Killara HIGH SCHOOL

CONSERVA PROGREDERE

STUDENT NUMBER

MATHEMATICS
COURSE

ASSESSMENT TASK NO 2
Date: 7 March 2012

TIME ALLOWED: *Reading time:* 5 *minutes*
Working time: 3 *hours*

INSTRUCTIONS:

- This paper consists of 10 Multiple Choice questions and 6 Free Response questions
- ANSWER all 16 questions
- Answer the Multiple Choice questions in the answer sheet provided
- Start each question in a new booklet
- A table of standard integrals is provided at the back of this paper
- All relevant working should be shown.

QUESTION NO	MARK
MC	/10
11	/15
12	/15
13	/15
14	/15
15	/15
16	/15
TOTAL	/100

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

OUTCOMES ASSESSED:

H1 - seeks to apply mathematical techniques to problems in a wide range of practical contexts
H2 - constructs arguments to prove and justify results
H4 - expresses practical problems in mathematical terms based on simple given models
H5 - applies appropriate techniques from the study of calculus and trigonometry to solve problems
H6 - uses the derivative to determine the features of the graph of a function
H7 - uses the features of a graph to deduce information about the derivative
H8 - uses techniques of integration to calculate areas and volumes
H9 - communicates using mathematical language, notation, diagrams and graphs

This assessment task constitutes 25% *of the Higher School Certificate Course Assessment.*

Section 1

10 marks

Attempt questions 1 – 10

Allow 15 minutes for this section

Use multiple choice answer sheet for Questions 1 – 10

1. The quadratic equation with 5 and −2 as roots is:

(A) $x^2 + 5x - 2 = 0$

(B) $x^2 - 2x + 5 = 0$

(C) $x^2 + 3x - 10 = 0$

(D) $x^2 - 3x - 10 = 0$

2. The domain of $y = \frac{1}{\sqrt{x^2 - 4}}$ is:

(A) $-2 < x < 2$

(B) $-2 \leq x \leq 2$

(C) $x < -2$ or $x > 2$

(D) $x \leq -2$ or $x \geq 2$

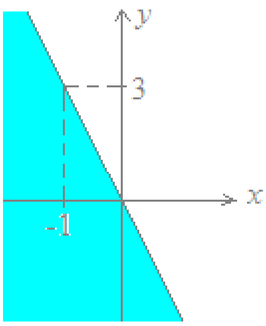
3. The shaded region is best described by which inequality:

(A) $y < -\frac{1}{3}x$

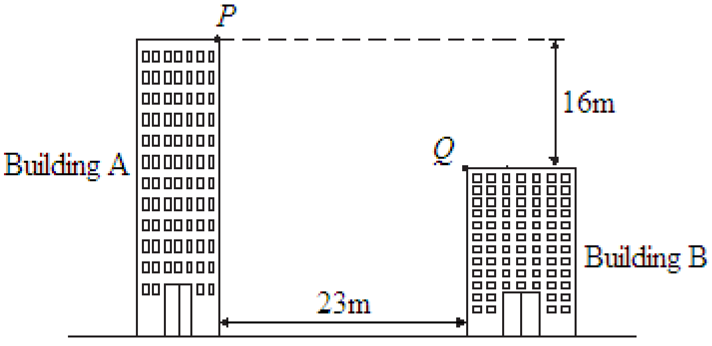
(B) $y \geq -3x$

(C) $y \leq -3x$

(D) $y \leq -\frac{1}{3}x$



4. The point Q on building B is visible from the point P on building A, as shown in the diagram above. Building A is 16m taller than building B.



The horizontal distance between P and Q is 23 metres.

The angle of depression of Q from P is closest to

(A) 35°

(B) 41°

(C) 44°

(D) 46°

5. If a curve is always increasing, which of the following <u>must</u> occur for all values of x?

(A) $\frac{dy}{dx} < 0$

(B) $\frac{dy}{dx} > 0$

(C) $\frac{d^2y}{dx^2} < 0$

(D) $\frac{d^2y}{dx^2} > 0$

6. Which of the following technique would you feel to be the best approach to

evaluate $\int_0^2 \sqrt{4-x^2}\ dx$?

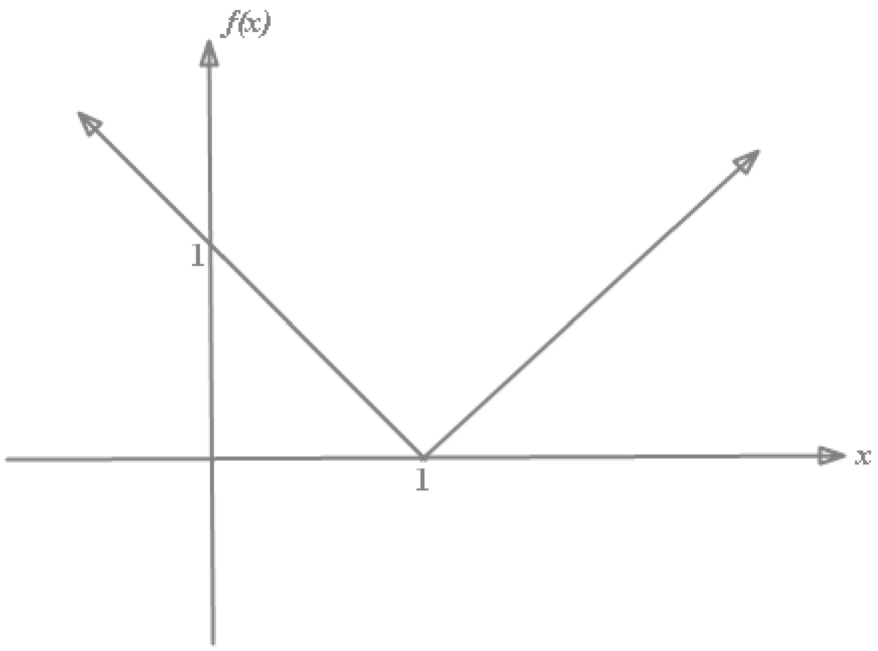
(A) Use substitution and integrate with respect to x

