Killara HIGH SCHOOL

STUDENT NUMBER

MATHEMATICS

ASSESSMENT TASK NO: 1

Date : 25/11/15

TIME ALLOWED: *Reading time: 5 minutes*
Working time: 1 hour

INSTRUCTIONS: This paper consists of 7 questions
ANSWER Multiple choice questions in the multiple choice sheet
Start each question (5 – 7) in a new booklet
Marks may not be given for untidy and poorly arranged work.
All relevant working should be shown.

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

OUTCOMES ASSESSED:

H1 - seeks to apply mathematical techniques to problems in a wide range of practical contexts
H4 - expresses practical problems in mathematical terms based on simple given models
H6 - uses the derivative to determine the features of the graph of a function
H7 - uses the features of a graph to deduce information about the derivative
H9 - communicates using mathematical language, notation, diagrams and graphs

	MARKS
QUESTION 1-4 Multiple Choice	/4
QUESTION 5 Differentiation	/12
QUESTION 6 Locus and the Parabola	/13
QUESTION 7 Geometrical Applications of Differentiation	/13
	/42

This assessment task constitutes 10% of the Higher School Certificate Course Assessment.

Section I

4 marks
Attempt Questions 1 – 4
Allow about 5 minutes for this section

Use the multiple-choice answer sheet for questions 1 – 4 (Detach from paper)

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1 What is the value of the derivative of $y = 2\sqrt{x}$ at the point where $x = 4$?

(A) 2

(B) 1

(C) $\frac{1}{16}$

(D) $\frac{1}{2}$

2 What is the second derivative of $y = \frac{1}{2}x^6 - x^4 + x^2$?

(A) $3x^5 - 5x^3 + 2x$

(B) $15x^4 - \cancel{15}x^2 + 2$

(C) $\frac{1}{2}x^4 - x^2 + 1$

(D) $10x^4 - 9x^3 + 1$

$y' = \frac{1}{2} \times 6 . x^5 - 4x^3 + 2x$
$y' = 3x^5 - 4x^3 + 2x$
$y'' = 15x^4 - 12x^2 + 2$

3 A point R moves such that the shortest distance from R to $y = x$ is equal to the shortest distance from R to $y = -x$. Which of the following is not true?

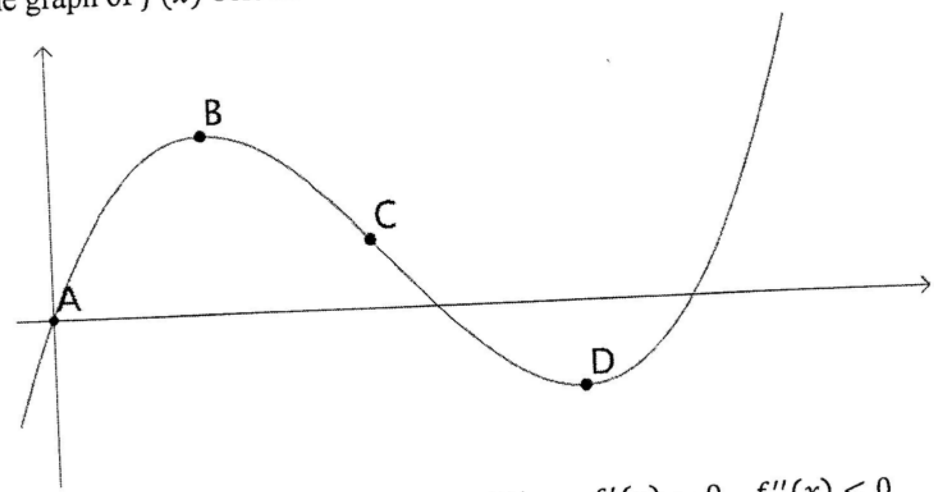
(A) The locus of R is a vertical straight line

(B) There exist more than one equation for the locus of R

(C) R would be fixed at the origin in order to satisfy the conditions

(D) The locus of R is a horizontal straight line

4 Given the graph of $f(x)$ below:



Which of the point(s) above satisfies the conditions: $f'(x) = 0\,,\ f''(x) < 0$

(A) Point A

(B) Point B

(C) Point D

(D) Not enough information to evaluate

End of Multiple Choice

Section II

36 marks

For Questions 5 – 7 allow about 55 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 5 (12 Marks) Start on a NEW page

Marks

a) Differentiate with respect to x.

