

STUDENT NUMBER

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MATHEMATICS

MARCH 2013

General Instructions

- Reading Time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Start a new booklet for each question

Total Marks - 100

- Attempt questions 1 - 16
- Answer questions 1 – 10 on the multiple choice answer sheet provided
- For questions 11-16, start each question in a new booklet

QUESTION NO	MARK
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
15	/15
16	/15
TOTAL	/100

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

5. The derivative of $(3x^2 - 2)^4$ is given by:

a) $24x(6x)^3$

b) $24x(3x^2 - 2)^3$

c) $4(3x^2 - 2)^3$

d) $12x^2(3x^2 - 2)^3$

6. The domain and range for the function $f(x) = |2x| - 5$ are:

a) $x \geq -5$ and all real y .

b) $x \geq -5$ and $y \geq -5$.

c) all real x and all real y .

d) all real x and $y \geq -5$.

7. If $\sin \theta = \frac{7}{8}$ and $\cos \theta < 0$ then the exact value of $\cos \theta$ is given by:

a) $\frac{\sqrt{15}}{8}$

b) $-\frac{\sqrt{15}}{8}$

c) $\frac{8}{15}$

d) $-\frac{8}{15}$

8. What is the value of $\int_1^6 \frac{1}{5x} dx$?

a) $\ln 30$

b) $\ln 25$

c) $\frac{1}{6} \ln 5$

d) $\frac{1}{5} \ln 6$

Section II

90 marks

Attempt Questions 11-16

Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet.

In Questions 11-16, your responses must include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE working booklet

Marks

- | | | | |
|----|-----------------------|---|---|
| a) | Factorise completely: | $6x^2 + 4x - 16$ | 2 |
| b) | Completely simplify: | $\frac{x}{x-y} - \frac{y}{x+y}$ | 2 |
| c) | Given | $f(x) = \begin{cases} x+3 & \text{for } x > -2. \\ (x+3)^2 & \text{for } x \leq -2 \end{cases}$ | |
| | Find | i. $f(-1)$ | 1 |
| | | ii. $f(-3)$ | 1 |
| d) | Solve for x : | $2 - 3x \leq 8$ | 2 |
| e) | Evaluate | $4^{2x} \times 16^{2-x}$ | 2 |

Question 12 (15 marks) Use a SEPARATE working booklet

Marks

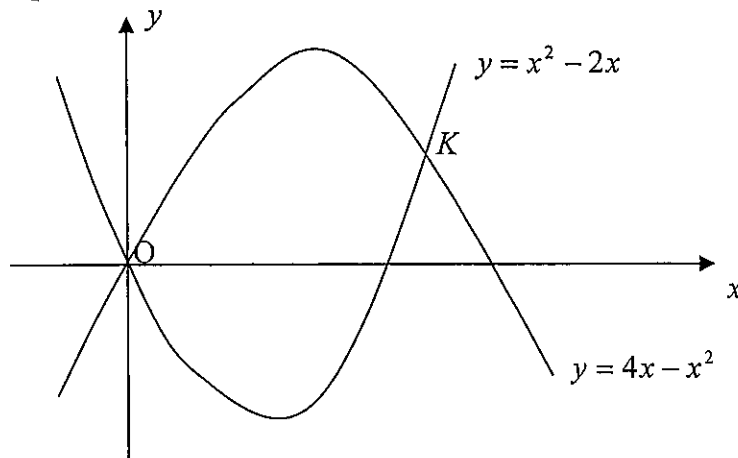
a) Find the values of x and y such that $\frac{1}{4-\sqrt{3}} = x + y\sqrt{3}$. 2

b) Solve $3\tan^2 2\theta - 1 = 0$ for $0 \leq \theta \leq 2\pi$. 3

c) Solve $|3x - 2| = 5x - 3$ 3

d) Differentiate $\tan x \cos x$ with respect to x . 1

e) The diagram shows the parabolas $y = x^2 - 2x$ and $y = 4x - x^2$. The parabolas intersect at the origin O and at the point K .