(B) Use Trapezoidal rule

(C) Use Simpson's rule

(D) Use geometrical formula

7. The graph of the function $y = f(x)$ is given below.



This function is not differentiable at $x = 1$ because

(A) $y = f(x)$ is not defined at $x = 1$

(B) $\lim\limits_{x \to 1} f'(x) \to 0$

(C) $\lim\limits_{x \to 1^-} f'(x) = \lim\limits_{x \to 1^+} f'(x) \to \infty$

(D) $\lim\limits_{x \to 1^-} f'(x) \neq \lim\limits_{x \to 1^+} f'(x)$

8. $\int_{-a}^{a} f(x)\,dx = 0$, then

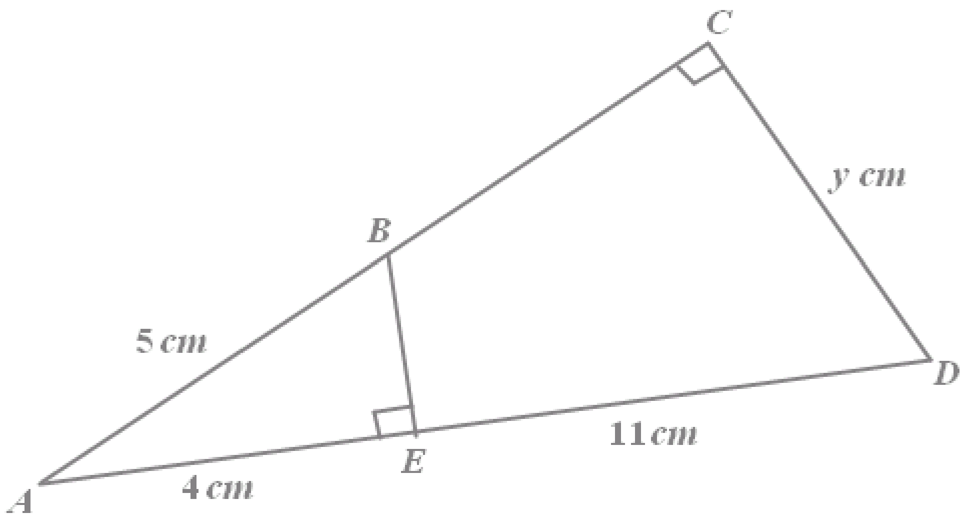
(A) $f(x)$ is a constant

(B) $f(x)$ is odd

(C) $f(x)$ is even

(D) $f(x)$ cannot be integrated

9.



In the diagram given above, the value of y is:

(A) 8

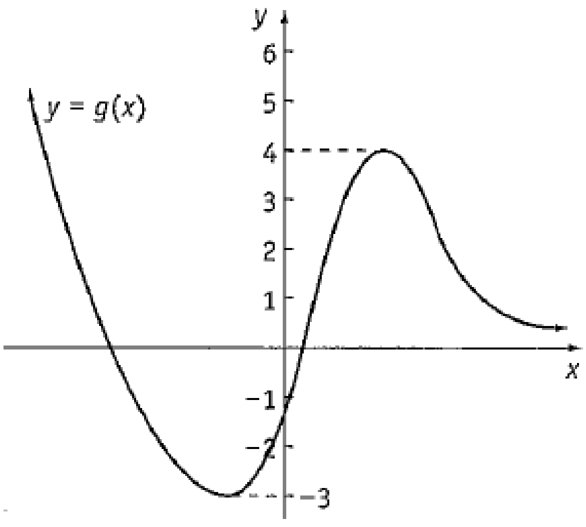
(B) 9

(C) 6

(D) 4

10. The diagram shows the graph of $y = g(x)$.

Which of the following show all values of k for which $g(x) = k$ have <u>*three*</u> solutions?



(A) $-3 \le k \le 4$

(B) $-3 \le k \le 0$

(C) $1 < k < 4$

(D) $0 < k < 4$

End of Multiple Choice Questions

Section 2

90 marks

Attempt Questions 11 – 16

Allow about 2 hours 45 minutes for this section

All necessary working should be shown for each question

Question 11 (15 marks) Use a SEPARATE working booklet **Marks**

a) Evaluate $\sum_{k=1}^{4} k^2$ **1**

b) Factorise fully: $3x^2 + x - 2$ **2**

c) Solve the equation $\frac{x}{3} - \frac{x-1}{2} = \frac{1}{2}$ **3**

d) Find the exact value of tan 120 **2**

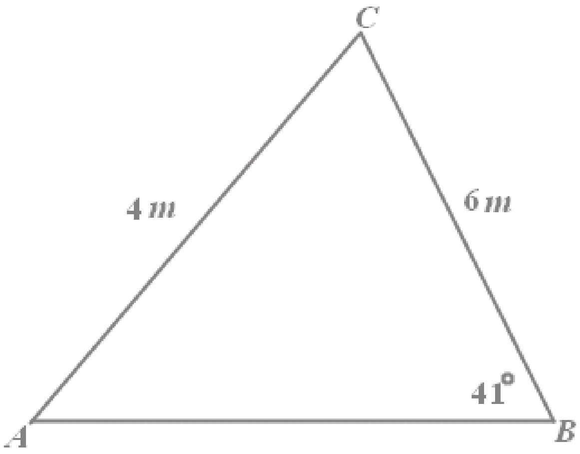
e) Find the values of x for which $|2x-1| \le 5$ **2**

Question 11 continued on page 8

f) (i) Express $3\sqrt{48} - 2\sqrt{27}$ in its simplest surd form. **2**

(ii) Hence, find p given that $3\sqrt{48} - 2\sqrt{27} = \sqrt{p}$ **1**

g) Find $\angle BAC$ in the following triangle to the nearest degree. **2**



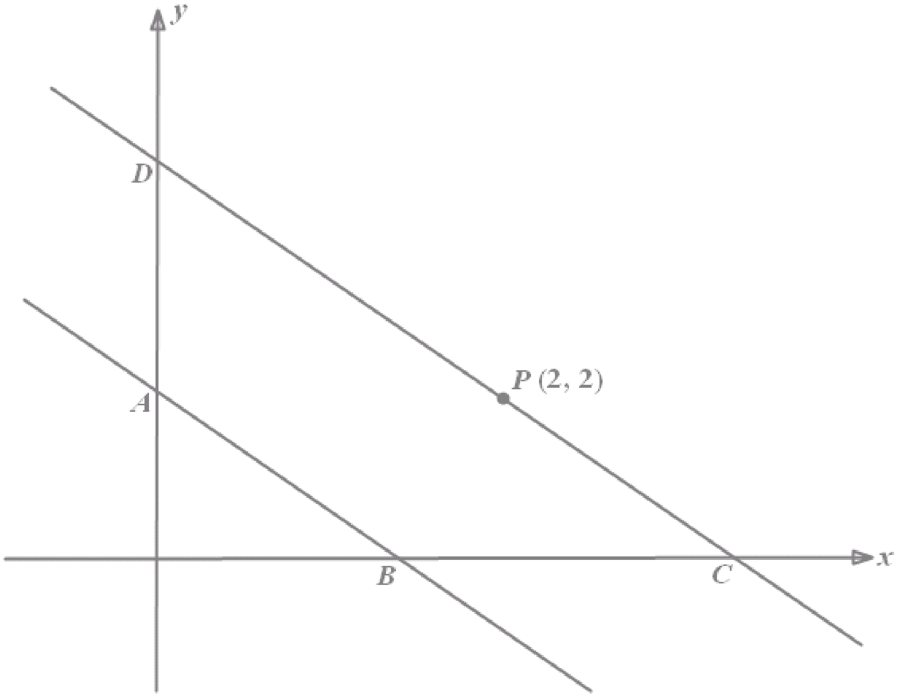
NOT TO SCALE

End of Question 11

Question 12 (15 marks) Use a SEPARATE working booklet **Marks**

a) The diagram shows the straight line AB with equation $x + 2y = 2$.

