Killara HIGH SCHOOL

STUDENT NUMBER

MATHEMATICS

ASSESSMENT TASK NO: 1

Date : 2nd December 2014

TIME ALLOWED: *Reading time: 5 minutes*
Working time: 1 hour

INSTRUCTIONS: This paper consists of 7 questions
ANSWER Multiple choice questions in the multiple choice sheet
Start each question (5 – 7) in a new booklet
Marks may not be given for untidy and poorly arranged work.
All relevant working should be shown.

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

OUTCOMES ASSESSED:

H1 - seeks to apply mathematical techniques to problems in a wide range of practical contexts
H4 - expresses practical problems in mathematical terms based on simple given models
H6 - uses the derivative to determine the features of the graph of a function
H7 - uses the features of a graph to deduce information about the derivative
H9 - communicates using mathematical language, notation, diagrams and graphs

	MARKS
QUESTION 1-4 Multiple Choice	/4
QUESTION 5 Introduction to Calculus/Locus	/15
QUESTION 6 Locus/Applications of differentiation	/16
QUESTION 7 Applications of differentiation	/15
	/50

This assessment task constitutes 10% of the Higher School Certificate Course Assessment.

Section I

4 marks
Attempt Questions 1 – 4
Allow about 5 minutes for this section

Use the multiple-choice answer sheet for questions 1 – 4 (Detach from paper)

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1 What are the coordinates of the *focus* of the parabola $(x-4)^2=8(y+3)$?

(A) $(-4, 1)$

(B) $(4, -1)$

(C) $(-3, -1)$

(D) $(-3, -4)$

2 Determine the points of discontinuity of $f(x)=\frac{3}{x}+\frac{2x}{x-4}$.

(A) $x = 0$ only

(B) $x=-4$ only

(C) $x=0,\ 4$

(D) No points of discontinuity

3 If $f(x)=-x^2+x$, then which of the following expressions represents $f'(x)$?

(A) $\displaystyle \lim_{h\to 0}\frac{\left(-x^2+x+h\right)-\left(-x^2+x\right)}{h}$

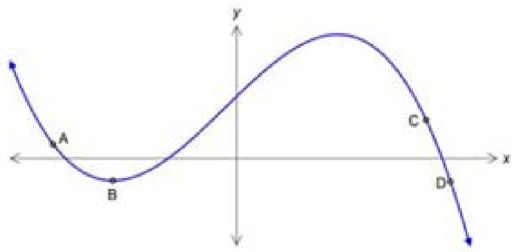
(B) $\displaystyle \lim_{h\to x}\frac{\left(-x^2+x+h\right)-\left(-x^2+x\right)}{h}$

(C) $\displaystyle \frac{-(x+h)^2+(x+h)-\left(-x^2+x\right)}{h}$

(D) $\displaystyle \lim_{h\to 0}\frac{-(x+h)^2+(x+h)-\left(-x^2+x\right)}{h}$

4 State which point on the diagram relates to the following:

$$y > 0, \quad \frac{dy}{dx} < 0, \quad \frac{d^2y}{dx^2} > 0$$



(A) A

(B) B

(C) C

(D) D

End of Multiple Choice

SectionII

46 marks
For Questions 5 – 7 allow about 55 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 5 (15 Marks) Start on a NEW page **Marks**

a) Differentiate with respect to x.

(i) $x^7 + 5$ **2**

(ii) $\sqrt{2x+7}$ **2**

(iii) $(3x-1)^2\left(x^2+4\right)$ **2**

(iv) $\dfrac{6x-7}{2x^2-3}$ **2**

b) Evaluate : $\lim\limits_{x\to\infty}\dfrac{x^3-2x+3}{6-4x^2-3x^3}$ **2**

c) (i) Draw the locus of all the points $P(x,y)$ in the number plane such that they are equidistant from x –axis and the y – axis. **1**

