded by the curve $y = x^2 + 1$, the line $y = x + 3$ and $y - axis$ is rotated about $y - axis$. **3**

(c) Sketch $y = 3sin2x - 1$ for $0 \leq x \leq \pi$, clearly marking important features. Write the amplitude and period. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Mathematics 2011 Half Yearly Assessment

Question 1

Award:

a) 4.96×10^{-2}

2 — 2 for correct answer; 1 correct rounding incorrect answer

b) $1-3x \leq 7$

$-3x \leq 6$

$x \geq -2$

2 — 2 for correct answer; 1 for $-3x \leq 6$

c) $x + \frac{2}{x} = \frac{9}{2}$

$\frac{2x^2+4}{2x} = \frac{9x}{2x}$

$2x^2 - 9x + 4 = 0$

$x^2 - 8x - x + 4 = 0$

$(2x(x-4) - (x-4) = 0$

$(2x-1)(x-4) = 0$

$x = \frac{1}{2}, 4$

2 — 2 for correct answer; 1 for equating the denom $2x^2 - 9x + 4 = 0$

d) $3^{2x} \times 9^{1-x} = 9^x \times 9^{1-x}$

$= 9$

OR $3^{2x} \times 3^{2-2x} = 3^2 = 9$

2 — 2 for correct answer; 1 for changing to the same base

e) $\frac{2}{2\sqrt{5}+3} \times \frac{2\sqrt{5}-3}{2\sqrt{5}-3} = \frac{4\sqrt{5}-6}{20-9}$

$= \frac{4\sqrt{5}-6}{11}$

2 — 2 for correct answer; 1 for rationalising the denominator

f) $\left(\frac{1}{3a^{-2}}\right)^2 = \left(\frac{a^2}{3}\right)^2$

$= \frac{a^4}{9}$

2 — 2 for correct answer; 1 for $\left(\frac{a^2}{3}\right)^2$

Question 2

Award:

a) $a^2 - b^2 + a + b = (a-b)(a+b) + (a+b)$

$= (a+b)(a-b+1)$

2 — 2 mark for the correct answer; 1 mark for factorising a^2-b^2

b) $|2x-3| = 1$

$2x-3=1$ $\qquad$ $2x-3=-1$

$2x=4$ $\qquad$ $2x=2$

$x=2$ $\qquad$ $x=1$

2 — 1 for each correct solution

c) $\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \lim_{x\to 2} \frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}}$

$= \lim_{x\to 2} (x+3)$

$= 5$

1 — 1 for correct answer

d) $a = a^3$

$a^3 - a = 0$

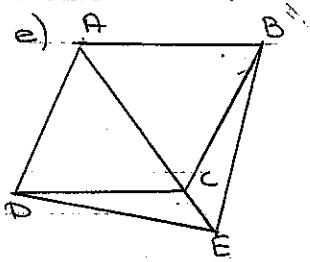
$a(a^2-1) = 0$

$a=0 \qquad a^2=1$

$a = \mp 1$

2 — 2 for 3 solutions; 1 for $a = \mp 1$; 1 for $a=0, a=1$; 0 for $a=1$; 0 for $a=0$

e)



i) $\angle ACD = \angle ACB$ (Diagonal bisect angle at the vertex) — 1 — 1 for correct comment

ii) $DC = BC$ (Equal sides of rhombus) — 3 — 3 marks for full solution; 2 for 2 reasons (1 for angle + 1 for any other)

$\angle DCE = 180° - \angle ACD$ (Supplem. angles)

$\angle BCE = 180° - \angle BCA$ (Supplemen. angles)

$\therefore \angle DCE = \angle BCE$ (as $\angle ACD = \angle ACB$)

CE common

$\triangle DCE \equiv \triangle BCE$ (SAS)

iii) $DE = EB$ (Corresponding sides of congruent triangles) — 1 — 1 for correct comment

