KILLARA HIGH SCHOOL

Year 12, 2 Unit Advanced Mathematics
Half-Yearly Examination

Reading time: 5 mins
Examination: 3 hours

Section I
Questions 1 to 10 – Multiple Choice (10 marks , 1 mark per question)
Allow 18 minutes for this section of the paper

Section II
Questions 11 to 16 – 6 questions of 15 marks each(90 marks).
Allow 27 minutes for each of the 6 questions in this section of the paper.
Each question must be answered in a separate writing booklet.

Instructions:

- Approved calculators may be used
- Answer Multiple choice questions on provided answer sheet (at back of this test)
- Standard Integral sheet is provided (at back of this test)
- You should use a separate booklet for each of questions 11 to 16
- Show all of your working clearly.
- Marks will only be awarded to answers that are recorded on the Multiple Choice Answer sheets, or the answer booklets. You will receive no marks for working on this examination paper.

Section I

Multiple Choice

1. Evaluate $\sqrt{\frac{235.1-4.15}{4+1.3}}$ correct to two significant figures

    A. 15.3

    B. 6.6

    C. 7.68

    D. 2.87

2. Factorise $x^3 - 27$

    A. $(x-3)(x^2+6x+9)$

    B. $(x+3)(x^2-6x+9)$

    C. $(x+3)(x^2-3x+9)$

    D. $(x-3)(x^2+3x+9)$

3. $(x-4)^2 + (y+1)^2 = 16$ is a

    A. Parabola, focus (4, -1)

    B. Circle, radius = 16, centre (4, -1)

    C. Circle, radius = 4, centre (-4, 1)

    D. Circle, radius = 4, centre (4, -1)

4. $\int x^5 + x^3 + 1\, dx$ is equal to

    A. $\frac{x^6}{6} + \frac{x^4}{4} + x + C$

    B. $\frac{x^6}{6} + \frac{x^4}{4} + x$

    C. $x^6 + x^4 + x + C$

    D. $x^6 + x^4 + x$

5. $\frac{d}{dx}$ $(x. sinx)$ is equal to

   A. cos^2x

   B. $x. sinx + cosx$

   C. $x. cosx + sinx$

   D. sin^2x

6. $\int sin\frac{x}{4}dx$ is equal to

   A. $\frac{1}{4}sin\frac{x}{4}$ + C

   B. $-\frac{1}{4}cos\frac{x}{4}$ + C

   C. $4sin\frac{x}{4}$ + C

   D. $-4cos\frac{x}{4}$ + C

7. The curve $y = 4cos3x$ has

   A. Amplitude 4 and Period $\frac{2\pi}{3}$

   B. Amplitude 4 and Period $\frac{\pi}{3}$

   C. Amplitude 3 and Period $\frac{\pi}{2}$

   D. Amplitude 3 and Period 4π

8. The curve $y = f(x)$ has gradient function $\frac{dy}{dx} = 3x^2 + 4x - 1$ and passes through the point B(0,1) . Its equation is given by

   A. $y = x^3 + 2x^2 - x + C$

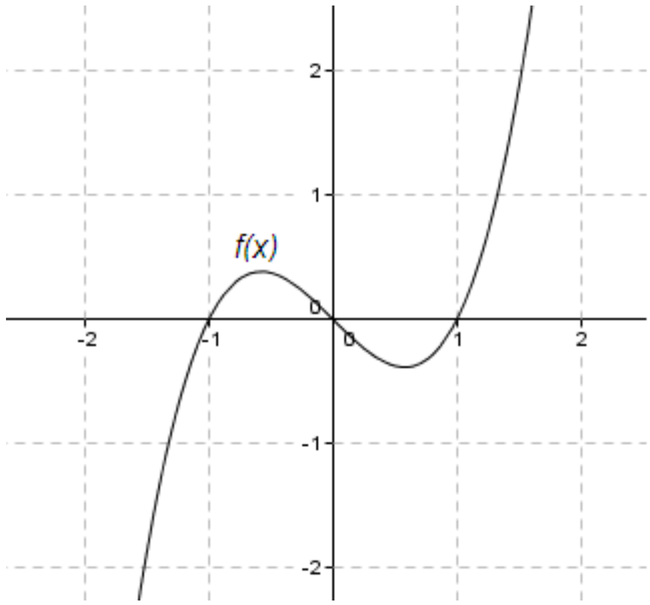
   B. $y = x^3 + 2x^2 - x + 2$

   C. $y = x^3 + 2x^2 - x + 1$

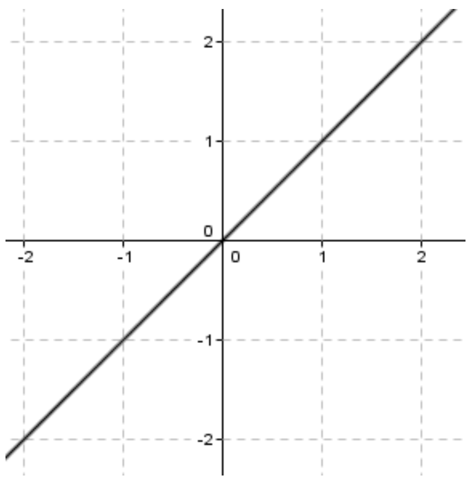
   D. $y = 6x + 4$

9. The point of inflexion of the curve $y = x^3 - 12x + 1$ is
   A. (2 , -15)
   B. (-2 , 17)
   C. (0 , 0)
   D. (0 , 1)

10. The curve *f(x)* is shown below, the graph of its gradient function is



A



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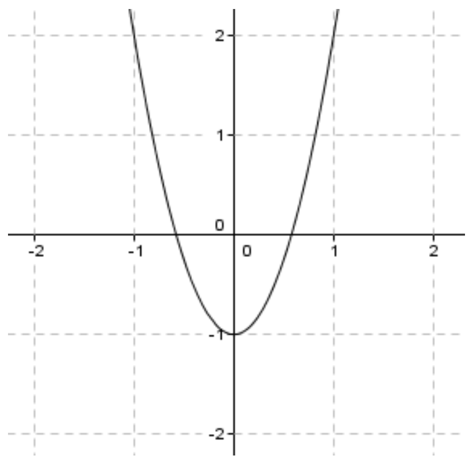

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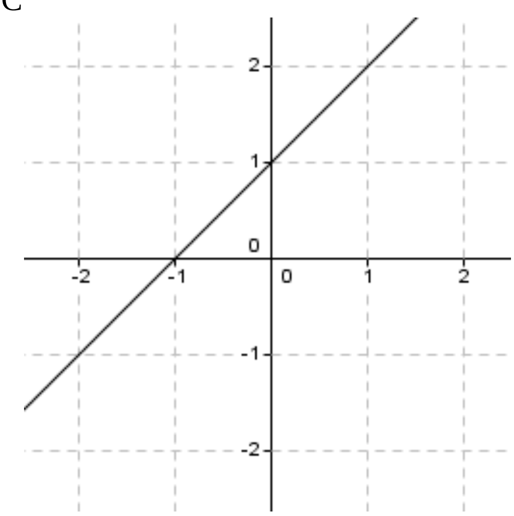

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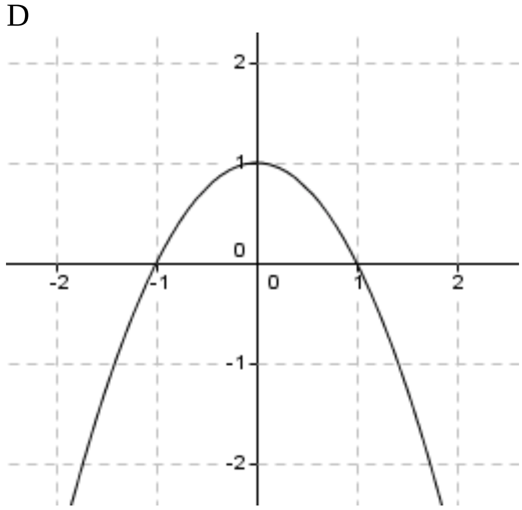