i. Find the coordinates of point K . 1

ii. Find the area of the region between the two curves. 2

f) A function is given by $f(x) = 2x^4 - 8x^2 - 7$. Find the coordinates of the stationary points of $f(x)$ and determine their nature. 3

End of Question 12

Question 13 (continued)

Marks

- b) A parabola has its focus at the point (4, 0) and its vertex at (1, 0).
- i) What is the focal length? **1**
 - ii) What is the equation of the directrix? **1**
 - ii) What is the equation of the parabola? **1**
- c) Show by shading on a sketch the region defined by the inequalities **3**
- $$2x + 3y < 6$$
- $$x - y \geq 1$$

End of Question 13

Question 15 (15 marks) Use a SEPARATE working booklet

Marks

a) Differentiate each of the following with respect to x .

i. $10x - 3x^6$ 1

ii. $5\sqrt{x}$ 1

iii. $\frac{1}{(4x+3)^5}$ 2

iv. $\sqrt{9-4x^3}$ 2

v. $(x^3-5)(4x^2+1)$ 2

b) Find the indefinite integral:

i. $\int \frac{3}{x^5} dx$ 1

ii. $\int (3x+2)^4 dx$ 2

iii. $\int \frac{x^3 - 3x^2 - 2x}{x} dx$ 2

d) Find the equation of the tangent to the curve $f(x) = 2x^3 - 3x^2 - 12x + 7$ 2
at the point where $x = 1$.

End of Question 15

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$



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Multiple Choice Answer Sheet

Completely fill the response oval representing the most correct answer.

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

7. A ○ B ○ C ○ D ○

8. A ○ B ○ C ○ D ○

9. A ○ B ○ C ○ D ○

10. A ○ B ○ C ○ D ○

Mathematics Year 12.
March 2013

Multiple choice.

1 b

2 b

3 d

4 c

5 b

6 d

7 b

8 d

9 b

10 a

Question 11

$$g) \quad \frac{x-2}{4} - \frac{x}{2} = \frac{3}{2}$$

$$\frac{x-2}{4} - \frac{2x}{4} = \frac{6}{4}$$

$$\therefore x-2-2x = 6$$

$$-x = 6+2$$

$$x = -8$$

1 for no fraction

1 answer.

Question 12

d) let $f(x) = \tan x \cos x$

$$f'(x) = \tan x (-\sin x) + \cos x (\sec^2 x) \\ = -\tan x \sin x + \cos x$$

1 for answer.

e) i sub $y = 4x - x^2$ into $y = x^2 - 2x$

Thus $4x - x^2 = x^2 - 2x$

$$2x^2 - 6x = 0$$

$$2x(x-3) = 0$$

$$x = 0 \text{ or } 3$$

when $x=3$, $y=3$

$\therefore K$ is $(3, 3)$

1 for K.

ii Area = $\int_0^3 [4x - x^2 - (x^2 - 2x)] dx$

$$= \int_0^3 (6x - 2x^2) dx$$

$$= \left[3x^2 - \frac{2x^3}{3} \right]_0^3$$

$$= 3(3)^2 - \frac{2(3)^3}{3} - 0$$

$$= 27 - 18$$

$$= 9 \text{ units}$$

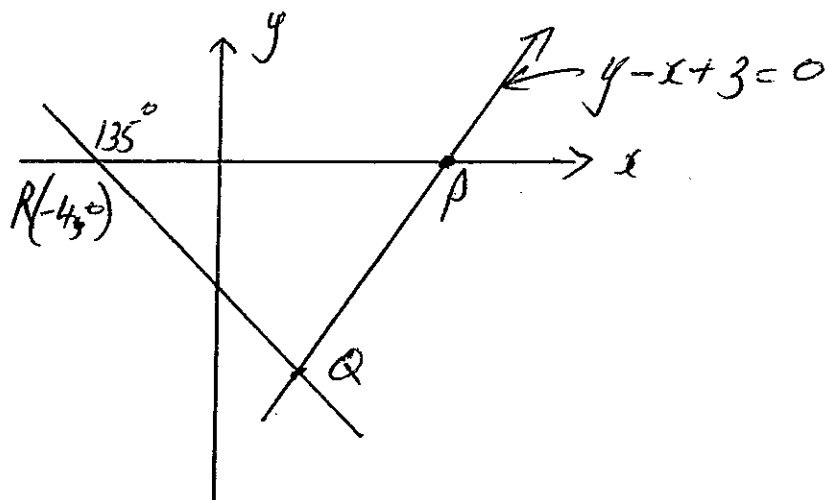
1 for $\int_0^3 [] dx$

1 for answer.