Question 3

a) $A(-1,b)$, $B(2,4)$, $d=\sqrt{18}$

$18=(2--1)^2+(4-b)^2$

$18=9+16-8b+b^2$

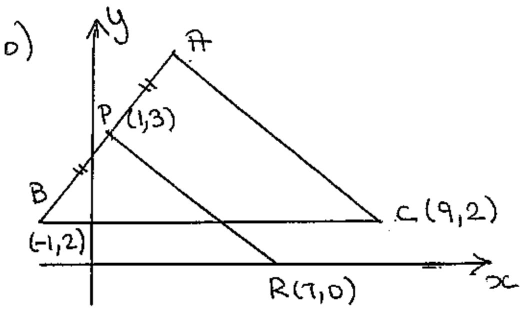
$b^2-8b+7=0$

$(b-7)(b-1)=0$

$b=1$ or 7

2 — Award: 2 for correct answ

1 for the substitutio correct

b) 

i) $\frac{x-1}{2}=1$ $\qquad$ $\frac{y+2}{2}=3$

$x-1=2$ $\qquad$ $y+2=6$

$x=3$ $\qquad$ $y=4$

$\therefore A(3,4)$

2 — 1 for each solutio

ii) $y=2$

1 — 1 for $y=2$

iii) $P(1,3)$, $R(7,0)$

$m=\frac{3-0}{1-7}$

$=-\frac{1}{2}$

$y-0=-\frac{1}{2}(x-7)$

$2y=-x+7$

$x+2y-7=0$ (Equation of PR)

2 — 2 for the correct equation of the line.

1 for finding gradient and substituting into $y-y_1=m(x-x_1)$

Question 3 continue

b iv) PR $x+2y-7=0$

y-intercept when $x=0$

$2y=7$ $\quad$ $y=3\frac{1}{2}$, $E(0,3\frac{1}{2})$

1 — Award: 1 for ycoordinate $3\frac{1}{2}$

v) $m_{AC}=\frac{2-4}{9-3}$

$=\frac{-2}{6}$

$=-\frac{1}{3}$

$m_{PR}=-\frac{1}{2}$ $\qquad$ $m_{PR}\neq m_{AC}$

$\therefore$ AC is not parallel to PR

2 — 2 for showing tha gradients are not equal

1 Finding gradient of PR

c) L_1 $2x+y+2=0$ $\qquad$ L_2 $3x+ky-1=0$

$y=-2x-2$ $\qquad$ $ky=-3x+1$

$m=-2$ $\qquad$ $y=-\frac{3}{k}x+1$

For perpendicular lines $m_1\times m_2=-1$

L_1 $m_1=-2$ $\qquad$ L_2 $m_2=-\frac{3}{k}$

$-2\times-\frac{3}{k}=-1$ $\qquad$ $\frac{6}{k}=-1$

$k=-6$

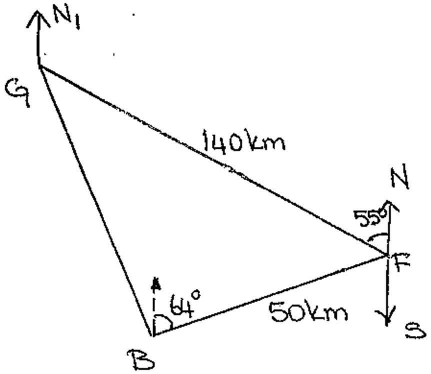
2 — 2 for finding $k=-6$

1 for finding the gradient of the lines.

Question 4

		Award
a) $\sin 210^\circ = \sin(180^\circ + 30^\circ)$ $\sin 30^\circ = \frac{1}{2}$	1	1 for $-\frac{1}{2}$
3rd Quadrant $\sin 210^\circ = -\frac{1}{2}$		

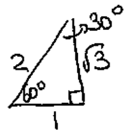
b)



i) $\angle BFS = 64^\circ$ (alternate angles) $\angle NFG = 360^\circ - 305^\circ$ (Angles at a point) $= 55^\circ$	2	2 for correct solutio. for correct
$\angle GFB = 180^\circ - 64^\circ - 55^\circ$ (Supplem. angles) $= 61^\circ$		1 for
ii) $BG^2 = 140^2 + 50^2 - 2 \times 140 \times 50 \times \cos 61^\circ$ $= 15\,312.7$ $BG = 123.7\text{km} \approx 124\text{km}$	2	2 For correct rule and working. 1 for using the rule and correct substitution
iii) $\frac{\sin \widehat{BGF}}{50} = \frac{\sin 61^\circ}{124\text{ km}}$ $\sin B\hat{G}F = \frac{\sin 61^\circ}{124} \times 50$ $= 0.3527$ $\angle BGF = 20.7^\circ$ $(20^\circ 39')$		3 for correct substitution, angle and bearing 2. Using sin rule and finding the angle in degrees