Section II

please answer each question in a new answer booklet Marks

11. (15 marks)

a) For the equation $5x^2 + 2x + k = 0$

i. What is the discriminant? 1

ii. For what values of k does $5x^2 + 2x + k = 0$ have real roots? 1

b) Using the substitution $u = x^2$ solve the equation $4x^4 - 5x^2 - 9 = 0$ 2

c) Find the coordinates of the focus of the parabola with equation $x^2 = 4(y-1)$. 1

d) Express $(2\sqrt{3}+1)(2-\sqrt{3})$ in the form $a\sqrt{3}+b$ 2

e) Solve the inequality $|3x - 5| \geq 2$ 2

f) Given the points A(1,3) and B(4,-2)

i. Determine the equation of the line AB 1

ii. What is the size of the acute angle that AB makes with the x axis (to the nearest degree)? 1

g) Prove $\frac{cot\theta}{cosec\theta-1} - \frac{cos\theta}{1+sin\theta} = 2\ tan\theta$ 2

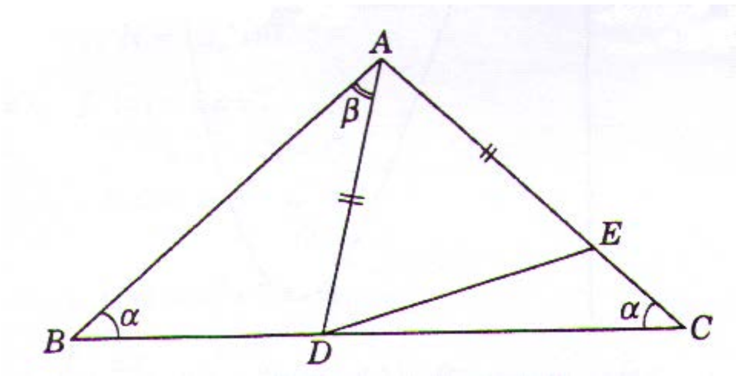
h) If $\sin\theta = -\frac{4}{11}$ and $\tan\theta > 0$ find the exact value of $\cos\theta$ 2

please answer each question in a new answer booklet Marks

12. (15 marks)

a) In the isosceles triangle ABC as shown below, $\angle ABC = \angle ACB = \alpha$.

The points D and E lie on BC and AC, so that AD = AE, as shown in the diagram. Let $\angle BAD = \beta$



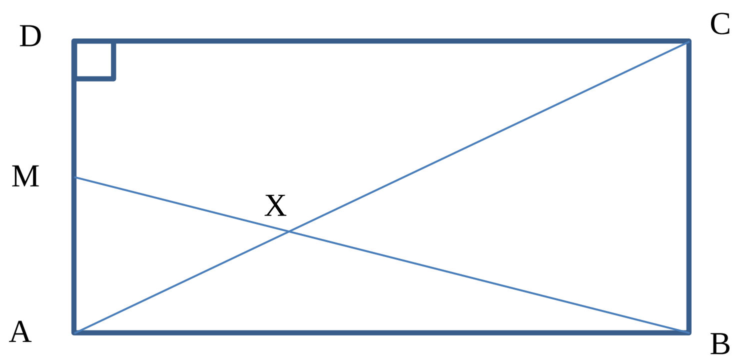
i. Explain why $\angle ADC = \alpha + \beta$ 1

ii. Find $\angle DAC$ in terms of α and β 1

iii. Hence, or otherwise, find $\angle EDC$ in terms of β 2

b) In the diagram below *ABCD* is a rectangle and *AB* = *2AD.*

The point M is the midpoint of AD. The line BM meets AC at X.



i. Show that $\Delta AXM \; ||| \; \Delta BXC$ 2

ii. Show that *3CX* = *2AC* 2

Marks

c) Differentiate $x^2 + x$ from first principles 2

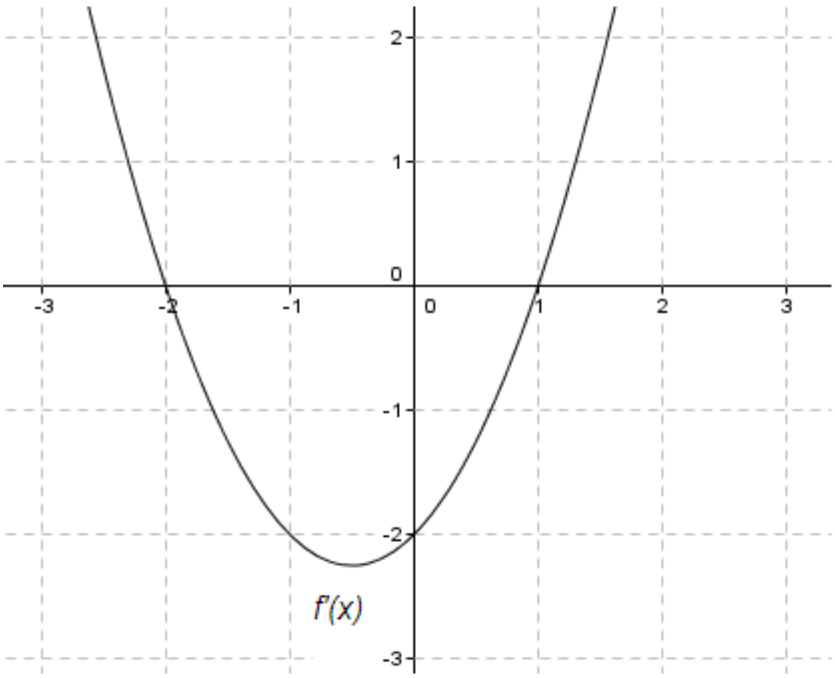
d) Find the equation of the tangent to the curve 2

$y = x^2 - 3x + 5$ when $x = 1$

e) Differentiate $\frac{\sin x}{(x+1)^2}$ 2

f) Sketch a primitive function *f(x)* given the graph of the derivative 1

function *f'(x)* as shown



please answer each question in a new answer booklet Marks

13. (15 marks)

a) Consider the curve given by $y = \frac{1}{3}x^3 - x^2$

i. Find any turning points and determine their nature 3

ii. Find any points of inflexion 1

iii. Sketch the curve in the domain -1 ≤ x ≤ *4* clearly indicating all important points. 2

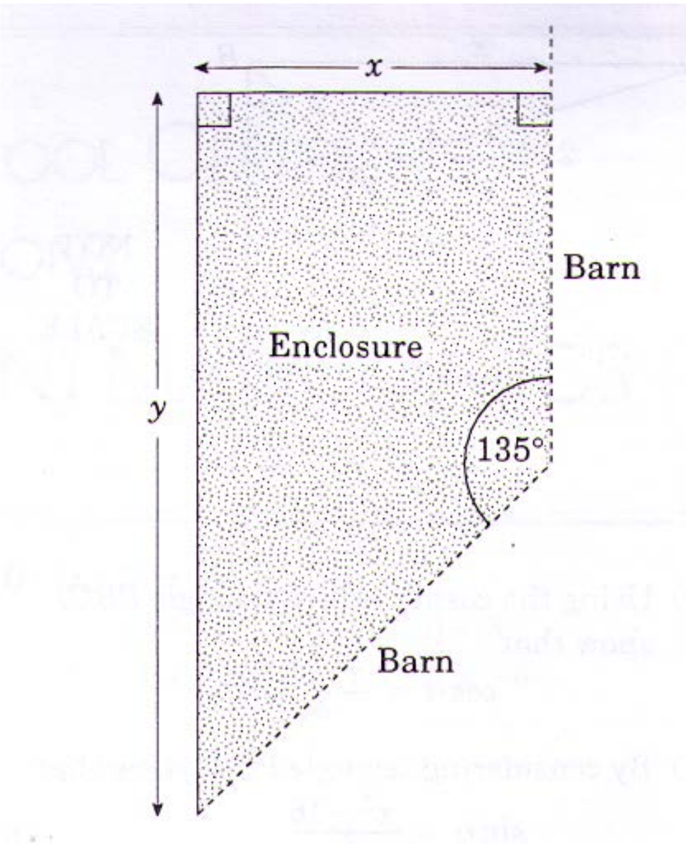
iv. For what values of x is the curve concave down? 1

b) The curve $y = 3x^2 - \frac{p}{x^2}$ has a turning point at $x = -1$. Find the constant p. 2

c) An enclosure is to be built adjoining a barn, as in the diagram shown below. The walls of the barn meet at 135^o, and 117 metres of fencing is available for the enclosure, so that $x + y = 117$ where x and y are as shown in the diagram. **Marks**

i. Show that the shaded area of the enclosure is given by $A = 117x - \frac{3}{2}x^2$ 3

ii. Show that the largest area of the enclosure occurs when $y = 2x$. 3



please answer each question in a new answer booklet Marks

14. ***(15 marks)***

a) Consider the function $f(x) = x^4 - x$.