Copy or trace the diagram on to your answer booklet.



(i) DC is parallel to AB and passes through (2,2).

Show that the equation of DC is $x + 2y = 6$ **2**

(ii) Calculate the perpendicular distance from P to AB. **1**

(iii) Show that the distance AB is $\sqrt{5}$. **3**

(iii) Calculate the area of the trapezium $ABCD$ **3**

Question 12 continued on page 10

b) M is the midpoint of $A(5, k)$ and $B\,(11, 8)$. **3**

Find k, if M lies on the line $x - 2y = 4$.

c) Consider the parabola with equation $y^2 = 4x - 8$. Find:

(i) the coordinates of the vertex **1**

(ii) the coordinates of the focus. **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE working booklet **Marks**

a) Differentiate:

(i) $y = \dfrac{3x+2}{4x-3}$ **2**

(ii) $y = x\sqrt{x}$ **2**

(iii) $y = (2x+1)(5x-3)^4$ **2**

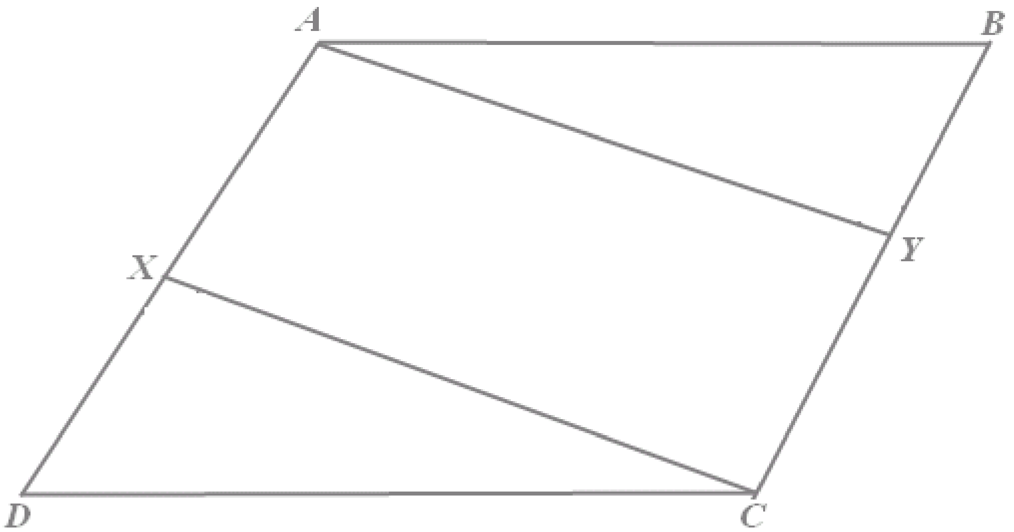
b) Derive the equation of the tangent to the curve $y = 2x^2 + 5$ at the point where $x = 2$. **2**

c) Sketch the continuous curve $y = f(x)$ which satisfies the following conditions. **3**

- $f'(x) < 0$ for $x < -1$
- $f'(-1) = f'(2) = 0$
- $f''(0) = 0$
- $f''(x) < 0$ for $0 < x \leq 5$
- $f(4) = 1$

Question 13 continued on page 12

d)



ABCD is a rhombus. *X* and *Y* lie on *AD* and *BC* such that $XC \perp AD$ and $AY \perp BC$.

Copy or trace the diagram on to your answer booklet.

(i) Prove that $\Delta ABY \equiv \Delta DXC$. **2**

(ii) If the side length of the rhombus is 10 cm and $\angle DAB = 120°$, find the length of diagonal *AC*. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE working booklet **Marks**

a) Consider the function $f(x) = 9x - 3x^2 - x^3$.

(i) Find the coordinates of the two stationary points. **2**

(ii) Determine the nature of the stationary points. **2**

(iii) Show that there is a point of inflection at $(-1, -11)$. **2**

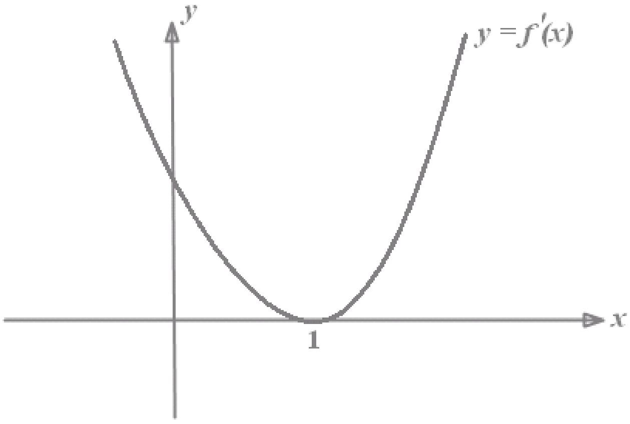
(iv) Sketch the curve $y = f(x)$ showing the stationary points and the point of inflection**.** **1**

(v) For what values of x is the curve concave up? **1**

Question 14 continues on page 14

Question 14 (Continued) **Marks**

b) Consider the function $y = f(x)$, whose gradient function is shown as the parabola in the sketch below.



(i) Comment on the sign of $\frac{dy}{dx}$ for all $x \neq 1$ **1**

(ii) There is a stationary point (a horizontal point of inflection) at $x = 1$.

Justify this statement with reference to the graph. **2**

(iii) Copy the graph above onto your answer sheet.

On the same set of axes, sketch the graph of $y = f''(x)$. **1**

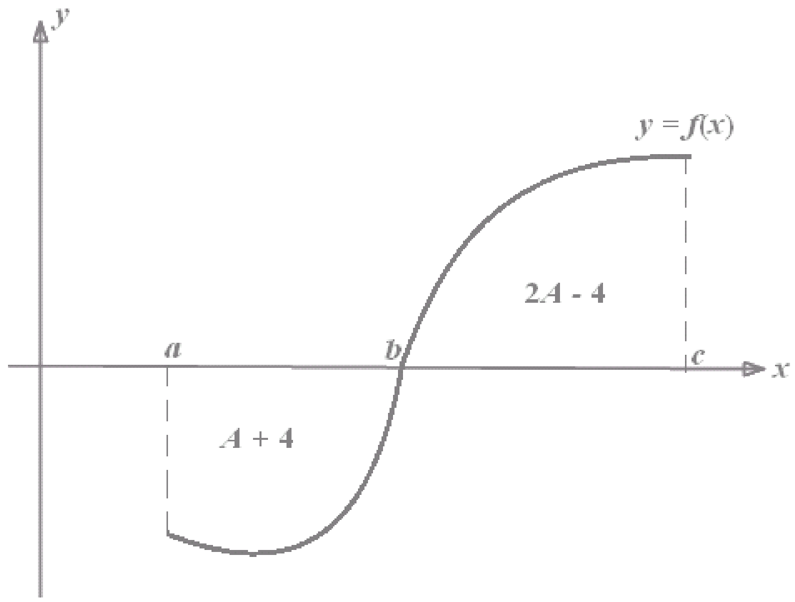
c) The function $y = ax^2 + bx + c$ has the gradient function $\frac{dy}{dx} = 4x + 2$. **3**

The function has a minimum value of 1. Find the values of a, b and c.