(i) $y = x^5 - 2x^4$ **2**

(ii) $y = (6-2x)^3$ **2**

(iii) $y = (x-1)(4x+1)$ **2**

(iv) $y = \dfrac{2x+1}{x^2-1}$ **2**

b) Differentiate $f(x) = x^2 - 5x + 1$ using first principles. **2**

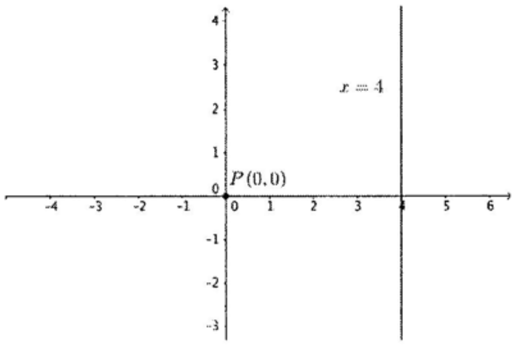
c) Find the equation of the tangent to the curve $y = x^3 - 2x^2 + 1$ at the point (2, 1). **2**

End of Question 5

Question 6 (13 Marks) Start on a NEW page **Marks**

a) Given $l_1: y = 2x$ and $l_2: y = 2x - 8$, point R moves such that the perpendicular distance from l_1 is equal to the perpendicular distance from l_2. Find the locus of R. **2**

b) P is a fixed point on the origin. Point R moves such that the distance PR is equal to the perpendicular distance of R from the line: $x = 4$.



(i) Find the locus of R. **2**

(ii) Explain why this is not a function. **1**

c) P is a fixed point on (2,4) and Q is a fixed point on (-2,2). Point R moves such that the lines PR and QR always intersect at right angles.

(i) Find the locus of R. **2**

(ii) Sketch the locus of R showing all important features. **2**

(iii) Show that R is always above the x axis. **1**

d) Given parabola: $x^2 = 4ay$:

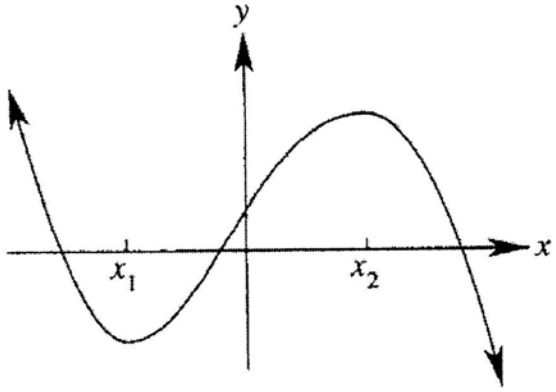
(i) Show that the latus rectum intersects the parabola at $(-2a, a)$ and $(2a, a)$ **1**

(ii) Show that the tangents on either ends of the latus rectum intersects at right angles. **2**

End of Question 6

Question 7 (13 Marks) Start on a NEW page **Marks**

a) The following is a graph of $f(x)$. Copy the graph into your answer booklet and on the same graph, draw the function $f'(x)$. **2**



b) Consider the curve $y = 2x^3 - 3x^2$

(i) Find the coordinates of the stationary points and determine their nature. **3**