i. Copy the table of values below into your writing booklet and calculate the missing values. 1

x	0	1	2	3	4
y	0				252

ii. Use these five values and the Trapezoidal rule to approximate the value of $\int_0^4 \mathrm{x}^4 - \mathrm{x} \quad dx$ 2

iii. Use these five values and Simpson's rule to approximate the value of $\int_0^4 \mathrm{x}^4 - \mathrm{x} \quad dx$ 2

iv. Calculate the exact value of $\int_0^4 \mathrm{x}^4 - \mathrm{x} \quad dx$ 2

v. Place in ascending order of accuracy your results for Simpson's rule, exact value and the Trapezoidal rule 1

vi. Briefly justify the ordering of your results in part v. 1

b) Considering the equations of $y = 2x$ and $y = \frac{1}{2}x^3$

i. Find the coordinates of the points of intersection 2

ii. Find the area between the 2 curves. 2

iii. Find the volume of the solid of revolution formed by rotating about the x-axis the area between the curves that is between the lines $x = 2$ and $x = 0$. 2

please answer each question in a new answer booklet Marks

15. (15 marks)

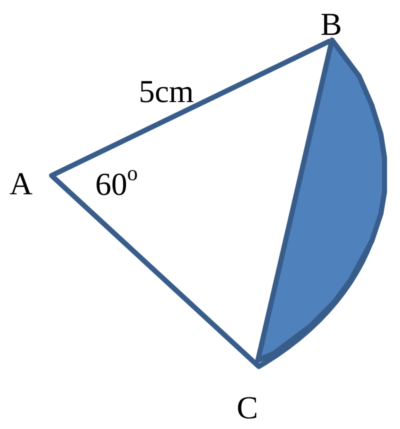
a) Sketch the curve $y = 1 - cos2x$ for $0 \leq x \leq \pi$ 3

Clearly show points at x = 0, $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$, and π

b) Solve $2sin^2x - 3sinx - 2 = 0$ for $0 \leq x \leq 2\pi$ 2

c) Calculate $\int_0^{\frac{\pi}{8}} sec^2 2x \, dx$ 2

d) For the sector of the circle shown below

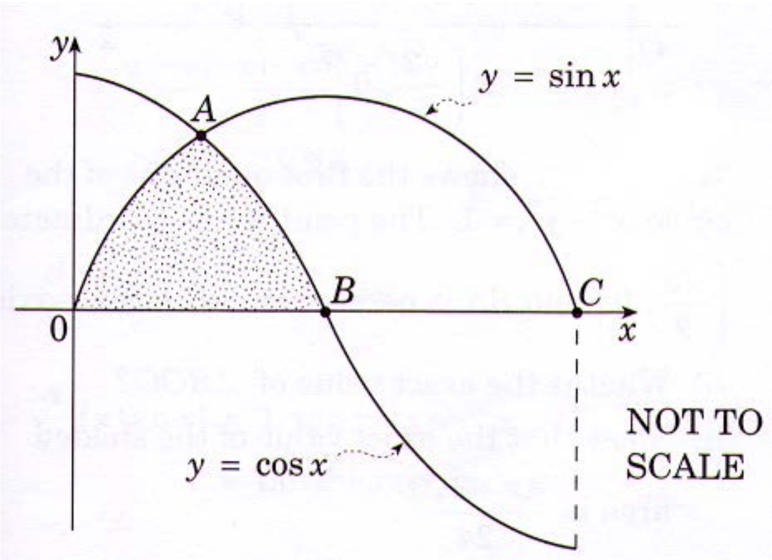


i. Calculate the exact arc length of the sector 1

ii. Calculate the exact Area of the shaded minor segment 3

Marks

e) The diagram below shows part of the curve of $y = sin\, x$ and $y = cos\, x$



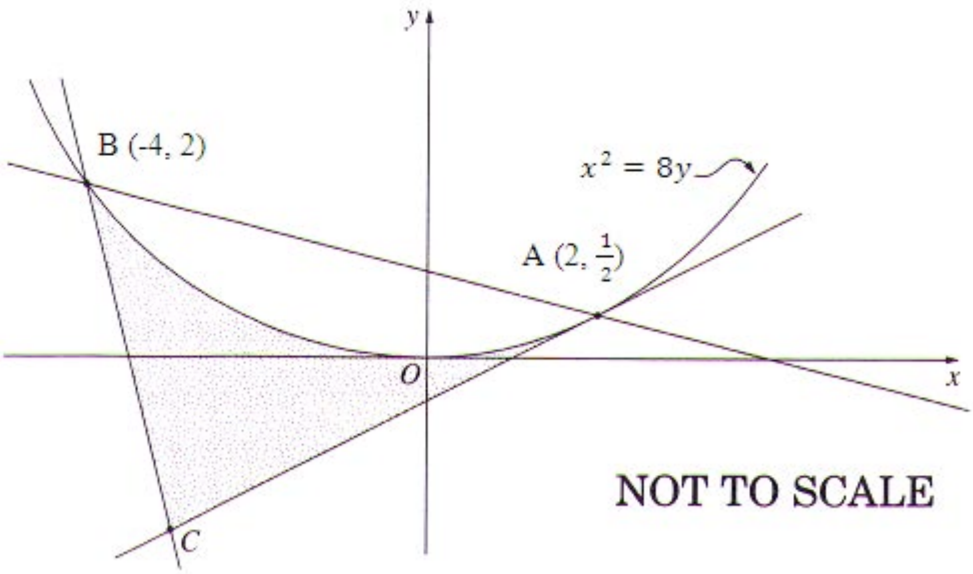
i. Find the coordinates of the points A and B 2

ii. Calculate the Area of the shaded region 2

please answer each question in a new answer booklet Marks

16. (15 marks)

a) The diagram below shows the graph of the parabola $x^2 = 8y$. The points A $(2, \frac{1}{2})$ and B (-4, 2) are on the parabola, and C is the point where the tangent to the parabola at A intersects the directrix.



i. Write down the equation of the directrix of the parabola $x^2 = 8y$ 1

ii. Show that the equation of the tangent to the parabola at point A is the line $x - 2y - 1 = 0$ 2

iii. Show that C is the point (-3, -2). 1

iv. Given that the equation of the line AB is $y = 1 - \frac{x}{4}$, find the area bounded by the line AB and the parabola. 2

v. Hence, or otherwise find the shaded Area bounded by the parabola, the tangent at A and the line BC. 3

b)

Marks

i. Show that $x = \frac{\pi}{6}$ is a solution of $\cot x = \tan 2x$ 1

ii. On the same set of axes, sketch the graphs of the functions $y = \cot x$ and $y = \tan 2x$ for $-\pi \leq x \leq \pi$. *Use a different pen colour, or different style of line for* $y = cot\, x$ *and* $y = tan\, 2x$. 3

iii. Hence find the number of solutions of $\cot x = \tan 2x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ 1

iv. Use your graphs to solve $\cot x \leq \tan 2x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ 1