End of Question 14

Question 15 (15 marks) Use a SEPARATE working booklet **Marks**

a) In the diagram below, the area between the curve $y = f(x)$, the x-axis and the line $x = a$ is equal to $(A+4)$ unit^2. The area between the curve $y = f(x)$, the x-axis and the line $x = c$ is equal to $(2A-4)$ unit^2.



Use the information to evaluate $\int_a^c f(x)\,dx$. **1**

b) Find:

(i) $\int \frac{4x^3 - 3x^2 + 5}{x^2}\,dx$ **2**

(ii) $\int_0^1 (2-5x)^4\,dx$ **2**

Question 15 continues on page 16

c) The function $f(x)$ is defined by the rule $f(x) = 5x\sqrt{x+1}$ in the domain $-1 \le x \le 1$

(i) Show that the derivative of $f(x)$ is $\dfrac{15x+10}{2\sqrt{x+1}}$. **1**

(ii) Use the derivative of $f(x)$ to find the turning point of $f(x)$ and determine its nature. Answer to one decimal place. **2**

(iii) Hence, draw a rough sketch of $f(x)$, showing clearly the turning points, and the values at the endpoints of the domain. **2**

(*You do not have to scale the diagram accurately and do not find the points of inflection*)

(iv) the area between the x-axis and the section of the curve $y = f(x)$ from $x = -1$ to $x = 0$ is rotated about the x-axis. **3**

Find the volume of the solid of revolution.

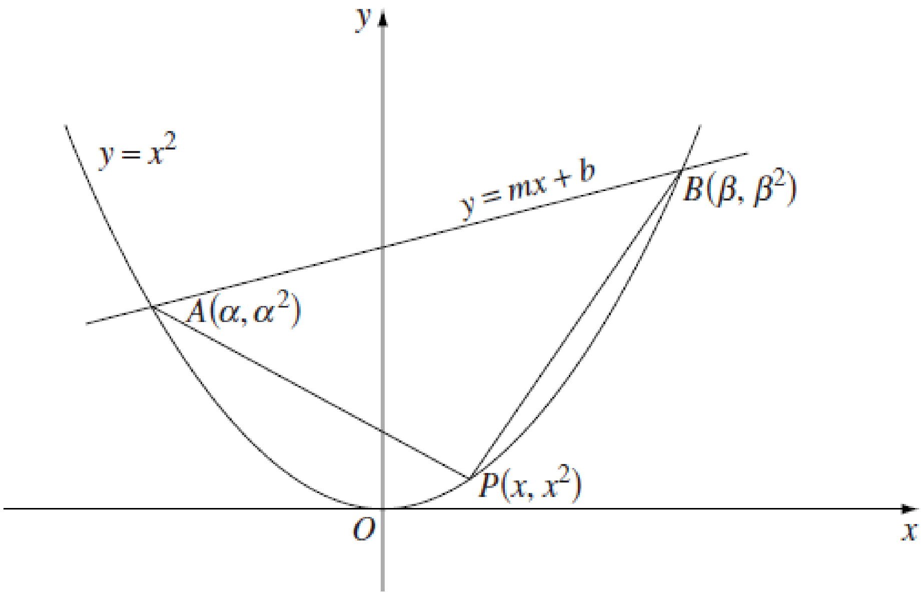
(v) Copy and complete the table below correct to *one decimal place* and apply Simpson's rule with five function values to give an estimate of $\int_{-1}^{1} 5x\sqrt{x+1}\,dx$. **2**

x	-1	-0.5	0	0.5	1
$f(x)$					

End of Question 15

Question 16 (15 marks) Use a SEPARATE working booklet **Marks**

a) The parabola $y = x^2$ and the line $y = mx + b$ intersect at the points $A\left(\alpha, \alpha^2\right)$ and $B\left(\beta, \beta^2\right)$ as shown in the diagram.



(i) Explain why $\alpha + \beta = m$ and $\alpha\beta = -b$. **1**

(ii) Show that $(\alpha - \beta)^2 + \left(\alpha^2 - \beta^2\right)^2 = (\alpha - \beta)^2 \left[1 + (\alpha + \beta)^2\right]$ **2**

(iii) Hence, Show that the distance **2**

$$AB = \sqrt{\left(m^2 + 4b\right)\left(1 + m^2\right)}$$

(iv) The point $P\left(x, x^2\right)$ lies on the parabola between A and B.

Show that the area of triangle ABP is given by **2**

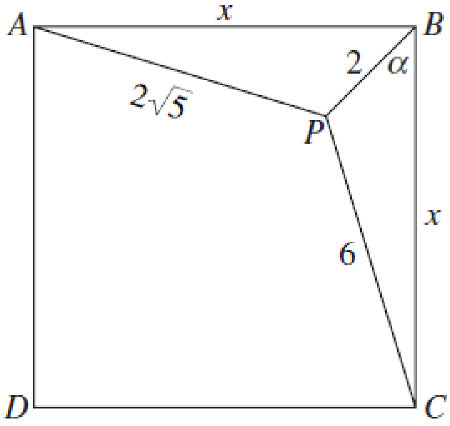
$$\frac{1}{2}\left(mx - x^2 + b\right)\sqrt{m^2 + 4b}$$

Question 16 continues on page 18

b) $ABCD$ is a square with side x cm, with P a point within the square such that

$PC = 6$ cm, $PB = 2$ cm and $AP = 2\sqrt{5}$ cm.

Let $\angle PBC = \alpha$.



NOT TO SCALE

(i) By using triangle PBC show that $\cos\alpha = \dfrac{x^2 - 32}{4x}$ **1**

(ii) By considering triangle PBA, show that $\sin\alpha = \dfrac{x^2 - 16}{4x}$ **2**

(iii) Hence, or otherwise, show that the value of x is a solution of **2**

$$x^4 - 56x^2 + 640 = 0$$

(iv) Find x. Give reason for your answer. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

STUDENT NUMBER

Multiple Choice Answer Sheet

Completely fill the response oval representing the most correct answer.