(ii) Write down the algebraic equation(s) representing this locus. **1**

d) The equation of a parabola is given by $y = x^2 - 2x + 5$

(i) Find the coordinates of its vertex **2**

(ii) State the focal length of the parabola. **1**

End of Question 5

Question 6 (16 Marks) Start on a NEW page

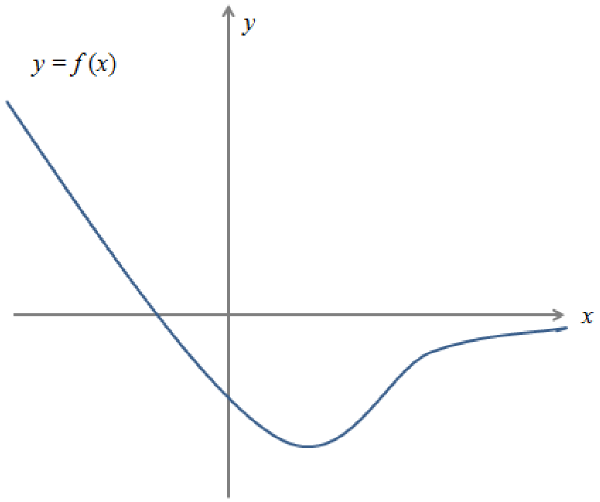
Marks

a) Write down the equation of the tangent to the curve $y = 3x^2 - 4x + 5$ at the point where $x = 1$. **2**

b) The coordinates of A and B are $(-2,1)$ and $(4,-6)$ respectively. **3**

Find the equation of the locus of all points $P(x, y)$ such that PA is perpendicular to PB, explaining the geometrical significance of this locus.

c) Copy the following graph into your answer booklet and on the same graph, draw the function $y = f'(x)$. **2**



d) Sketch a continuous smooth curve for $x \geq 0$, where: **2**

$$f(0) = 1$$

$$f'(x) < 0 \text{ and } f''(x) > 0 \text{ for } 0 < x < 2,$$

$$f'(2) = 0,$$

$$f(2) = -2$$

$$f'(x) > 0 \text{ and } f''(x) > 0 \text{ for } x > 2.$$

Question 6 continued on page 6

e) Consider the function $y = 1 + 3x - x^3$, for $-2 \le x \le 3$.

(i) Find all stationary points and determine their nature. **3**

(ii) Find the point of inflection. **1**

(iii) Sketch the curve for $-2 \le x \le 3$. Do not find the x- intercepts. **2**

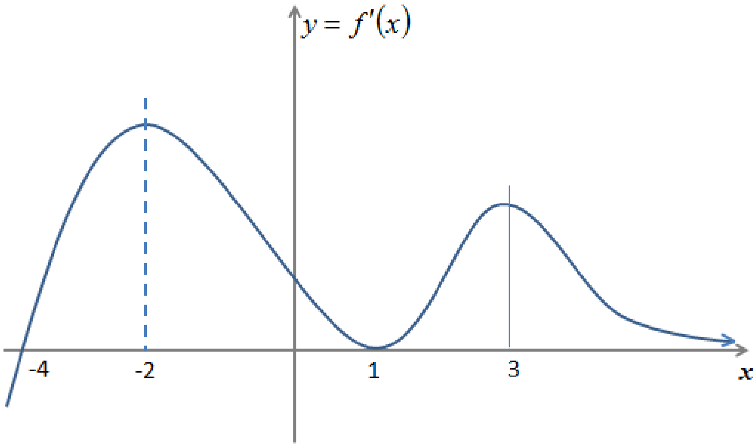
(iv) What is the minimum value for the curve over the stated domain? **1**

End of Question 6

Question 7 (15 Marks) Start on a NEW page **Marks**

a) Find the primitive function of $x^7 + 3$. **2**

b) The graph of $y = f'(x)$ is shown below.



(i) There are two stationary points on $y = f(x)$. **3**
Determine their x- coordinates and the nature of each of these stationary points.