END OF EXAMINATION

McCartney Q 13

2014

~~2013~~ - Killara - 2 Unit Half Yearly Solutions

Section 1

1 B

2 D

3 D

4 A

5 C

6 D

7 A

8 C

9 D

10 B

Worked solutions to Section 1

1 $\sqrt{\frac{(235.1-4.15)}{(4+1.3)}} = \sqrt{\frac{230.95}{5.3}}$

$= 6.60117\ldots$

$= 6.6$ 2 sig fig

$=$ B option

2 x^3-27

$= x^3-3^3$

$= (x-3)(x^2+3x+9)$

$=$ D option

3 $(x-4)^2+(y+1)^2=16$

$=(x-4)^2+(y+1)^2=4^2$

circle centre $(4,-1)$, $r=4$

$=$ option D

4 $\int x^5+x^3+1\,dx$

$= \frac{x^6}{6}+\frac{x^4}{4}+x+C$

$=$ option A

5 $y=x.\sin x$

$y'=u'v+v'u$

$y'=1.\sin x+\cos x.x$

$y'=x.\cos x+\sin x$

$=$ option C

6 $\int \sin\frac{x}{4}\,dx$

we know if $y=-\cos\frac{x}{4}$

$y'=\frac{1}{4}\sin\frac{x}{4}$

we need -4, out the fron

$\cos\frac{x}{4}$ $\therefore$ $y=-4\cos\frac{x}{4}+c$

Section 1 answers continued...

7 $y = 4\cos 3x$

amplitude

period $= 2\pi \div$

$= \frac{2\pi}{3}$

$\therefore$ option A

8 $y' = 3x^2 + 4x - 1$

$y = x^3 + 2x^2 - x + c$

sub $(0,1)$

$1 = 0^3 + 2\times 0^2 - 0 + c$

$\therefore c = 1$

$\therefore y = x^3 + 2x^2 - x + 1$

$\therefore$ option C

9 $y = x^3 - 12x + 1$

$y' = 3x^2 - 12$

$y'' = 6x$

when $y'' = 0$, $x = 0$

sub $x = 0$ into

$y = x^3 - 12x + 1$

$y = 0^3 - 12\times 0 + 1$

$\therefore y = 1$

$\therefore$ point is $(0,1)$

check a point of inflexion.

at $x = -1$, $y'' = -6$

$x = 1$, $y'' = 6$

$\therefore (0,1)$ is point of inflexion

$\therefore$ option D

10 option B is the answer

Note: crosses x-axis in graph B at $x = \pm\frac{1}{2}$ where turning points are on $f(x)$ i.e gradient $= 0$.

Q11

a i $\Delta = b^2 - 4ac$

$\Delta = 2^2 - 4 \times 5 \times k$

$\Delta = 4 - 20k$

$\Delta = 4(1-5k)$

ii when $\Delta \geq 0$ we have real roots

$1 - 5k \geq 0$, $-5k \geq -1$

$k \leq \frac{1}{5}$

b $4x^4 - 5x^2 - 9 = 0$

let $u = x^2$

$4u^2 - 5u - 9 = 0$

$(4u-9)(u+1) = 0$

$\therefore u = \frac{9}{4}$, $u = -1$ (1 mark for solving correctly for u

when $u = \frac{9}{4}$, $x^2 = \frac{9}{4}$

$x = \pm \frac{3}{2}$

when $u = -1$, $x^2 = -1$

$\therefore$ no real values for x

$\therefore$ Solutions are $x = \pm \frac{3}{2}$ (1 mark for correct solution, or ECF solution from their u values)

c $x^2 = 4(y-1)$

vertex $= (0,1)$

$x^2 = 4a(y-1)$ $\therefore a = 1$

$\therefore$ Focus $= (0,2)$

d $(2\sqrt{3}+1)(2-\sqrt{3})$

$= 4\sqrt{3} - 6 + 2 - \sqrt{3}$

$= 3\sqrt{3} - 4$

in form $a\sqrt{3} + b$

$\therefore a = 3,\ b = -4$ (1 mark for a)

(1 mark for b)

e $|3x - 5| \geq 2$

$3x - 5 \leq -2$ or $3x - 5 \geq 2$

$3x \leq 3$ $\quad$ $3x \geq 7$

$x \leq 1$ $\quad$ $x \geq \frac{7}{3}$

(1 mark) (1 mark)

f $A(1,3)$; $B(4,-2)$

i gradient of AB $= \frac{3 - -2}{1-4}$

$= -\frac{5}{3}$

ii equation of AB $y - y_1 = m(x - x_1)$

$y - 3 = -\frac{5}{3}(x-1)$

$3y - 9 = -5x + 5$

$5x + 3y - 14 = 0$

iii obtuse angle $= 180° - \tan^{-1}\left(\frac{5}{3}\right)$

$= 180° - 59°$

$= 121°$

g LHS $= \dfrac{\cot\theta}{\operatorname{cosec}\theta - 1} - \dfrac{\cos\theta}{1+\sin\theta}$

$\because \operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\cos\theta}{1-\sin\theta} - \dfrac{\cos\theta}{1+\sin\theta}$

$= \dfrac{\cos\theta(1+\sin\theta) - \cos\theta(1-\sin\theta)}{1-\sin^2\theta}$

$= \dfrac{\cos\theta}{1-\sin^2\theta}(2\sin\theta)$ (1 mark).

$= \dfrac{\cos\theta \cdot 2\sin\theta}{\cos^2\theta}$ $\because \sin^2\theta + \cos^2\theta = 1$

$= \dfrac{2\sin\theta}{\cos\theta}$ (1 mark)

$= 2\tan\theta = $ RHS.

OR alternatively...

LHS $\dfrac{\cot\theta}{\operatorname{cosec}\theta - 1} \times \dfrac{\operatorname{cosec}\theta + 1}{\operatorname{cosec}\theta + 1} - \dfrac{\cos\theta}{1+\sin\theta} \times \dfrac{1-\sin\theta}{1-\sin\theta}$

$= \dfrac{\cot\theta(\operatorname{cosec}\theta + 1)}{\operatorname{cosec}^2\theta - 1} + \dfrac{-\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$

(1 mark) $= \dfrac{\cancel{\cot\theta}(\operatorname{cosec}\theta + 1)}{\cot^2\theta} + \dfrac{-\cancel{\cos\theta}(1-\sin\theta)}{\cos^2\theta}$ $\because 1+\cot^2\theta = \operatorname{cosec}^2\theta$

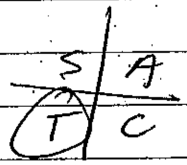
$1-\sin^2\theta = \cos^2\theta$

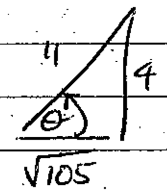
$= \dfrac{\operatorname{cosec}\theta}{\cot\theta} + \dfrac{1}{\cot\theta} - \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1}{\cancel{\sin\theta}} \times \dfrac{\cancel{\sin\theta}}{\cos\theta} - \dfrac{1}{\cos\theta} + \tan\theta + \tan\theta$ (1 mark).

$= 2\tan\theta = $ RHS.

h) $\sin\theta = \frac{-4}{11}$ and $\tan\theta > 0$





$\pi < \theta < \frac{3\pi}{2}$

$\cos\theta < 0$

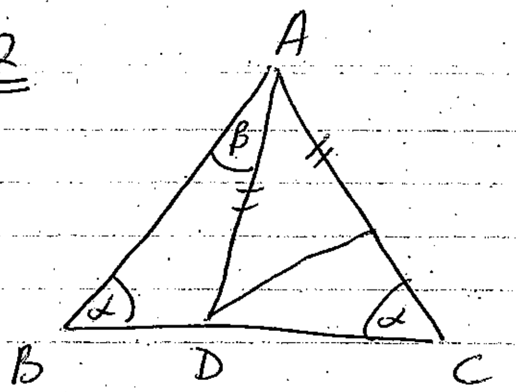
$\therefore \cos\theta = \frac{-\sqrt{105}}{11}$

(1 mark quadrant & sign).