1. A ⬭ B ⬭ C ⬭ D ⬭
2. A ⬭ B ⬭ C ⬭ D ⬭
3. A ⬭ B ⬭ C ⬭ D ⬭
4. A ⬭ B ⬭ C ⬭ D ⬭
5. A ⬭ B ⬭ C ⬭ D ⬭
6. A ⬭ B ⬭ C ⬭ D ⬭
7. A ⬭ B ⬭ C ⬭ D ⬭
8. A ⬭ B ⬭ C ⬭ D ⬭
9. A ⬭ B ⬭ C ⬭ D ⬭
10. A ⬭ B ⬭ C ⬭ D ⬭

2012 HSC Half yearly marking scheme

1. D	2. C	3. C	4. A	5. B
6. D	7. D	8. B	9. B	10. D

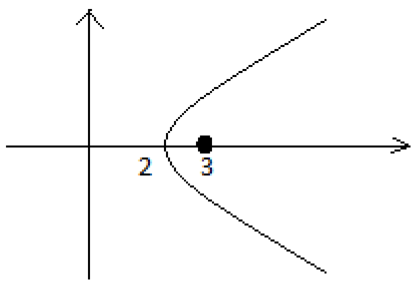
Question 11

a) $\sum_{k=1}^{4} k^2 =$ 1 +4 +9 + 16 = 30	1 mark: correct answer	
b) $3x^2 + x - 2$ $= (x+1)(3x-2)$	1 mark	
c) $\frac{x}{3} - \frac{x-1}{2} = \frac{1}{2}$ $\frac{2x}{3\times 2} - \frac{3(x-1)}{2\times 3} = \frac{1\times 3}{2\times 3}$ $2x - 3(x-1)\ 2x = 3$ **1 mark** $2x - 3x + 3 = 3$ **1 mark** $-x = 0$ $x = 0$	1 mark: manipulates equation to remove the denominators 1 mark: removes brackets and simplifies correctly 1 mark: correctly solves the equation	
d) tan 120 = −tan 60 $= -\sqrt{3}$	1 mark: expresses in terms of the acute angle –tan 60 1 mark: gives correct exact value	
e) $\|2x-1\| \le 5$ $-5 \le 2x - 1 \le 5$ **1 mark** $-4 \le 2x \le 6$ $-2 \le x \le 3$ **1 mark**	1 mark: removes the absolute value sign and gives the correct inequalities. 1 mark: correct solution	
f) (i) $3\sqrt{48} - 2\sqrt{27}$ $= 3\times 4\sqrt{3} - 2\times 3\sqrt{3}$ **1 mark** $= 12\sqrt{3} - 6\sqrt{3}$ $= 6\sqrt{3}$ **1mark**	1 mark: correctly simplifies the surds $\sqrt{48}$ and $\sqrt{27}$ 1 mark: correct answer	
f)(ii) $3\sqrt{48} - 2\sqrt{27} = \sqrt{p}$ using (i) $6\sqrt{3} = \sqrt{p}$ $\sqrt{108} = \sqrt{p}$ $\therefore p = 108$	1 mark: correct answer	

1(g) $\frac{\sin\theta}{6} = \frac{\sin 41}{4}$ Sin θ = $\frac{\sin 41}{4} \times 6 = 0.98408\ldots$ Θ = $\sin^{-1}(0.98408\ldots)$ = 79.765… = 80°	1 mark: correctly applies the Sine rule 1 mark: correct answer rounded to the nearest degrees from the calculation of the sine rule.	

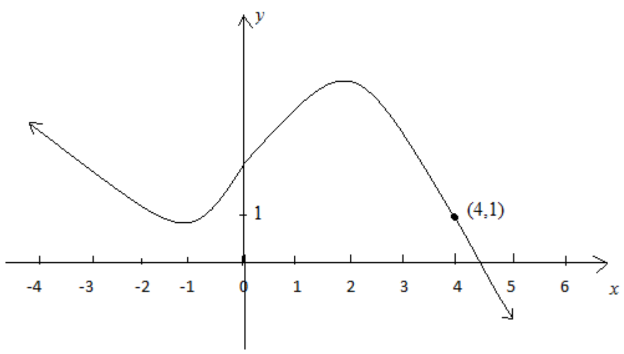
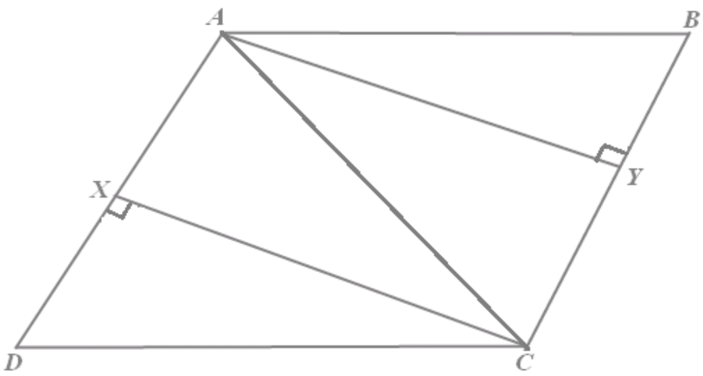
Question 12

a)(i) DC ∥ AB ∴ DC has equation x + 2y + k = 0 **1 mark** (2, 2) lies on it. 2 + 2 ×2 + k = 0 ∴ k = -6 ∴DC has equation x + 2y - 6 = 0 x + 2y = 6 **1 mark**	1 mark: for giving the correct gradient of DC 1 mark: substitutes (2, 2) to get the correct equation	
b) P(2, 2) x + 2y – 2 =0 $\perp$ distance = $\left\lvert \frac{2+2\times 2-2}{\sqrt{1^2+2^2}} \right\rvert$ $= \frac{4}{\sqrt{5}}$	1 mark: correct solution	
c) x + 2y – 2 = 0 For A, x = 0 ∴ 2y = 2 y = 1 (0, 1) **1 mark** For B, y = 0 ∴ x = 2 (2, 0) **1 mark** AB = $\sqrt{1^2+2^2} = \sqrt{5}$ **1 mark**	1 mark: correct y intercept 1 mark: correct x intercept 1 mark: uses the distance formula or Pythagoras’ theorem to evaluate AB	
d) Finding length of CD x + 2y – 6 = 0 For D, x = 0 ∴ y = 3 For C, y = 0 ∴ x = 6 CD = $\sqrt{3^2+6^2} = \sqrt{45}$ $= 3\sqrt{5}$	1 mark: finds length of CD (giving $\sqrt{45}$ is enough)	

Area of trapezium = $\frac{1}{2}\times\frac{4}{\sqrt{5}}\times(\sqrt{5}+3\sqrt{5})$ = 8 unit^2 **1 mark**	1 mark: substitutes into correct formula 1 mark: evaluates area correctly.	
e) $M\left(\frac{11+5}{2},\frac{k+8}{2}\right)$ = **1 mark** $M\left(8,\frac{k+8}{2}\right)$ This point lies on $x - 2y = 4$ $8 - 2\times\left(\frac{k+8}{2}\right) = 4$ **1 mark** $8 - k - 8 = 4$ ´$\therefore -k = 4$ $\therefore k = -4$ **1 mark**	1 mark: uses the midpoint formula to get the correct expression for midpoint 1 mark: substitutes the coordinates of M in to the equation 1 mark: solves for k.	
c)(i) $y^2 = 4x - 8$ $y^2 = 4(x-2)$ Vertex (2,0) **1 mark** (ii) $4a = 4$ $a = 1$ **1 mark** focus (3, 0) **1 mark** 	1 mark: expresses the equation in the form $(y-k) = 4a(x-h)$ and gives the correct vertex (ii) 1 mark: calculates the focal length correctly 1 mark: calculates the correct focus	