(ii) What happens to $f(x)$ for large values of x? **1**

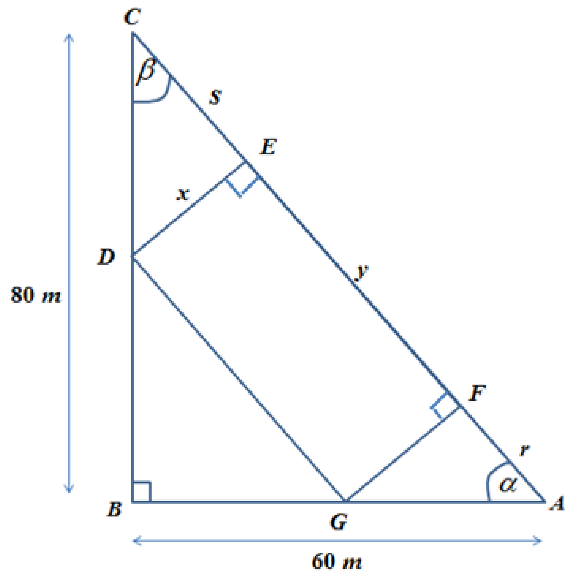
c) $f(x) = x^3 + ax^2 + bx$. Given that the maximum turning point occurs at $x = 1$ and a point of inflection at $x = 2$ on the graph of $y = f(x)$.

(i) Show that $a = -6$. **2**

(ii) Hence, find the value of b. **1**

Question 7 continued on page 8

d) A lot of land has the form of a right triangle, with perpendicular sides 60 and 80 metres long. *DEFG* is a rectangular slab of width x and length y metres inside the triangular block.



(i) Show that $r = \frac{3}{4}x$ and $s = \frac{4}{3}x$. **2**

(ii) Show that $y = 100 - \frac{25}{12}x$. **1**

(iii) Use Calculus to find the length and width of the largest rectangular base area on which the building can be erected, facing the hypotenuse of the triangle. **3**

End of paper

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Multiple Choice Answer Sheet

Completely fill the response oval representing the most correct answer.

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

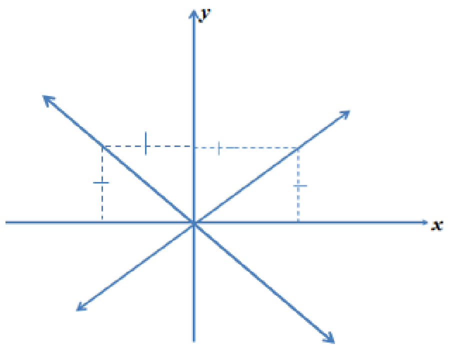
$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x + \sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x + \sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

1.	B	
2.	C	
3.	D	
4.	A	
5.a)	$y = x^7 + 5$ $\frac{dy}{dx} = 7x^6$	2 marks: Correct answer 1 mark: one mistake
(ii)	$y = \sqrt{2x+7}$ $y' = \frac{1}{2\sqrt{2x+7}} \times 2$ $= \frac{1}{\sqrt{2x+7}}$	2 marks: correct answer 1 mark: gives the answer as $\frac{1}{2\sqrt{2x+7}}$ but does not multiply the derivative
(iii)	$y = (3x-1)^2 (x^2+4)$ $u = (3x-1)^2 \quad u' = 2(3x-1)\times 3$ $v = (x^2+4) \quad v' = 2x$ **1 mark** $\frac{dy}{dx} = (x^2+4)6(3x-1) + 2x(3x-1)^2$ **1 mark** OR $2(3x-1)(6x^2 - x + 12)$	1 mark: Correct u, u', v, v' 1 mark: correctly applies the product rule using their u, u', v, v'
(iv)	$y = \frac{6x-7}{2x^2-3}$ $u = 6x - 7 \quad u' = 6$	