(1 mark positive or negative version of $\frac{\sqrt{105}}{11}$)

Q12

a



i in ΔABD

ext $\angle ADC$ = sum of the interior opposite $\angle$'s

$= \alpha + \beta$

ii in ΔADC

$\angle ADC + \angle DCA + \angle DAC = 180°$ (sum of angles in a Δ)

$(\alpha+\beta) + \alpha + \angle DAC = 180°$

$\angle DAC = 180° - (2\alpha+\beta)$

iii In ΔADE

$\angle DAC = 180° - (2\alpha+\beta)$ from ii

$\angle ADE = \dfrac{180 - \angle DAC}{2}$

$= \dfrac{180 - [180 - (2\alpha+\beta)]}{2}$

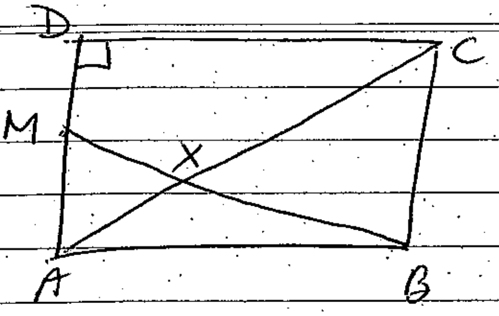
$= \alpha + \dfrac{\beta}{2}$ (1 mark) validly show $\angle ADE = \alpha + \dfrac{\beta}{2}$ or similar step to answe

$\angle ADE + \angle EDC = \angle ADC = \alpha + \beta$

$\alpha + \dfrac{\beta}{2} + \angle EDC = \alpha + \beta$

$\therefore \angle EDC = \dfrac{\beta}{2}$ (1 mark) makes a conclusion and gets $\angle EDC$.

12 b



i In ΔAXM and ΔBXC

(1 mark body)

$\angle AXM = \angle BXC$ (vertically opposite $\angle$'s)

$\angle XMA = \angle CBX$ (alternate $\angle$'s of parallel lines, $AD \parallel BC$).

(1 mark conclusion) $\therefore \Delta AXM \parallel\parallel \Delta BXC$ (Equiangular)

ii $\dfrac{AX}{CX} = \dfrac{AM}{BC}$ (matching sides of similar triangles)

$\dfrac{AX}{CX} + 1 = \dfrac{AM}{BC} + 1$

(1 mark) shows $AC = AX + XC$

$\dfrac{AX + CX}{CX} = \dfrac{AM + BC}{BC}$ $\because BC = 2AM$, $AC = AX + XC$

$\dfrac{AC}{CX} = \dfrac{AM + 2AM}{2AM}$

$\dfrac{AC}{CX} = \dfrac{3}{2}$

$\therefore 3CX = 2AC$ (1 mark) correctly uses ratios to get answer.

OR $CX : AC$

$CX : AX + XC$

$2 : 1+2$ $\because AC = AX + XC$

and as $BC : AM = 2 : 1$ as M is midpoint other sides in ratio $2:1$ of similar Δ's } 1 mark

$\therefore CX : AC$
$= 2 : 3$

$\dfrac{CX}{AC} = \dfrac{2}{3}$ cross multiply

$3CX = 2AC$ (1 mark)

12c Let $f(x) = x^2 + x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ must show

$$f'(x) = \lim_{h \to 0} \frac{\left[(x+h)^2 + x + h\right] - \left[x^2 + x\right]}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$$ (1 mark)

$$f'(x) = \frac{\cancel{h}(2x + 1 + h)}{\cancel{h}} \lim_{h \to 0}$$

$$f'(x) = 2x + 1$$ (1 mark) final answer.

Note 1 mark given out of 2 for good process but errors made

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d $y = x^2 - 3x + 5$

$y' = 2x - 3$

$M_T = 2(1) - 3 = -1$ at $x = 1$

when $x = 1$, $y = 1 - 3 + 5 = 3$

$\therefore M_T = -1$, point is $(1, 3)$ (1 mark)

$y - y_1 = m(x - x_1)$

$y - 3 = -1(x - 1)$

$y - 3 = -x + 1$

$x + y - 4 = 0$ (1 mark)

or if correct equation for their incorrect gradient.

e $\quad y = \dfrac{\sin x}{(x+1)^2}$ $\qquad \left[\begin{array}{l} y = \dfrac{u}{v} \\ y' = \dfrac{vu' - uv'}{v^2} \end{array}\right.$

$$y' = \frac{(x+1)^2 \cdot \cos x - 2(x+1) \cdot \sin x}{(x+1)^4}$$ (1 mark) substitutes into quotient rule

$$y' = \frac{\left[(x+1) \cdot \cos x - 2\sin x\right] \cdot \cancel{(x+1)}}{(x+1)^{\cancel{4}\,3}}$$

$$\therefore y' = \frac{(x+1) \cdot \cos x - 2\sin x}{(x+1)^3}$$ (1 mark) final answer.

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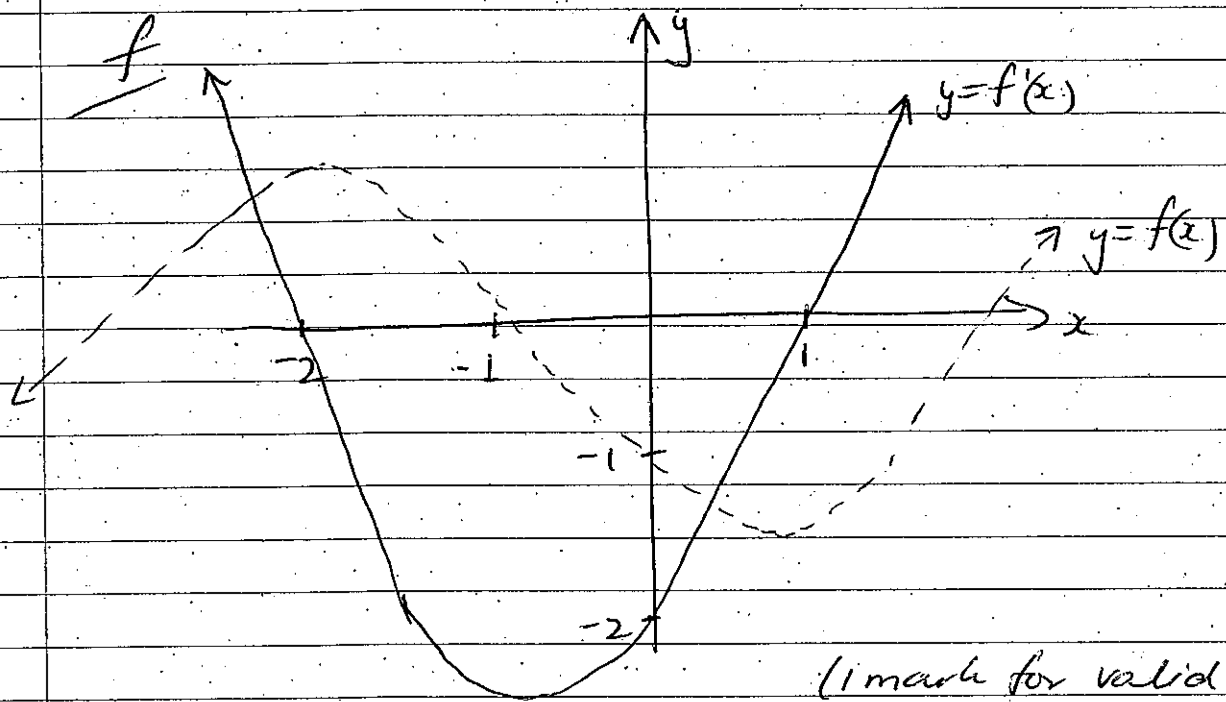
f



(1 mark for valid attempt, must have turning points at -2 and 1

13 a $y = \frac{1}{3}x^3 - x^2$

i $y' = x^2 - 2x$

$y' = x(x-2)$

$\therefore y' = 0$ when $x = 0,\ x = 2$

[redacted] $\therefore$ stationary points are $(0,0)$ and $\left(2, \frac{-4}{3}\right)$

$y'' = 2x - 2$

(1 mark) for max — when $x = 0$ $\frac{d^2y}{dx^2} < 0$ $\therefore (0,0)$ is a maximum turning point

ECF o.k (1 mark) [redacted] — when $x = 2$ $\frac{d^2y}{dx^2} > 0$ $\therefore \left(2, \frac{-4}{3}\right)$ is a minimum turning point

Note: must show actual values if y' used to determine min & max.