Question 4 continue

		Award
b iii) $\angle N_1GF = 180^\circ - 55^\circ$ (Co-interior angles) $= 125^\circ$ Bearing of B from G: $125^\circ + 21^\circ$ $= 146^\circ$ T		1 for using sin rule and substitution
c) $1 + \sec^2\alpha - \tan^2\alpha = 2$ $LHS = 1 + \sec^2\alpha - \tan^2\alpha$ $= 1 + \frac{1}{\cos^2\alpha} - \frac{\sin^2\alpha}{\cos^2\alpha}$ $= \frac{\cos^2\alpha + (1 - \sin^2\alpha)}{\cos^2\alpha}$ $(\sin^2\alpha + \cos^2\alpha = 1)$ $(\cos^2\alpha = 1 - \sin^2\alpha)$ $= \frac{\cos^2\alpha + \cos^2\alpha}{\cos^2\alpha}$ $= \frac{2\cos^2\alpha}{\cos^2\alpha}$ $= 2$ LHS = RHS $\therefore$ $1 + \sec^2\alpha - \tan^2\alpha = 2$	2	2 for correct working substitution and answer 1 toward halfway solving
d) $\sqrt{3}\cos\theta = \sin\theta$ $\sqrt{3} = \frac{\sin\theta}{\cos\theta}$ $\tan\theta = \sqrt{3}$ $\tan 60^\circ = \sqrt{3}$ $\therefore 60^\circ = \frac{\pi}{3}$ In the domain $0 \leq \theta \leq 2\pi$ Tangent is positive in first and third quadr. $\therefore$ $60^\circ, 240^\circ$ $\frac{\pi}{3}, \pi + \frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}, \frac{4\pi}{3}$	2	2 for finding both angles 1 for finding 1 angle.



Question 5

a) $3a^3-24=3(a^3-8)$

$=3(a-2)(a^2+2a+4)$

	Award
2	2 for full factorising
	1 for $3(a-2)(\ldots\ldots)$

b) $f(x)=3x^2-4x+(k+1)$

For 2 distinct real roots $\Delta>0$

$\Delta=b^2-4ac$

$=(-4)^2-4(3)(k+1)$

$=16-12k-12$

$=4-12k$

$\Delta>0 \quad 4-12k>0$

$-12k>-4$

$k<\frac{1}{3}$

2 — 2 for finding correct value of k from correct working

1 for finding $4-12k>$

c) $y=2x^2-4x+1$

$\alpha+\beta=-\frac{b}{a}$ $\quad$ $\alpha\beta=\frac{c}{a}$

$=-\frac{-4}{2}$ $\quad$ $=\frac{1}{2}$

$=2$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}$

$=\frac{2}{\frac{1}{2}}$

$\frac{1}{\alpha}+\frac{1}{\beta}=4$

3 — 3 for finding $\alpha+\beta$, $\alpha\beta$ and answer

2 for finding $\alpha+\beta$, $\alpha\beta$ substitution

1 for finding $\alpha+\beta$, $\alpha\beta$

d) $2x^2+9x+4\equiv A(x+3)^2+B(x+3)+C$

RHS $=A(x^2+6x+9)+Bx+3B+C$

$=Ax^2+6Ax+9A+Bx+3B+C$

$=Ax^2+(6A+B)x+9A+3B+C$

2 — 2 For finding A, B, C through correct working.

1 For for writing * (next page)

Question 5 continue.

d) continue

Equate the coefficients.