	$v = 2x^2 - 3 \qquad v' = 4x$ **1 mark** $\frac{dy}{dx} = \frac{(2x^2-3)6-(6x-7)4x}{(2x^2-3)^2}$ **1 mark** OR $\frac{dy}{dx} = \frac{-12x^2+28x-18}{(2x^2-3)^2}$	
b)	$\lim_{x\to\infty} \frac{x^3-2x+3}{6-4x^2-3x^3}$ $= \lim_{x\to\infty} \frac{\frac{x^3}{x^3}-\frac{2x}{x^3}+\frac{3}{x^3}}{\frac{6}{x^3}-\frac{4x^2}{x^3}-\frac{3x^3}{x^3}}$ $= \lim_{x\to\infty} \frac{1-0+0}{0-0-3}$ $= \frac{-1}{3}$	2 marks: correct answer with working 1 mark: divides by x^3 both numerator and denominator OR 1 mark: Shows that $\lim_{x\to\infty} \frac{1}{x} = 0$

c)(i)		1 mark

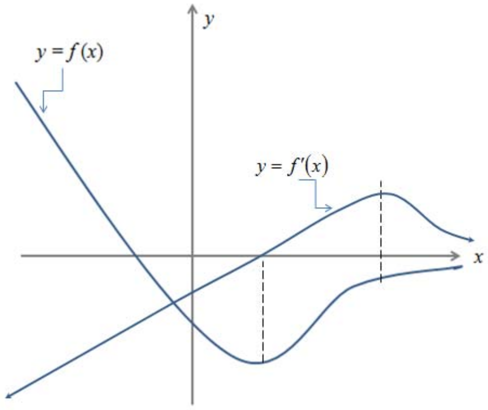
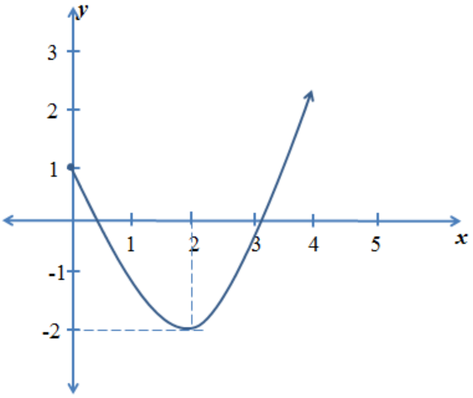
(ii)	$y = \pm x$ OR $y = \pm\lvert x\rvert$	1 mark (writes the equation of the graph they have drawn in (i)
d)(i)	$y = x^2 - 2x + 5$ $y - 5 + 1 = x^2 - 2x + 1$ $(x-1)^2 = y - 4$ Vertex $(1,4)$	2 marks: correct answer from correct working 1 mark: attempts to complete the square or any other method of equivalent merit (sets $\frac{dy}{dx} = 0$ etc)
(ii)	4a = 1 $\therefore a = \frac{1}{4}$	1 mark: correct answer

Question 6

a)	$y = 3x^2 - 4x + 5$ $\frac{dy}{dx} = 6x - 4$ At $x = 1$, $\frac{dy}{dx} = 6(1) - 4 = 2$ **1 mark** When x = 1, y = 4 $\therefore$ equation of the tangent is	**1 mark: correctly calculates the gradient of the tangent** **1 mark: correctly gives the equation of the tangent using their gradient and point.**

	$y-4=2(x-1)$ **1 mark** $2x-y+2=0$	
b)	$A(-2,1)$ and $B(4,-6)$ $PA \perp PB \qquad \therefore m_{PA} \times m_{PB} = -1$ $\dfrac{y-1}{x+2} \times \dfrac{y+6}{x-4} = -1$ **1 mark** $(y-1)(y+6) = -1(x+2)(x-4)$ $y^2+5y-6 = -x^2+2x+8$	**1 mark: applies** $\therefore m_{PA} \times m_{PB} = -1$