3

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ii when $y'' = 0$ $y'' = 2(x-1)$

$x = 1$

x	0.9	1	1.1
y''	−0.2	0	+0.2

$\therefore$ change in concavity at $x = 1$

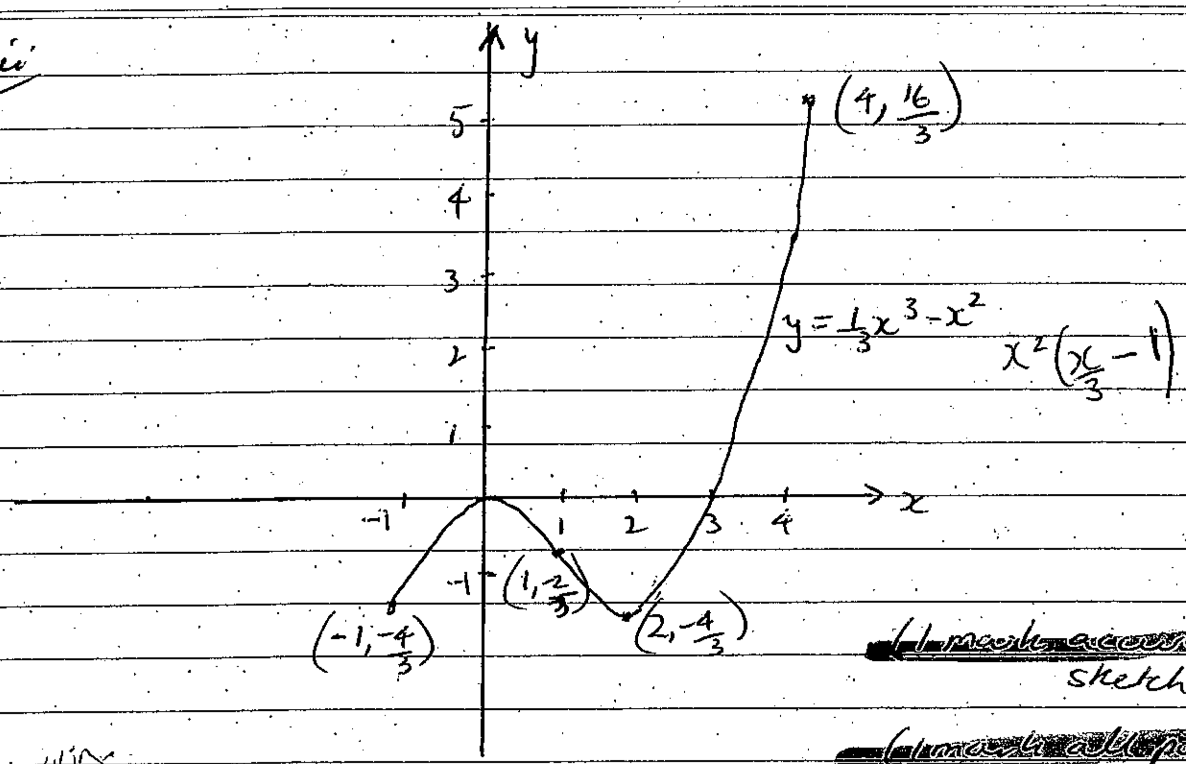
Must show change in concavity

1 [redacted]

m

Y

iii



[redacted] sketch)

[redacted] indicated)

2

-1 if error or some info missing

iv curve is concave down

for $-1 < x < 1$ (from the graph)

$x < 1$ ok. ~~must have domain restriction~~.

$x \leq 1$ is not.

1

13b $y = 3x^2 - \frac{p}{x^2}$

$y' = 6x + \frac{2p}{x^3}$ [redacted] first derivative

when $x = -1$, $y' = 0$

$\therefore 0 = 6 \times -1 + \frac{2p}{-1}$ $\quad 0 = -6 - 2p$

$2p = -6$

$p = -3$

$(0 = -6 - 2p) \times -1$

$2p + 6 = 0$

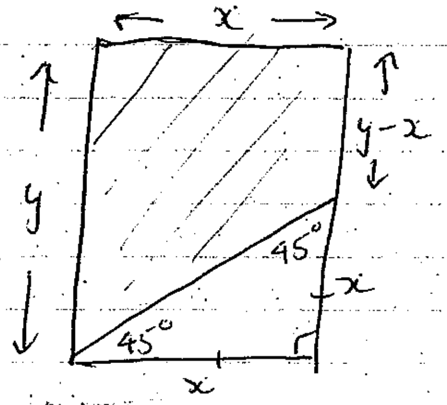
$2p = -6$

$\therefore p = -3$ [redacted] final answer

2

13a)

i)



$y - (y-x)$

$x + y = 117$

Area of unshaded Δ

[redacted]

Area of $\square = xy$

[redacted]

$A = xy - \frac{1}{2}x^2$

$A = x(117 - x) - \frac{1}{2}x^2$

[redacted]

/3

ii)

$\frac{dA}{dx} = 117 - 3x$

when $\frac{dA}{dx} = 0$ $\quad$ $0 = 117 - 3x$, $3x = 117$ $\}$ 1 mark

[redacted]

$\frac{d^2A}{dx^2} = -3$ which is < 0 $\quad$ Max when $x = 39$.

(1 mark) shows maximum correctly

when $x = 39$, $y = 117 - 39$, $y = 78$ $\quad$ (1 mark)

$\therefore y = 2x$. $\quad$ shows $y = 2x$ with good reasoning

$\therefore x = 39,\ y = 78$

3

Q14 a $f(x) = x^4 - x$

i

x	0	1	2	3	4
y	0	0	14	78	252

(1 mark)

ii Trapezoidal Rule

$$\int_0^4 x^4 - x \, dx = \frac{h}{2}\left[y_0 + y_4 + 2(y_1 + y_2 + y_3)\right]$$

$$h = \frac{4-0}{4} = 1$$

(1 mark uses correct h value and substitution)

$$= \frac{1}{2}\left[0 + 252 + 2(0 + 14 + 78)\right]$$

$$= 218 \text{ units}^2$$ (1 mark)

iii Simpson's Rule

$$\int_0^4 x^4 - x \, dx = \frac{h}{3}\left[(y_0 + y_4) + 4(y_1 + y_3) + 2y_2\right]$$

$$h = 1$$

(1 mark h value correct and substitution)

$$= \frac{1}{3}\left[(0 + 252) + 4(0 + 78) + 2 \times 14\right]$$

$$= 197\tfrac{1}{3} \text{ units}^2$$ (1 mark)

iv $$\int_0^4 x^4 - x \, dx$$

$$= \left[\frac{x^5}{5} - \frac{x^2}{2}\right]_0^4$$ (1 mark correct integration)

$$= \left[\frac{4^5}{5} - \frac{4^2}{2}\right] - 0$$

$$= 204.8 - 8$$

$$= 196.8 \text{ units}^2$$ (1 mark answer)

v. In ascending order of accuracy
trapezoidal rule, simpson's rule, exact

vi. Exact is done by integration and is most precise.

simpson's rule approximates areas using curves
so matches this curve well.

trapezoidal rule, use multiple trapeziums
so doesn't match curve well.