Question13

a)(i) $y = \dfrac{3x+2}{4x-3}$ $u = 3x + 2 \quad u' = 3$ $v = 4x - 3 \quad v' = 4$ **1 mark** $\dfrac{dy}{dx} = \dfrac{(4x-3)\times 3-(3x+2)\times 4}{(4x-3)^2}$ $= \dfrac{12x-9-12x-8}{(4x-3)^2} = \dfrac{-17}{(4x-3)^2}$	1 mark: correctly differentiates 3x + 2 and 4x – 3. 1 mark: applies the quotient rule correctly	
(ii) $y = x\sqrt{x} = x^{3/2}$ **1 mark** $\dfrac{dy}{dx} = \dfrac{3}{2}x^{1/2}$ **1 mark**	1 mark: writes $x\sqrt{x}$ in index form 1 mark: correctly differentiates	
(iii) $y = (2x+1)(5x-3)^4$ $u = 2x + 1 \quad u' = 2$ $v = (5x-3)^4$ $v' = 4(5x-3)^3 \times 5$ **1 mark** $\dfrac{dy}{dx} = (2x+1)\times 4(5x-3)^3\times 5 +$ $(5x-3)^4 \times 2$ $= 2(5x-3)^3[20x+10+5x-3]$ $= 2(5x-3)^3[25x+7]$	1 mark: applies chain rule to differentiate $(5x-3)^4$ 1 mark: applies product rule	
b) $y = 2x^2 + 5$ $\dfrac{dy}{dx} = 4x$ When $x = 2$, $y = 2 \times 2^2 + 5 = 13$ (2, 13) $\dfrac{dy}{dx}$ at $x = 2$, is $4 \times 2 = 8$ **1 mark** Equation of the tangent is $y - 13 = 8(x - 2)$ $8x - y - 3 = 0$ is the equation of the tangent **1 mark**	1 mark: correctly evaluates the gradient of the tangent ($\dfrac{dy}{dx} = 8$) 1 mark: uses their gradient and the point (2, 13) to find the equation of the line.	

c)  	1 mark: correct shape for x < 0. Correct gradient for $x < -1$, stat. point at x = 1, correct concavity required 1 mark: concave down for $0 < x \le 5$, including point of inflection at x = 0 1 mark: max. turning point at x = 2 and the point (4, 1) is plotted.	
d)  (i) AB = CD (sides of a rhombus are equal) $\angle AYB = 90°$ $\angle AXC = 90°$ ($XC \perp AD$ and $AY \perp BC$.) **1 mark** $\angle B = \angle D$ (opposite angles of a rhombus are equal) $\therefore \Delta ABY \equiv \Delta DXC$ (AAS angle-angle side)**1 mark**		
(ii) Construct AC. $\angle DAC = 60°$ $\angle ADC = 60°$ (the diagonal bisects the vertices of a rhombus) **1 mark** ΔADC is equilateral (each of the vertices of the triangle = $60°$. <u>Sides opposite equal angles are equal</u> *must have this*) $\therefore$AC 10cm **1 mark**		

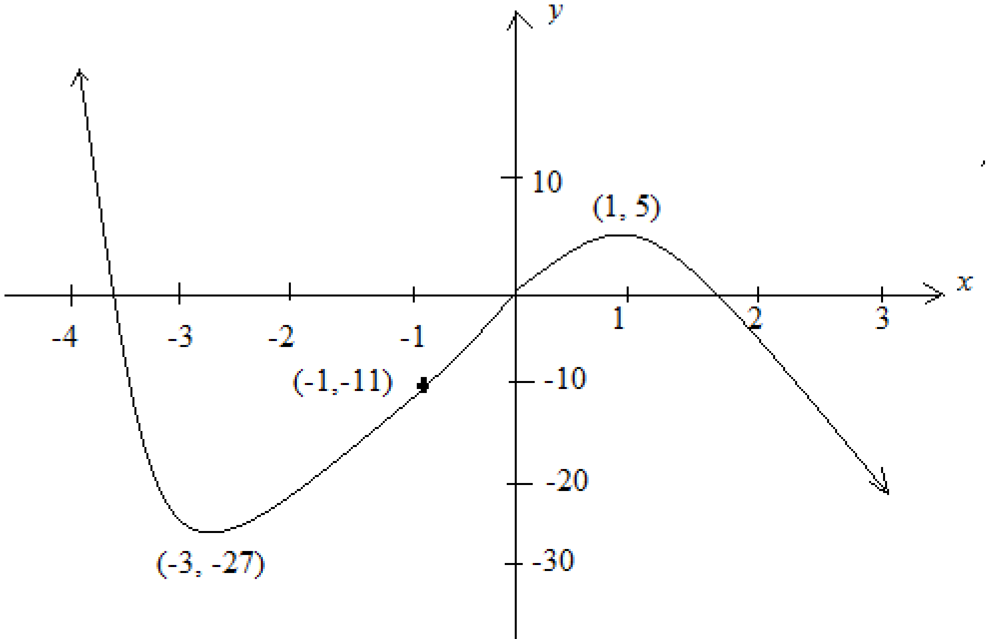
Question 14

<table>
<tr>
<td>a) $f(x) = 9x - 3x^2 - x^3$.
$f'(x) = 9 - 6x - 3x^2$.
For stationary points, $f'(x) = 0$
$9 - 6x - 3x^2 = 0$
$-3(x^2 + 2x - 3) = 0$
$(x + 3)(x - 1) = 0$
$X = -3,\ x = 1$
When $x = -3$, $y = -27$
When $x = 1$, $y = 5$
Stationary points are $(-3, -27)$ and $(1, 5)$.</td>
<td>1 mark for each of the stationary points</td>
<td></td>
</tr>
<tr>
<td>$f''(x) = -6 - 6x$
When $x = -3$, $f''(-3) = 12 > 0$
$\therefore$ concave up;
Minimum turning point
$(-3, -27)$ 1 mark
When $x = 1$, $f''(1) = -6 - 6 < 0$
$\therefore$ concave down;;
Maximum turning point
$(1, 5)$ 1 mark</td>
<td>1 mark: proves that there is a minimum turning point at $(-3, -27)$

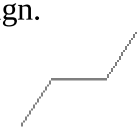
1 mark: proves that there is a maximum turning point at $(1, 5)$</td>
<td></td>
</tr>
<tr>
<td>(iii) For points of inflection
$f''(x) = 0$
$\therefore -6 - 6x = 0$
$X = -1$
When $x = -1$, $f(x) = -11$
Checking for concavity:
<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td></tr><tr><td>$f''(x)$
= -6-6x</td><td>6</td><td>0</td><td>-6</td></tr></table>
Change in concavity and therefore point of inflection at $(-1, -11)$ must have ths</td>
<td>1mark: sets $f''(x) = 0$ and finds the possible point of inflection.