	$x^2-2x-8+y^2+5y-6=0$ $x^2-2x+y^2+5y=8+6$ $x^2-2x+1+y^2+5y+\left(\dfrac{5}{2}\right)^2 = 8+6+\left(\dfrac{5}{2}\right)^2$ $(x-1)^2+\left(y+\dfrac{5}{2}\right)^2 = \dfrac{81}{4}$ **1 mark** Which is a circle with centre $\left(1,-\dfrac{5}{2}\right)$ and radius $\dfrac{9}{2}$ **1 mark**	1 mark: gives the correct equation 1 mark: Gives the geometrical significance of the locus

c)		2 marks: correct shape, correct positions for x-intercept, turning point 1 mark: Any one of the following is missing or incorrect
d)		2 mark: correct shape, correct y-intercept and turning point clearly shown and labelled. (Labelled and scaled axes required) 1 mark: correct shape, intercept and turning point shown.
e)(i)	$y=1+3x-x^3$ $\frac{dy}{dx}=3-3x^2$ For stationary points $\frac{dy}{dx}=0$ $\frac{dy}{dx}=3-3x^2=0$ $x=1$ or $x=-1$ **1 mark** $\frac{d^2y}{dx^2}=-6x$	1 mark: correctly calculates the x-values of the stationary points

<table>
<tr>
<td></td>
<td>When $x=1$, $\frac{d^2y}{dx^2}= -6\times1=-6<0$

Maximum turning point at $(1,3)$

When $x=-1$, $\frac{d^2y}{dx^2}= -6\times-1=6>0$

Minimum turning point at $(-1,-1)$</td>
<td>1 mark: correctly determines the nature of each of the stationary <u>points</u>.

(ie. One mark for each point)</td>
</tr>
<tr>
<td>(ii)</td>
<td>For possible points of inflection,

$\frac{d^2y}{dx^2}= -6x = 0$

$\therefore x=0$

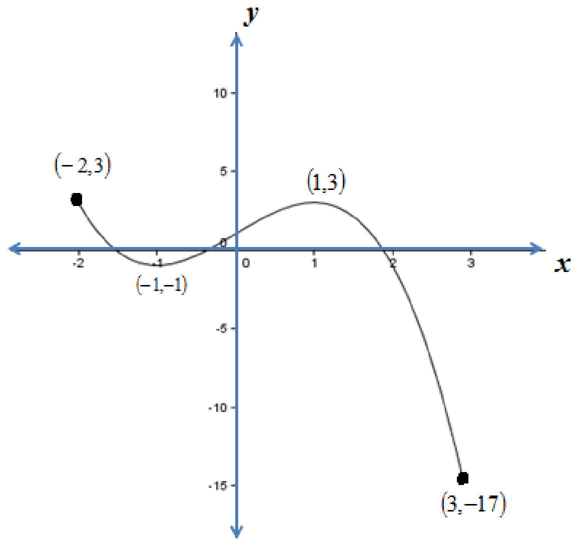
When x = 0, y = 1

<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td></tr>
<tr><td>$\frac{d^2y}{dx^2}$</td><td>6</td><td>0</td><td>-6</td></tr>
</table>

Change in concavity, hence point of

inflection at $(0,1)$</td>
<td>1 mark: sets $\frac{d^2y}{dx^2}= 0$ and tests for concavity to establish point of inflection.</td>
</tr>
<tr>
<td>(iii)</td>
<td>X = -2, y = 3 $(-2,3)$

X= 3, y = -17 $(3,-17)$</td>
<td></td>
</tr>
</table>



(iv)	-17	1 mark: correct answer

Question 7

(a)	Primitive = $\dfrac{x^8}{8} + 3x + C$	**1 mark: for** $\dfrac{x^8}{8} + 3x$ 1 mark: for +C
(b)(i)	$\dfrac{dy}{dx} = 0$ at $x = -4$ and $x = 1$ **1 mark**	1 mark: correctly gives the two stationary points

<table>
<tr>
<td></td>
<td>
<table>
<tr><td>x</td><td>-4^-</td><td>-4</td><td>-4^+</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>-</td><td>0</td><td>+</td></tr>
<tr><td></td><td>\</td><td>—</td><td>/</td></tr>
</table>

Minimum turning point at $x = -4$.