$2x^2+9x+4\equiv Ax^2+(6A+B)x+9A+3B+C$ *

$A=2$ (1)

$6A+B=9$ (2)

$9A+3B+C=4$ (3)

sub $A=2$ into (2)

$6\times2+B=9 \quad B=-3$

Sub $A=2$, $B=-3$ into (3)

$9\times2+3\times(-3)+C=4$

$18-9+C=4$

$C=-5$

$\therefore 2x^2+9x+4=2(x+3)^2-3(x+3)-5$

1 mark up to *

e) $y=3-x-x^2$

y-intercept when $x=0$ $y=3$

$y'=-1-2x$ when $x=0$ $y'=-1$

Gradient of tangent $m_t=-1$

Gradient of normal $m_1\times m_2=-1$

$\therefore m_N=1$

Point-gradient formula

$(0,3)$, $m=1$

$y-3=1(x-0)$

$y-3=x$

$y=x+3$ Equation of normal

3 for finding equation of normal through correct working

2 finding gradient of normal

1 for y-intercept &

Question 6

a i) $\frac{d}{dx} 4x^2(3-6x)^2 = (3-6x)^2 \cdot 8x + 4x^2 \times 2(3-6x) \times -6$

$= 8x(3-6x)^2 - 48x^2(3-6x)$

$= 8x(3-6x)[(3-6x)-6x]$

$= 8x(3-6x)(3-12x)$

$= 72x(1-2x)(1-4x)$

Award:

2 — 2 for full correct differentiation

1 for first correct line

ii) $\frac{d}{dx}\left(2\sqrt{x} + \frac{1}{x^2}\right) = \frac{d}{dx}\left(2x^{\frac{1}{2}} + x^{-2}\right)$

$= 2 \times \frac{1}{2} x^{-\frac{1}{2}} + (-2)x^{-3}$

$= \frac{1}{\sqrt{x}} - \frac{2}{x^3}$

2 — 2 for full correct differentiation

1 for $x^{-\frac{1}{2}}$ or $-2x^{-3}$

iii) $\frac{d}{dx} \frac{x^2}{3x-1} = \frac{2x(3x-1) - x^2 \times 3}{(3x-1)^2}$ *