(ii) Show that the point $(0.5, -0.5)$ is a point of inflexion. **2**

(iii) Sketch a graph of the function, indicating all important features. **3**

(iv) For what values of x is the curve concave up? **1**

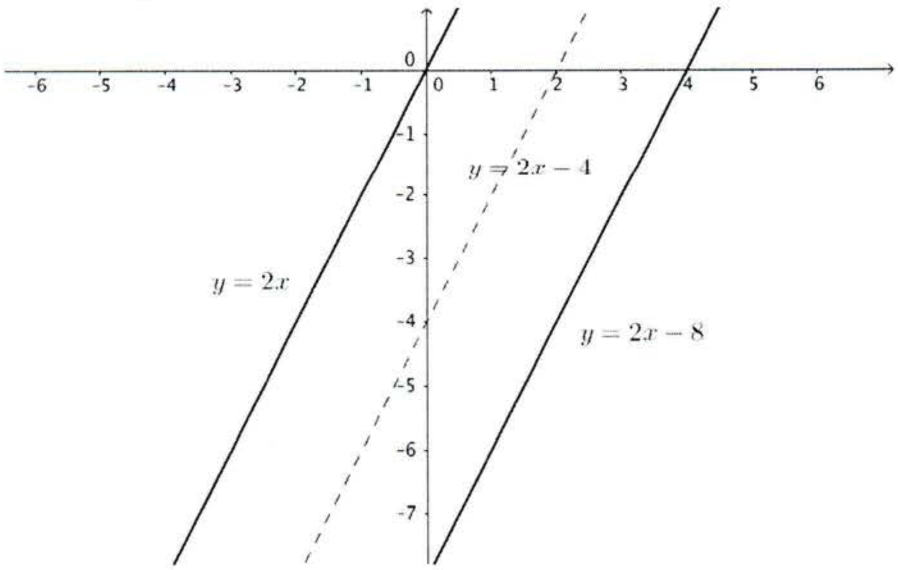
c) Find the values of x for which $y = 2x^3 - 3x^2 - 12x + 8$ is increasing. **2**

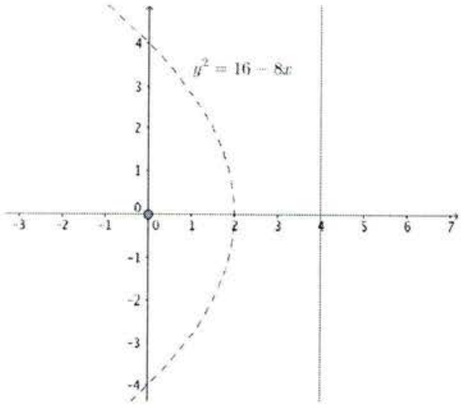
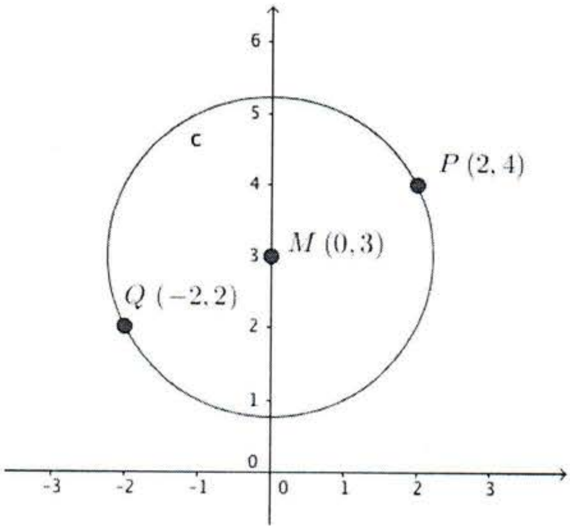
End of paper

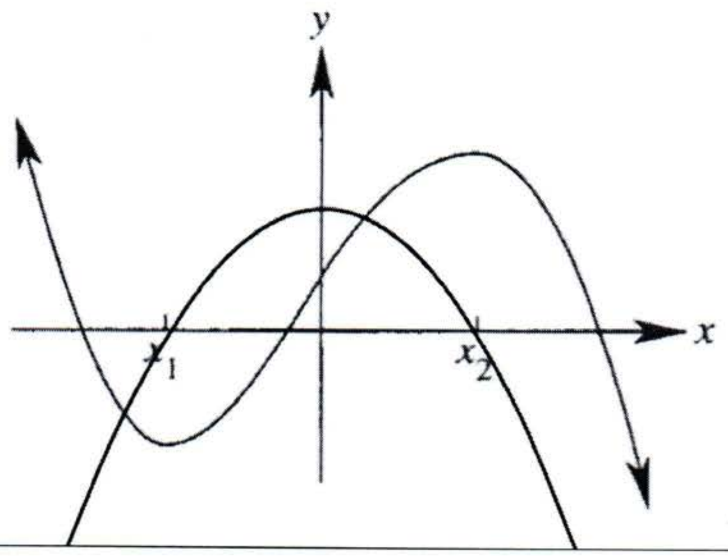
4/

 mark Q5 :)