<table>
<tr><td>x</td><td>1^-</td><td>1</td><td>1^+</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>/</td><td>—</td><td>/</td></tr>
<tr><td>$\frac{d^2y}{dx^2}$</td><td>–</td><td>0</td><td>+</td></tr>
</table>

Point of inflection at $x = 1$
</td>
<td>1 mark each: correctly determines the nature of each of the stationary points</td>
</tr>
<tr>
<td>b)(ii)</td>
<td>As $x \to \infty$, $f(x)$ approaches a horizontal asymptote.

($f'(x)$approaches a limiting value zero)</td>
<td>1 mark: correct answer</td>
</tr>
<tr>
<td>c)(i)</td>
<td>$f(x) = x^3 + ax^2 + bx.$

$f'(x) = 3x^2 + 2ax + b.$

$f'(1) = 3 + 2a + b = 0$ $\qquad 2a + b = -3$

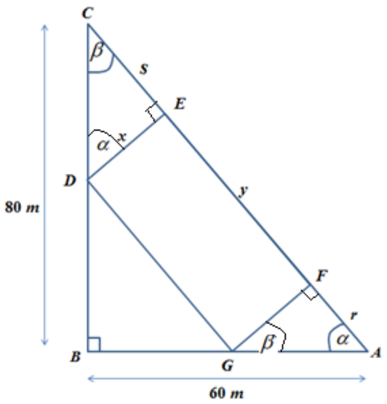
$f''(x) = 6x + 2a$

$f''(2) = 12 + 2a = 0$

$\therefore a = -6$</td>
<td>2 mark: correctly obtains the two simultaneous equations and evaluates a.

1 mark: gives the correct $f'(x)$, $f''(x)$ and attempts to find the value of a .</td>
</tr>
<tr>
<td>(ii)</td>
<td>$2a + b = -3$ $\qquad 2(-6) + b = -3$

$b = 9$</td>
<td>1 mark: substitutes the value of a to correctly evaluate b.</td>
</tr>
</table>

d)(i)		

	$\Delta AFG \,\|\|\|\, \Delta ABC$ $\frac{AF}{AB} = \frac{AG}{BC} = \frac{FG}{BC}$ $\frac{r}{60} = \frac{x}{80}$ $r = \frac{60}{80}x = \frac{3}{4}x$ **Similarly,** $\Delta CED \,\|\|\|\, \Delta CBA$ $\frac{CE}{CB} = \frac{ED}{BA}$ $\frac{s}{80} = \frac{x}{60}$ $rs = \frac{80}{60}x = \frac{4}{3}x$	**1 mark: for each proof**

(ii)	**Using Pythagoras' Theorem,** **AC = 100** $r+s+y=100$ $\therefore y=100-s-r$ $\therefore y=100-\frac{4}{3}x-\frac{3}{4}x$ $=100-\frac{25}{12}x$	**1 mark: gives the result** $r+s+y=100$ and uses result from (I) to complete the proof
(iii)	$A=xy$ $A=x\left(100-\frac{25}{12}x\right)$ $=100x-\frac{25}{12}x^2$ **1 mark** $\frac{dA}{dx}=100-\frac{25}{12}\times 2x$ = 0 for stationary points. $100=\frac{25}{12}\times 2x$ $\therefore x=24$ **1 mark** $\frac{d^2A}{dx^2}=-\frac{25}{6}<0$ Ie. Area is maximum when x = 24 m $\therefore y=100-\frac{25}{12}\times 24$ = 50m	**1 mark: derives the correct expression for area of the rectangle in terms of *x*.** **1 mark: sets** $\frac{dA}{dx}$ = 0 and calculates the value of x. 1 mark: establishes that the value of x = 24 gives the minimum area and gives the corresponding y.