b)

Question 13

a)



- i Gradient of QR is -1
 Gradient of PQ is 1 (from $y = x - 3$)
 $m_1 \times m_2 = -1 \times 1 = -1$
 $\therefore$ PQ is perpendicular to QR

- ii gradient, $m = -1$ it passes through $(3, 0)$
 $\therefore$ equation through P // to QR is
 $y - 0 = -1(x - 3)$
 $y = -x + 3$

- iii R is $(-4, 0)$ line parallel to PQ has $m = 1$
 $\therefore$ equation is
 $y - 0 = 1(x - (-4))$
 $y = x + 4$
 $x - y + 4 = 0$

- iv sub $y = -x + 3$ into $x - y + 4 = 0$
 This gives $x - (-x + 3) + 4 = 0$
 $x + x - 3 + 4 = 0$
 $2x = -1$
 $x = -\frac{1}{2}$ sub into $y = -x + 3$

$$\therefore y = \frac{1}{2} + 3 = 3\frac{1}{2}$$

$$\therefore H is = (-\frac{1}{2}, 3\frac{1}{2})$$

- v HPQR is a rectangle. Opposite sides are parallel and QR $\perp$ PQ.

1 for showing
 $m_1 \times m_2 = -1 \times 1 = -1$

1 for $m = -1$
 and $(3, 0)$
 1 for answer

1 for R
 and $m = 1$
 1 for showing
 steps to
 answer

1 for $x = -\frac{1}{2}$
 1 for H

1 for rectangle
 1 for reason

Question 14

$$\begin{aligned} a) \quad & a(x-3)^2 + b(x+2) + c \\ & = a(x^2 - 6x + 9) + bx + 2b + c \\ & = ax^2 - 6ax + 9a + bx + 2b + c \\ & = ax^2 + (b - 6a)x + (9a + 2b + c) \\ & = 4x^2 + 0x - 9 \end{aligned}$$

← 1 for this line

$$\begin{aligned} \therefore a &= 4, \quad b - 6a = 0, \quad 9a + 2b + c = -9 \\ a &= 4, \quad b - 6 \times 4 = 0 \\ & \quad b = 24 \end{aligned}$$

1 for equating

$$\text{and } 9a + 2b + c = -9$$

$$9 \times 4 + 2 \times 24 + c = -9$$

$$36 + 48 + c = -9$$

$$c = -9 - 84$$

$$c = -93$$

$$\therefore a = 4, \quad b = 24 \text{ and } c = -93$$

1 for final answers.

$$b) \text{ let the roots be } \beta \text{ and } 2\beta$$

$$\therefore \beta + 2\beta = -\frac{b}{a} = \frac{9}{2} \text{ and } \beta \times 2\beta = \frac{c}{a}$$