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b i. $y = 2x$, $y = \frac{1}{2}x^3$

$2x = \frac{1}{2}x^3$

$\frac{1}{2}x^3 - 2x = 0$

$x\left(\frac{1}{2}x^2 - 2\right) = 0$ (1 mark) factorised correctly.

$\therefore$ intersect when $x = 0$ and $\frac{1}{2}x^2 - 2 = 0$

$\frac{1}{2}x^2 = 2$

$x^2 = 4$

$x = \pm 2$

$\therefore$ points are $(0,0)$, $(2,4)$, $(-2,-4)$ (1 mark)

---

ii. $A = 2\int_0^2 2x - \frac{1}{2}x^3\,dx$

$A = 2\left[x^2 - \frac{x^4}{8}\right]_0^2$ (correct integration 1 mark)

$A = 2\left\{\left[4 - \frac{16}{8}\right] - 0\right\}$, $A = 2 \times 2 = 4$ units^2
(1 mark).

Q 14 b iii $V_1 = \pi \int_0^2 y^2\,dx$ where $y = 2x$

$$V_1 = \pi \int_0^2 (2x)^2\,dx$$

$$V_1 = \pi \int_0^2 4x^2\,dx$$

$$V_1 = \pi \left[\frac{4x^3}{3}\right]_0^2$$

$$V_1 = \pi . \frac{32}{3} \text{ units}^3$$

$V_2 = \pi \int_0^2 y^2\,dx$ where $y = \frac{1}{2}x^3$

$$V_2 = \pi \int_0^2 \left(\frac{1}{2}x^3\right)^2 dx$$

$V_2 = \pi \int_0^2 \frac{1}{4}x^6\,dx$ (1 mark V_1 or V_2 calculated, accurately) ✓

$$V_2 = \pi \left[\frac{x^7}{28}\right]_0^2$$

$$V_2 = \pi . \frac{128}{28} = \frac{32\pi}{7}$$

$\therefore V = V_1 - V_2$

$$V = \pi\left(\frac{32}{3} - \frac{32}{7}\right)$$

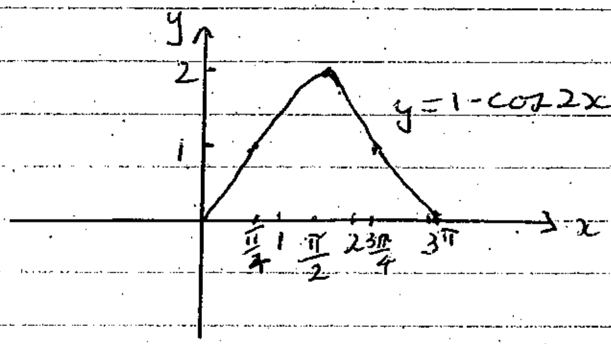
$V = \pi \frac{128}{21}$ units^3 or $6\frac{2}{21}\pi$ units^3 (1 mark final answer), or correct V for their V_1 and V_2 i.e. correct.

15a $y = 1 - \cos 2x$
for $0 \le x \le \pi$

(1 mark for correct shape)

(1 mark for curve between $y=0$ and 2)

(1 mark, clearly shows values for $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$)



b. $2\sin^2 x - 3\sin x - 2 = 0$

$(2\sin x + 1)(\sin x - 2) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = 2$ (1 mark)

no solution as $y = \sin x$ lies between -1 and 1

S	A
T	C

$\therefore x = \frac{\pi}{6}$ in 3rd & 4th quadrants

$\therefore x = \frac{7\pi}{6}, \frac{11\pi}{6}$ when $0 \le x \le 2\pi$ (1 mark)

15c/ $\displaystyle\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$

$= \left[\frac{1}{2}\tan 2x\right]_0^{\frac{\pi}{8}}$ (1 mark) correct integration

$= \left(\frac{1}{2} \times \tan\frac{\pi}{4}\right) - \left(\frac{1}{2} \times \tan 0\right)$

$= \frac{1}{2} \times 1 - 0$

$= \frac{1}{2}$ (1 mark)

d/ i/ $l = r\theta$

$l = 5 \times \frac{\pi}{3} = \frac{5\pi}{3}$ cm

ii/ sector Area

$A_s = \frac{1}{2} r^2 \theta$

$\therefore A_s = \frac{1}{2} \times 5^2 \times \frac{\pi}{3} = \frac{25\pi}{6}$ cm^2 (1 mark) Area of sector.

triangle ABC Area

$A_{ABC} = \frac{1}{2} ab \cdot \sin C$

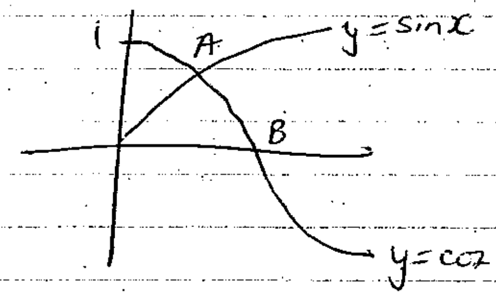
$= \frac{1}{2} \times 5 \times 5 \times \sin\frac{\pi}{3} = \frac{25}{2} \times \frac{\sqrt{3}}{2} = \frac{25\sqrt{3}}{4}$ cm^2 (1 mark) Area of Δ

$\therefore$ Area $= A_s - A_{ABC}$

$= \frac{25\pi}{6} - \frac{25\sqrt{3}}{4}$

$= \frac{50\pi}{12} - \frac{75\sqrt{3}}{12} = 25\left(\frac{2\pi - 3\sqrt{3}}{12}\right)$ cm^2 (1 mark). final Area or ECF with correct method.

15e i $y = \cos x$, $y = \sin x$



$\cos x = 0$, when $x = \frac{\pi}{2}$

let $\cos x = \sin x$

$\cos x - \sin x = 0$

$\therefore \cos x = \sin(90 - x)$

$\sin(90 - x) - \sin x = 0$

$\therefore 90 - x = x$

$90 = 2x$

$\therefore x = 45° = \frac{\pi}{4}$

(1 mark working that gets A and B x-values)

$\therefore A = \left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$, $B = \left(\frac{\pi}{2}, 0\right)$. (1 mark, final answers)

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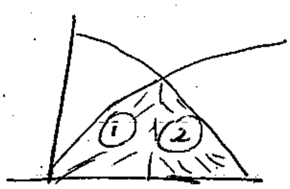
ii