$= \frac{6x^2 - 2x - 3x^2}{(3x-1)^2}$

$= \frac{3x^2 - 2x}{(3x-1)^2}$

2 — 2 for correct differentiation & answer

1 for 1st line *

b) $f(x) = (x-1)^3$

$f'(x) = 3(x-1)^2$

$f'(x) > 0$ $\therefore$ curve $(x-1)^3$ is always increasing

as $(x-1)^2 \geqslant 0$

1 — 1 for correct reasoning as $(x-1)^2 \geqslant 0$

c) $y = 3x - x^3$

$y' = 3 - 3x^2$

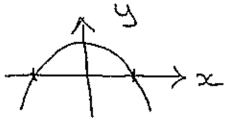
$y' > 0$ $\quad 3 - 3x^2 > 0$

$3(1-x^2) > 0$

$1 - x^2 > 0$

For solution $1 - x^2 = 0$

$x = \mp 1$



Check $x = 0$ $\quad 1 - 0^2 > 0$

$1 > 0$ Yes

$\therefore -1 < x < 1$

2 — 2 for finding $-1 < x < 1$

1 for $y' > 0$ & $3 - 3x^2 > 0$

Question 6 continue

d) $2\sin^2 x - 3\sin x + 1 = 0$

Let $a = \sin x$

$2a^2 - 3a + 1 = 0$

$2a^2 - 2a - a + 1 = 0$

$2a(a-1) - (a-1) = 0$

$(2a-1)(a-1) = 0$

$a = \frac{1}{2}$ $\quad a = 1$

$\therefore \sin x = \frac{1}{2}$, $\sin x = 1$

sin is positive in 1st & 2nd quadrant

$\sin x = \frac{1}{2}$ $\qquad \sin x = 1$

$\sin x = \sin 30$ $\qquad \sin 90° = 1$

$\therefore x = 30°, 180° - 30°$

$\therefore x = 30°, 90°, 150°$

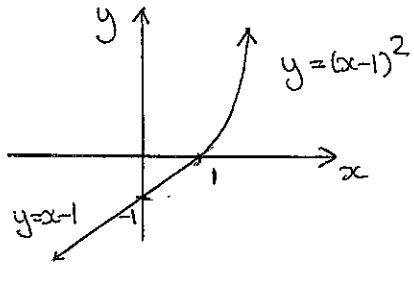
Award

2 — 2 for correct answer

1 for $30°, 90°$

Question 7

a) i) $f(x) = \begin{cases} x-1 & x \leqslant 1 \\ (x-1)^2 & x > 1 \end{cases}$



2 — For correct drawing of $y = x-1$, $y = (x-1)^2$ in the correct domain

1 for drawing 1 graph

ii) $f(2) + f(0) = (2-1)^2 + (0-1)$

$= 1 - 1$

$= 0$

1 — 1 mark for correct answer

Question 7

Award

b) $y=\frac{1}{\sqrt{2-x}}$ $\quad 2-x>0$

$-x>-2$

$x<2$

1 — 1 for $x<2$

c) $f(x)=x^3-3x^2+5$

i) Stationary points when $y'=0$

$f'(x)=3x^2-6x$

$f'(x)=0 \quad 3x^2-6x=0$

$3x(x-2)=0$

$3x=0 \qquad x-2=0$

$x=0 \qquad x=2$

$\therefore x=0 \qquad x=2 \quad y=8-3\times4+5$

$y=5 \qquad\qquad =1$

$\therefore$ Stationary points $(0,5)$, $(2,1)$

Nature of stationary points:

$y''=f''(x)=6x-6$

$x=0 \quad y''=-6$ concave down maximum pt

$x=2 \quad y''=6$ concave up minimum pt

OR a table.

3 — 3 for finding the stationary points (correctly) & determining the natu

2 for finding the stationary points

1 for $3x^2-6x=0$

ii) $y''=0$ when $6x-6=0 \quad x=1$

Check concavity at $x=1$

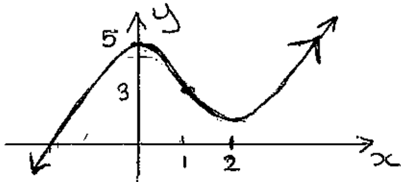
x	0.5	1	1.5
$f''(x)$	$-$	0	$+$
	⌒	—	⌣

Concavity changes $x=1, y=3 \therefore (1,3)$ pt of inflexion

2 — 2 for finding points and determining nature

1 for finding the pt of inflexion

iii)



2 — 2 for correct drawing and critical points marked

1 for correct drawing

iv) Minimum value when $x=2$, $y=3$.

1 — 1 for $y=3$

Question 8

Award

a) i) $\frac{dy}{dx}=(x+2)(x^2-3)$

$=x^3+2x^2-3x-6$

$y=\frac{x^4}{4}+2\frac{x^3}{3}-3\frac{x^2}{2}-6x+C$

2 — 2 for correct answ with "c"

1 for answer withou "c"

ii) $\frac{dy}{dx}=\frac{x^3-5x^2+x}{x}$

$=x^2-5x+1$

$y=\frac{x^3}{3}-5\frac{x^2}{2}+x+C$

2 — 2 for correct answer

1 for answer withou "c"

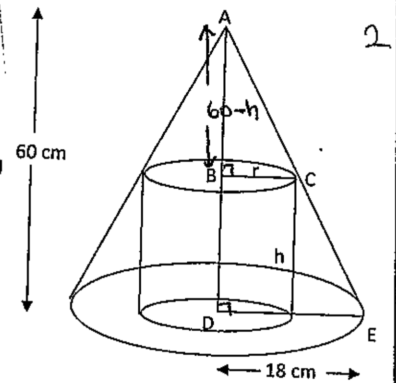
b) i

$\angle ABC=\angle ADE=90°$ Given

$\angle BAC=\angle DAE$ Common

$\angle ACB=\angle AED$ (Angle sum of a Δ)

$\therefore \Delta ABC \quad \Delta ADE$ (AAA)



2 — 2 for full proof

ii) In similar triangles sides are in the same ratio.

$\frac{60-h}{60}=\frac{r}{18}$

$60-h=60\frac{r}{18}$

$60-\frac{10}{3}r=h$

OR $h=60-\frac{10}{3}r$

2 — 2 for full working

1 for writing the ratio

Question 8 continue | Award

b) iii) $V = \pi r^2 h$.

$V = \pi r^2\left[60 - \frac{10}{3}r\right]$

$V = 60\pi r^2 - \frac{10}{3}\pi r^3$

2 | 2 for correct working for correct answer

1 Correct substitution

iv) For maximum volume $V' = 0$.