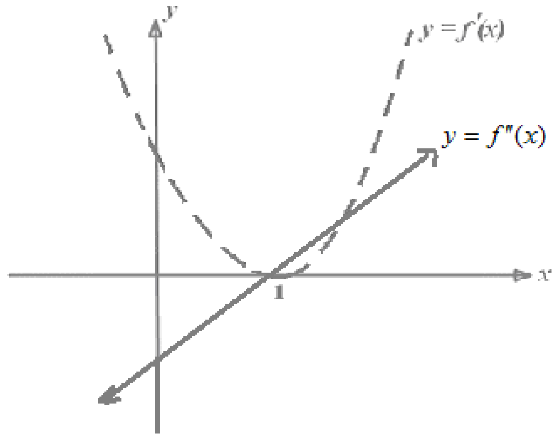
1 mark: shows working to establish that there is a change in concavity and verbal explanation is required</td>
<td></td>
</tr>
</table>

(iv)



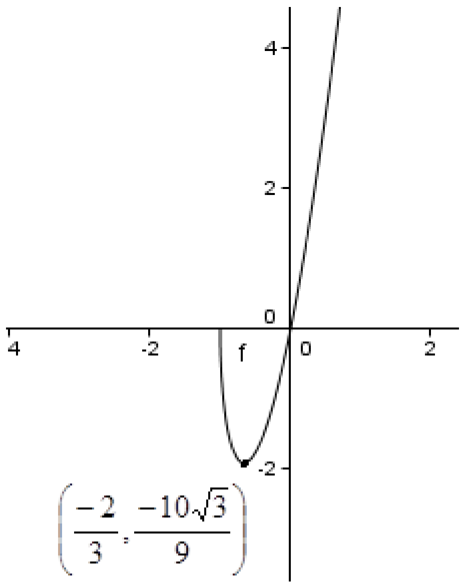
(iv)	1 mark: correct shape with the turning points, point of inflection and the x-intercept shown	
(v) The graph is concave up for $x < -1$	1 mark: correctly writes the region from their graph Or sets $f''(x) < 0$ and solves the inequality	

(b)(i) $\dfrac{dy}{dx} > 0$ for all x, $x \neq 1$	Correct answer, must say $x \neq 1$ 1 mark	
(ii) $\dfrac{dy}{dx} = 0$ at $x = 1$ $\therefore$ stationary point.**1 mark** $\dfrac{dy}{dx}$ is positive on both sides of $x = 1$. it does not change sign.  or $\dfrac{d^2y}{dx^2} = 0$ at $x = 1$; $\dfrac{d^2y}{dx^2} < 0$ for x < 1 and $\dfrac{d^2y}{dx^2} > 0$ for $x > 1$. Change in concavity, ie point of inflection at $x = 1$.	1 mark for identifying the stationary point at $x = 1$ 1 mark for establishing point of inflection.	

$\therefore\ x = 1$ is a horizontal point of inflection.		
(iii) 	1 mark for correct diagram of $y = f''(x)$	
c) y = $ax^2 + bx + c$ $\frac{dy}{dx}$ = 2ax + b = 4x + 2 Comparing coefficients, $\therefore$ 2a = 4 a = 2 **1 mark** b = 2 **1 mark** $\therefore$ y = $2x^2 + 2x + c$ minimum value of 1. X = $\frac{-b}{2a}$ = $\frac{-1}{2}$ X = $\frac{-1}{2}$, y =1 $1 = 2\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) + c$ $c = \frac{3}{2}$ $\therefore$ a = 2, b= 2, c = $\frac{3}{2}$	1 mark each for a and b for setting $\frac{dy}{dx}$ = 4x + 2 and comparing and evaluating coefficients 1 mark: for the value of c. (must show working)	

Question 15

(a) $-\left(A+4)\right)+\left(2A-4)\right)$ $= -A - 4 + 2A - 4$ $= A - 8$	1 mark: Subtracts the area from below the x-axis from the area above the x-axis.	
b)(i) $\int \frac{4x^3-3x^2+5}{x^2}\,dx$ $= \int 4x-3+5x^{-2}\,dx$ $= \frac{4x^2}{2}-3x+\frac{5x^{-1}}{-1}+C$ $= 2x^2-3x-\frac{5}{x}+C$	1 mark: performs term by term division correctly 1 mark: correct integration	
b)(ii) $\int_0^1 (2-5x)^4\,dx$ $\left[\frac{(2-5x)^5}{5\times -5}\right]_0^1$ **1 mark** $= \frac{-1}{25}\{(2-5)^5-\left(2-0)^5\right)\}$ $= \frac{-275}{-25} = 11$	1 mark: correct integration 1 mark: correct evaluation	
c) (i) *f(x)* = $5x\sqrt{x+1}$ u = 5x u´ = 5 v = $\sqrt{x+1}$ v´= $\frac{1}{2\sqrt{x+1}}$ $f'(x) = \frac{5x}{2\sqrt{x+1}}+5\sqrt{x+1}$ $= \frac{5x+2(x+1)}{2\sqrt{x+1}}$ $= \frac{15x+10}{2\sqrt{x+1}}$	1 mark: correct proof. Every step of working is required.	
c)(ii) $f'(x)=0$ $\frac{15x+10}{2\sqrt{x+1}} = 0$ 15x + 10 = 0 $\therefore$ x = $\frac{-2}{3}$ **1 mark** when x = $\frac{-2}{3}$, y = $\frac{-10\sqrt{3}}{9}$ = 1.9	1 mark: sets $f'(x)=0$ and solves to get the x-value of the turning point	

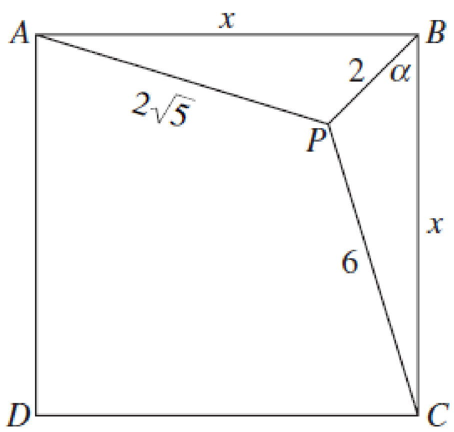
$\begin{array}{c\|ccc} x & -0.8 & \frac{-2}{3} & 0 \\ \hline f'(x) & -\sqrt{5} & 0 & 5 \\ & \searrow & \text{—} & \nearrow \end{array}$ $\therefore$ minimum turning point at $(\frac{-2}{3}, 1.9)$	1 mark: Uses gradient test or second derivative to determine the nature of the stationary point. *Must give the turning point as a coordinate pair*	
(iii)  		
(iv) Volume = $\int_{-1}^{0} \pi \times 25x^2(x+1)dx$ **1 mark** $= 25\pi \int_{-1}^{0} x^3 + x^2 \; dx$ $= 25\pi \left[\frac{x^4}{4} + \frac{x^3}{3}\right]_{-1}^{0}$ **1 mark** $= \left\|25\pi\left(0 - \left(\frac{1}{4} - \frac{1}{3}\right)\right)\right\|$ $= \frac{25\pi}{12}\text{unit}^3$ **1 mark**	1 mark: correct formula to evaluate the area 1 mark: correctly integrates 1 mark: correct evaluation. Unit required.	