Area = Area ① + Area ②

$$\text{Area ①} = \int_0^{\frac{\pi}{4}} \sin x \, dx$$

$$= \left[-\cos x\right]_0^{\frac{\pi}{4}}$$

$$= \frac{-1}{\sqrt{2}} - -1 = 1 - \frac{1}{\sqrt{2}} \text{ units}^2.$$

(1 mark correct method split area into 2 areas)

$$\text{Area ②} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx$$

$$= \left[\sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

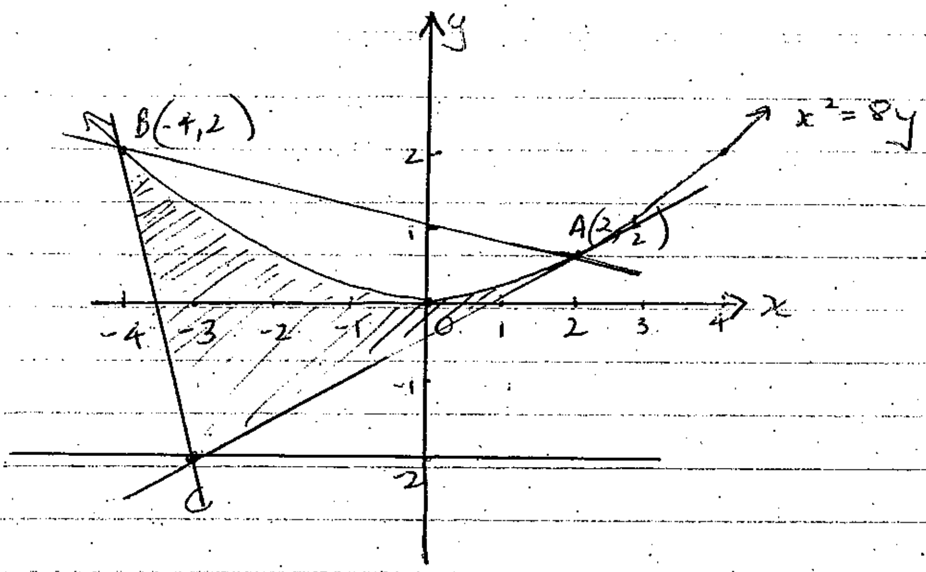
$$= 1 - \frac{1}{\sqrt{2}}$$

Area = Area ① + Area ②

$$= 2\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$= 2 - \sqrt{2} \text{ units}^2$$ (1 mark correct answer)

Q16.



a) i $x^2=8y$

$x^2=4ay$ $\therefore a=2$ $\therefore y=-2$ is directrix

ii $y=\frac{x^2}{8}$

$y'=\frac{x}{4}$

at $x=2$ $m_T=\frac{1}{2}$ (1 mark for gradient of tangent

$y-y_1=m(x-x_1)$

$\left[y-\frac{1}{2}=\frac{1}{2}(x-2)\right]\times 2$

$2y-1=x-2$

$\therefore x-2y-1=0$ (1 mark, correctly shows by any method).

iii let $C=(x,-2)$, C lies on $x-2y-1=0$

$x-2y-1=0$

$x_1-2(-2)-1=0$

$x_1+4-1=0$

$\therefore x_1=-3$

$\therefore C=(-3,-2)$

iv Area $= \int_{-4}^{2} \left(1 - \frac{x}{4}\right) - \frac{x^2}{8}\, dx$

$= \left[x - \frac{x^2}{8} - \frac{x^3}{24}\right]_{-4}^{2}$ (1 mark correct integration.

$= \left[2 - \frac{2^2}{8} - \frac{2^3}{24}\right] - \left[-4 - \frac{(-4)^2}{8} - \frac{(-4)^3}{24}\right]$

$= \frac{7}{6} - -\frac{10}{3} = \frac{9}{2}$

$= 4\frac{1}{2}$ units^2 (1 mark) final answer.

v Area of ΔCBA

distance AC $A = (2, \frac{1}{2})$, $C = (-3, -2)$

$d = \sqrt{(2 - -3)^2 + (\frac{1}{2} - -2)^2}$

$d = \sqrt{5^2 + (\frac{5}{2})^2}$

$d = \sqrt{25 + \frac{25}{4}} = \sqrt{\frac{125}{4}} = \frac{\sqrt{125}}{2} = \frac{5\sqrt{5}}{2}$ units

perp-distance from $B(-4, 2)$ to $x - 2y - 1 = 0$

$d_p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|1 \times -4 + -2 \times 2 + -1|}{\sqrt{1^2 + (-2)^2}}$

$d_p = \frac{|-4 + -4 + -1|}{\sqrt{5}} = \frac{9}{\sqrt{5}}$ units (1 mark perp distance

$\therefore$ Area $\Delta CBA = \frac{1}{2} \times \frac{9}{\sqrt{5}} \times \frac{5\sqrt{5}}{2} = \frac{45}{4}$ units^2 (1 mark)

from iv Shaded Area $= \Delta CBA -$ Area between AB and curve

$= \frac{45}{4} - 4\frac{1}{2}$

$= 6\frac{3}{4}$ units^2 (1 mark correct answer or ECF

Q16 b i $\cot x = \tan 2x$

sub $x = \frac{\pi}{6}$

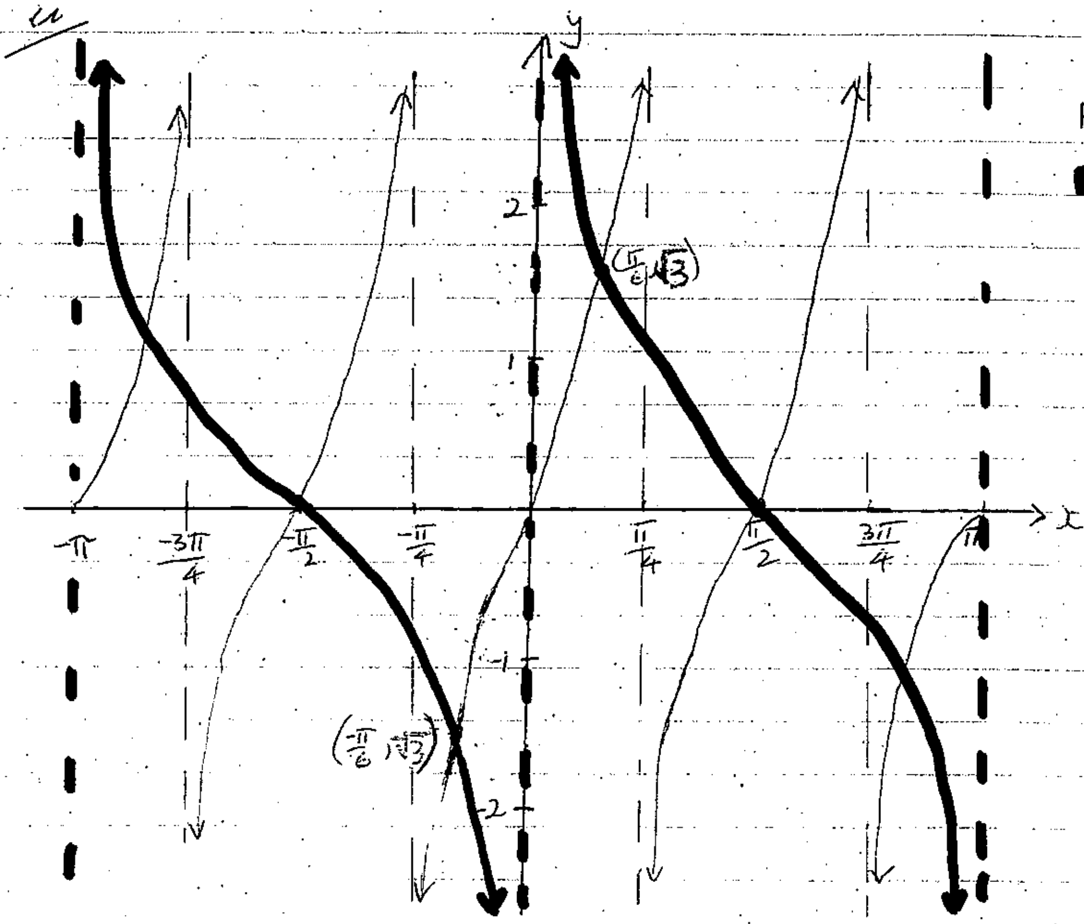
LHS = $\cot \frac{\pi}{6} = \sqrt{3}$

RHS = $\tan \frac{\pi}{3} = \sqrt{3}$

$\therefore x = \frac{\pi}{6}$ is a solution

(correctly shown by any valid method)

ii



(1 mark cotx)

(1 mark tan2x)

(1 mark asymptotes)

iii $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{-\pi}{2}, \frac{-\pi}{6}$ from graph.

iv $\frac{\pi}{6} \le x < \frac{\pi}{4}$ and $\frac{-\pi}{6} \le x < 0$ and $\frac{-\pi}{2} \le x < \frac{-\pi}{4}$