$\frac{dV}{dr} = 120\pi r - 10\pi r^2$

$\frac{dV}{dr} = 0 \quad 120\pi r - 10\pi r^2 = 0$

$10\pi r(12 - r) = 0$

$r = 0 \quad r = 12$

$r = 0$ not a solution.

Check the nature of the turning point

r	10	12	13
V'	+ 628.3	0	− 408.1

$r = 12$ is a maximum turning point

$\therefore$ $r = 12$ Volume is maximum

2 | 2 for finding $r = 12$ determining the nature.

1 for finding $r = 12$

Accurate values of V' for $r = 10, 12, 13$ is a MUST.

Question 9

a) i) $\int_0^1 (\sqrt{x} - 3x^2)dx = \int_0^1 (x^{\frac{1}{2}} - 3x^2)dx$

$= \left[\frac{2x^{\frac{3}{2}}}{3} - \frac{\cancel{3}x^3}{\cancel{3}}\right]_0^1$

$= \left[\frac{2}{3}1 - 1\right] - 0$

$= -\frac{1}{3}$

2 | 2 for correct solution

1. for integrating

Question 9 continue | Award

b) $\frac{d^2y}{dx^2} = 2x \quad \frac{dy}{dx} = 2\frac{x^2}{2} + C$

$\tan 45 = m = 1$

$\therefore 1 = x^2 + c$ at $(3, 6)$

$x = 3 \quad 1 = 9 + c \quad c = -8$

$\therefore y' = x^2 - 8$

$y = \frac{x^3}{3} - 8x + C_1$

Sub $(3,6)$ $6 = \frac{27}{3} - 24 + C_1 \Rightarrow C_1 = 21$

Equation of the curve $y = \frac{x^3}{3} - 8x + 21$

3 | 3 marks for correct solution

2 for $y' = x^2 - 8$ finding $m = 1$ & substituting

1 for differentiating $y' = x^2 + c$

c)

x	1	1.5	2	2.5	3.0	3.5	4
$\frac{1}{x^2}$	1	0.4	0.25	0.16	0.11	0.08	0.0625
OR	1	$\frac{4}{9}$	$\frac{1}{4}$	$\frac{4}{25}$	$\frac{1}{9}$	$\frac{4}{49}$	$\frac{1}{16}$

$h = 0.5$

$\int_1^4 \frac{1}{x^2}dx = \frac{0.5}{3}\left[1 + 4\times0.4 + 2\times0.25 + 4\times0.16 \cdots + 2\times0.1 + 4\times0.08 + 0.0625\right]$

$= 0.72$ square units

2 | 2 for correct answer

1 for completing the table and $h = 0.5$

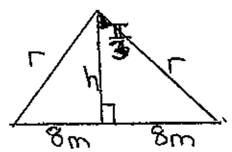
d) i) $\frac{\pi}{3} = 60°$

$\tan 60° = \frac{8}{h}$

$h = \frac{8}{\tan 60°}$

$h = 4.62\text{m} \Leftarrow \left[h = \frac{8}{\sqrt{3}}\text{m}\right]$

(evaluate this)

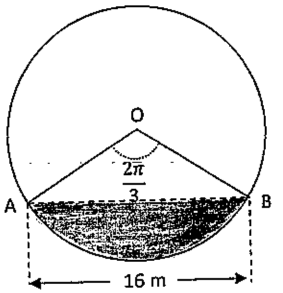


2 | 2 Correct solution

1 writing $\tan 60° = \frac{8}{h}$

Question 9 continue

d ii) $\triangle AOB$



$A_{\triangle AOB} = \frac{1}{2}ab\sin C$

$\sin 60^\circ = \frac{8}{r}$

$r = \frac{8}{\sin 60^\circ}$

$= 9.2\,m$

$A_{\triangle AOB} = \frac{1}{2} \times 9.2^2 \times \sin 60^\circ$

$= 36.95\,m^2$

[OR $A_{\triangle AOB} = \frac{1}{2} \times 16 \times \cancel{4.6}$

$= 33.6\,m^2$]