(iv) <table><tr><td>x</td><td>-1</td><td>-0.5</td><td>0</td><td>0.5</td><td>1</td></tr><tr><td>$f(x)$</td><td>0</td><td>-1.8</td><td>0</td><td>3.06</td><td>7.07</td></tr></table> $\int_{-1}^{1} 5x\sqrt{x+1}\,dx \approx$ $\frac{0.5}{3}\left(0+4\times-1.8+2\times0+4\times3.06+7.07\right)$ $= -2.018\ldots. \approx -2.0$ (1 d.p)	1 mark: correctly completes the table. 1 mark: correct substitution and evaluation into the correct formula	

Question 16

a)(i) From the diagram, the roots of the simultaneous equations $y = x^2$ and $y = mx + b$ are $x = \alpha$ and $x = \beta$. Now $x^2 = mx + b$ $x^2 - mx - b = 0$ sum of roots $(\alpha + \beta) = \frac{-b}{a}$ $\alpha + \beta = \frac{-(-m)}{1} = m$ **1 mark** Product of roots $(\alpha\beta) = \frac{c}{a}$ $= \frac{-b}{1} = -b$ **1 mark**	1 mark: develops the correct quadratic equation, $x^2 - mx - b = 0$ and gives sum of roots $(\alpha + \beta) = \frac{-b}{a}$ and product of roots $(\alpha\beta) = \frac{c}{a}$	
(ii) $(\alpha-\beta)^2 + (\alpha^2-\beta^2)^2 =$ $(\alpha-\beta)^2 + [(\alpha+\beta)(\alpha-\beta)]^2$ $= (\alpha-\beta)^2 + (\alpha+\beta)^2(\alpha-\beta)^2$ **1 mark** $= (\alpha-\beta)^2\left[1+(\alpha+\beta)^2\right]$ **1 mark**	1 mark: correctly factorises and expands $(\alpha^2-\beta^2)^2$ as $(\alpha+\beta)^2(\alpha-\beta)^2$ 1 mark: correctly factorises to get the required result	
(iii) By distance formula $AB = \sqrt{(\alpha-\beta)^2 + (\alpha^2-\beta^2)^2}$ $= \sqrt{(\alpha-\beta)^2\left[1+(\alpha+\beta)^2\right]}$ Now $(\alpha-\beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$ $= \alpha^2 + 2\alpha\beta + \beta^2 - 4\alpha\beta$ $= (\alpha+\beta)^2 - 4\alpha\beta$ $= m^2 - 4(-b)$ $= m^2 + 4b$	2 marks: for correct solution Award only 1 mark if: Writes $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$ Or Application of distance formula and uses the result from (ii)	

AB = $\sqrt{(\alpha-\beta)^2\left[1+(\alpha+\beta)^2\right]}$ $= \sqrt{(m^2+4b)\left[1+(m)^2\right]}$ $= \sqrt{(m^2+4b)(1+m^2)}$		
(iv) perpendicular distance from *P* (x, x^2) to the line mx – y + b = 0 is given by: h $= \dfrac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ $= \dfrac{\lvert mx-x^2+c\rvert}{\sqrt{m^2+1}}$ $= \dfrac{mx-x^2+b}{\sqrt{m^2+1}}$ (from the diagram, mx + b > x^2) Area of triangle ABP, $= \frac{1}{2}\times AB\times perp.\ distance\ from\ AB$ $= \frac{1}{2}\times\sqrt{(m^2+4b)(1+m^2)}\times\dfrac{mx-x^2+b}{\sqrt{m^2+1}}$ = $\frac{1}{2}\times\sqrt{(m^2+4b)}\times\sqrt{(1+m^2)}\times\dfrac{mx-x^2+b}{\sqrt{m^2+1}}$ $= \frac{1}{2}(mx-x^2+b)\sqrt{m^2+4b}$	2 marks: correct solution Award only **1 mark**: attempts to calculate perpendicular from P to AB or attempts to find the area of the Δ by writing A = ½ bh and applies their base length AB from (iii).	



16(b)(i) In ΔPBC, $\cos\alpha = \dfrac{PB^2 + CB^2 - PC^2}{2 \times PB \times CB}$ $= \dfrac{2^2 + x^2 - 6^2}{2 \times 2 \times x}$ $= \dfrac{x^2 - 32}{4x}$	1 mark: applies cosine rule correctly (*Every step of working must be shown*)	
(ii) Since $\angle ABC$ is a right angle in a square and $\angle PBC = \alpha$, then $\angle ABP = 90° - \alpha$. Using cosine rule in ΔABP, $\cos\angle ABP = \dfrac{x^2 + 2^2 - (2\sqrt{5})^2}{2 \times x \times 2}$ $= \dfrac{x^2 - 16}{4x}$ But $\cos(90° - \alpha) = \sin\alpha$ $\therefore \sin\alpha = \dfrac{x^2 - 16}{4x}$	1 mark: Recognises $\angle ABP = 90° - \alpha$ and applies cosine rule correctly 1 mark: Uses the property $\cos(90° - \alpha) = \sin\alpha$ and proves the required result	
(iii) From (i), $\cos\alpha = \dfrac{x^2 - 32}{4x}$ From (ii), $\sin\alpha = \dfrac{x^2 - 16}{4x}$ But, $\sin^2\alpha + \cos^2\alpha = 1$ $\therefore \left(\dfrac{x^2 - 16}{4x}\right)^2 + \left(\dfrac{x^2 - 32}{4x}\right)^2 = 1$ $\therefore$ $x^4 - 64x^2 + 1024 + x^4 - 32x^2 + 256 = 16x^2$ $\therefore \quad 2x^4 - 112x^2 + 1280 = 0$ $\therefore \quad x^4 - 56x^2 + 640 = 0$ That ie, X is a solution of $x^4 - 56x^2 + 640 = 0$	2 mark: correct solution with all steps of working Award only **1 mark** if uses the results from (i) and (ii) and attempts to eliminate $\sin\alpha$ and $\cos\alpha$.	

(iv) $x^4 - 56x^2 + 640 = 0$ ie. $(x^2 - 16)(x^2 - 40) = 0$ $\therefore\ x = \pm 4$ or $x = \pm\sqrt{40} = \pm 2\sqrt{10}$ **1 mark** But x cm is the length of the side of a square, so $x > 0$. $\therefore\ x = 4$ or $2\sqrt{10}$ **1 mark** If $x = 4$, cos α = $\dfrac{4^2 - 32}{4 \times 4} < 0$. But α is an acute angle, so cos α > 0. If $x = 2\sqrt{10}$, Cos α = $\dfrac{\left(2\sqrt{10}\right)^2 - 32}{4 \times \left(2\sqrt{10}\right)} > 0$ $\therefore$ solution is $x = 2\sqrt{10}$. **1 mark**	1 mark: Correctly solves the equation 1 mark: Recognises x is the length of a side. $\therefore\ > 0$ 1 mark: Explains why x = 4 cannot be a solution and proves x = $2\sqrt{10}$ is the only solution	