$$3\beta = \frac{9}{2}$$

$$\beta = \frac{9}{6} = \frac{3}{2} \text{ --- (1)}$$

$$2\beta^2 = \frac{b}{a}$$

$$2\left(\frac{3}{2}\right)^2 = \frac{b}{a} \text{ from (1)}$$

$$2 \times \frac{9}{4} \times 2 = b$$

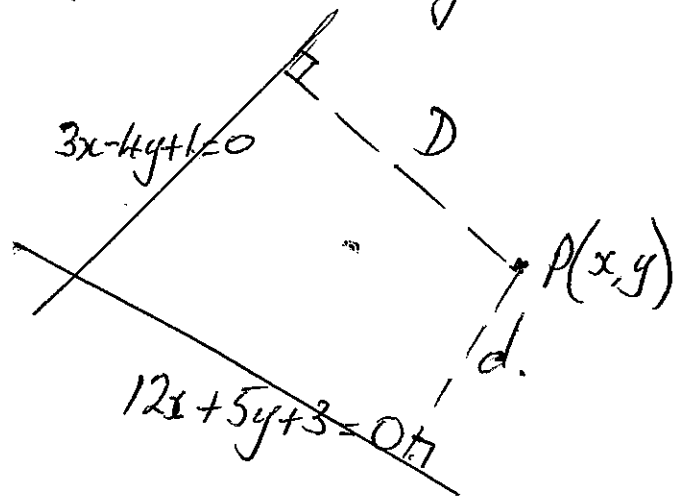
$$b = 9$$

1 for $3\beta =$
and $2\beta^2 =$

1 for answer

Question 14.

e) schematic view only.



$$3d = D.$$

$$\therefore 3 \frac{|12x + 5y + 3|}{\sqrt{12^2 + 5^2}} = \frac{|3x - 4y + 1|}{\sqrt{3^2 + 4^2}}$$

$$\frac{3(12x + 5y + 3)}{13} = \frac{3x - 4y + 1}{5}$$

$$15(12x + 5y + 3) = 13(3x - 4y + 1)$$

$$180x + 75y + 45 = 39x - 52y + 13$$

$$141x + 127y + 32 = 0$$

1 for this line

← 1 for this

← 1 for final answer

Question 15

b) ii $\int (3x+2)^4 dx = \frac{(3x+2)^5}{3 \times 5} + C$
 $= \frac{(3x+2)^5}{15} + C$

1 This line
1 for answer
(1 off for no C)

iii $\int \frac{x^3 - 3x^2 - 2x}{x} dx$

$$= \int x^2 - 3x - 2 dx$$
$$= \frac{x^3}{3} - \frac{3x^2}{2} - 2x + C$$

1 for this line
1 for answer

d) $f(x) = 2x^3 - 3x^2 - 12x + 7$

$$f'(x) = 6x^2 - 6x - 12$$

when $x=1$, $f(1) = -6$, $f'(1) = -12$

$\therefore$ the equation of the tangent is

$$y - y_1 = m(x - x_1)$$

~~$$y - 7 = -12(x - 1)$$~~

$$y - -6 = -12(x - 1)$$

$$y + 6 = -12x + 12$$

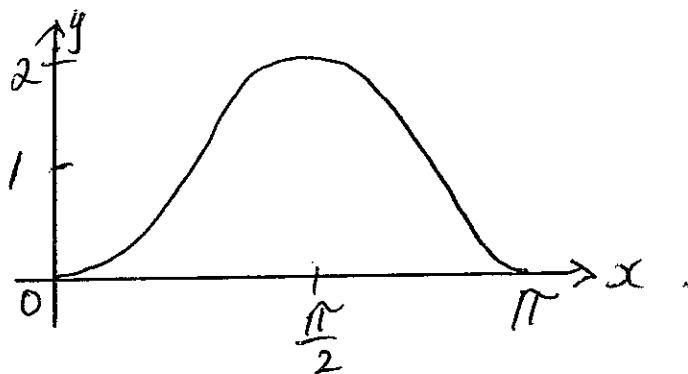
$$y = -12x + 6$$

1 for $f(1) =$
and $f'(1) =$

1 for final
answer

Question 16.

e) i



ii $\int_0^{\frac{\pi}{2}} (1 - \cos 2x) dx$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
y	0	1	2

$$= \frac{h}{2} [y_0 + 2y_1 + y_2]$$

$$= \frac{\pi}{8} [0 + 2(1) + 2]$$

$$= \frac{\pi}{2}$$

$$\approx 1.6$$

iii $\int_0^{\frac{\pi}{2}} (1 - \cos 2x) dx = \left[x - \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{2}}$

$$= \left[\frac{\pi}{2} - \frac{1}{2} \sin \pi - \left(0 - \frac{1}{2} \sin 0 \right) \right]$$

$$= \frac{\pi}{2}$$

1 for shape

1 for correct marking on axis.

(1 off for poor shape)

(1 off for not measuring axes)

1 for this line

1 for answer.

1 for integration

1 for answer.