Area of major sector

$A = \frac{\theta r^2}{2}$ $\quad \theta = \frac{4\pi}{3}$

$A = \frac{4\pi \times 9.2^2}{3 \times 2}$

$= 177.3\,m^2$

$A_{crosssection} = 177.3 + 36.95$
$= 214.2\,m^2$

3 | 3 for correct solution for correct answer

2 for finding area of $\triangle AOB$ [correctly finding r and substituting]

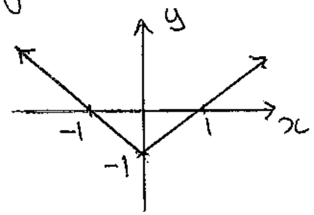
2 for finding $A_{minorsegment}$

1 for finding "r"

1 for finding $A_{minorsector}$

Question 10

a) $y = |x| - 1$



$x = 0,\ y = 1$

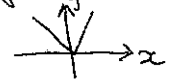
$x = -1 \quad y = 0$

$x = 1 \quad y = 0$

Award

2 for correct drawing and marking axis

1 for basic shape



b) i) $y = x^2 + 1$, $y = x + 3$

$x^2 + 1 = x + 3$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \qquad x = -1$

$y = 5 \qquad y = 2$

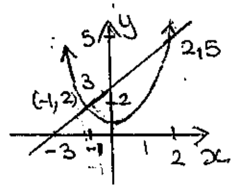
$\therefore (2,5)$ and $(-1,2)$

2 | 2 for finding (2,5) and (-1,2)

1 for $x^2 - x - 2 = 0$
$x = 2,\ x = -1$

ii)

$A = \int_{-1}^{2} [(x+3) - (x^2+1)]\,dx$



$= \int_{-1}^{2} (-x^2 + x + 2)\,dx$

$= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{2}$

$= \left[-\frac{8}{3} + \frac{4}{2} + 4\right] - \left[-\frac{-1}{3} + \frac{1}{2} - 2\right]$

$= \frac{20}{6} - -\frac{7}{6}$

$= \frac{27}{6}$

$= 4\frac{1}{2}$ square units

2 | 2 for correct integration and area value

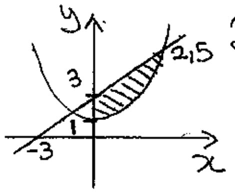
1 for integrating

Question 10 continue | Award

b ii) $y=x^2+1 \Rightarrow x^2=y-1$

$y=x+3 \Rightarrow x=y-3$

$x^2=y^2-6y+9$



3

$V=\pi\int_1^5 (y-1)\,dy - \pi\int_3^5 (y^2-6y+9)\,dy$

$V=\pi\left[\frac{y^2}{2}-y\right]_1^5 - \pi\left(\frac{y^3}{3}-6\frac{y^2}{2}+9y\right]_3^5$

$V=\pi\left[\left(\frac{25}{2}-5\right)-\left(\frac{1}{2}-1\right)\right]-\pi\left[\left(\frac{5^3}{3}-3\times25+45\right)\right.$

$\left.-\left(\frac{27}{3}-27+27\right)\right]$

$V=8\pi-\pi\frac{8}{3}$

$V=\frac{16\pi}{3}$

$=16.8$ cubic units

3 for correct integration and solution

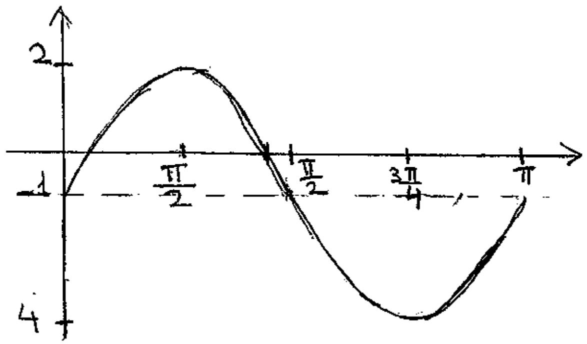
2 for correct boundary and $\left[\begin{array}{l} x^2=y-1 \\ x^2=(y-3)^2 \end{array}\right]$

and correct substitution

1. for $x^2=y-1$
$x^2=(y-3)^2$

c) $y=3\sin 2x-1 \quad 0\leq x\leq\pi$ 3

$A=3$ period π



3 for correct informat and graph

2 for A & P and sketching $3\sin 2x$

1 for A & P