	Solution	Marking Scheme
1.	$y = 2\sqrt{x}$ $y' = \frac{2}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$ at $x = 4$, $y' = \frac{1}{2}$ (D)	Correct answer (1)
2.	$y' = \frac{6}{2}x^5 - 4x^3 + 2x$ $y'' = \frac{30}{2}x^4 - 12x^2 + 2$ $y'' = 15x^4 - 12x^2 + 2$ (B) Question cut as answer incorrect on exam.	Correct answer (1)
3.	Both $y = 0$ and $x = 0$ would satisfy the condition. (C)	Correct answer (1)
4.	From the information, the point is a max turning point (B)	Correct answer (1)
5. (a)	(i) $y = x^5 - 2x^4$ $y' = 5x^4 - 8x^3$ (ii) $y = (6 - 2x)^3$ $y' = 3(6 - 2x)^2 \times (-2)$ $= -6(6 - 2x)^2$ (iii) $y = (x - 1)(4x + 1)$ $y' = (1)(4x + 1) + (x - 1)(4)$ $= 4x + 1 + 4x - 4 = 8x - 3$ (iv) $y = \frac{2x+1}{x^2-1}$ $y' = \frac{(2)(x^2-1)-(2x)(2x+1)}{(x^2-1)^2} = \frac{2x^2-2-4x^2-2x}{(x^2-1)^2} = \frac{-2x^2-2x-2}{(x^2-1)^2}$	One correct term (1) ✓ Correct solution (1) ✓ $3(6 - 2x)^2$ (1) Correct solution (1) Evidence of using ~~chain~~ product correctly rule (1) or correctly expan + get 8x or - Correct solution (1) ECF if Evidence of using correctly quotient rule (1) Correct solution (1)

5 (b)	$f(x) = x^2 - 5x + 1$ $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim\limits_{h \to 0} \frac{(x+h)^2 - 5(x+h) + 1 - (x^2 - 5x + 1)}{h}$ $= \lim\limits_{h \to 0} \frac{2xh + h^2 - 5h}{h}$ $= \lim\limits_{h \to 0} 2x + h^{\cancel{2}} - 5$ $= 2x - 5$	Substitute correctly into the $f'(x)$ formula (1) Lose 1 mark if lim h→0 disappears Correct Solution (1)
5 (c)	$y = x^3 - 2x^2 + 1$ $y' = 3x^2 - 4x$ $f'(2) = 3(2)^2 - 4(2) = 12 - 8 = 4$ $\boxed{\text{Gradient} = 4}$ Using the Gradient-Point Form: $y - y_1 = m(x - x_1)$ $y - 1 = 4(x - 2)$ $y = 4x - 7$ OR $4x - y - 7 = 0$	Correct gradient (1) ECF (1) if gradient incorrect Correct Solution (1)
6 (a)	Draw a diagram:  Using the diagram, it is clear that the solution is $y = 2x - 4$	Correct diagram (1) Or Reasonable attempt (1) Correct equation (1)

6 (b)	(i) Equate the distances: $\sqrt{x^2+y^2} = 4 - x$ $x^2 + y^2 = (4-x)^2$ $y^2 = 16 - 8x + x^2 - x^2$ $y = \pm\sqrt{16-8x}$  (ii) For some values of x, $f(x)$ is not unique. (i.e. there exist multiple y-values)	Reasonable attempt (1) Correct Solution (1) Correct Solution (1)
6 (c)	(i) The locus of R is a circle because angles in a semi circle are always 90°. Finding the mid point of $(2,4)$ and $(-2,2)$ will give the centre of the circle. $M = \left(\frac{2-2}{2}, \frac{4+2}{2}\right) = (0,3)$ The radius of the circle is QM: $r = \sqrt{(2-0)^2 + (4-3)^2} = \sqrt{4+1} = \sqrt{5}$ units Therefore the locus of R is: $x^2 + (y-3)^2 = 5$ (ii)  (iii) Since the Centre is at $(0,3)$ and the radius is $\sqrt{5} \sim 2.24$ units Therefore the locus of R is always above the $x-$ axis.	Correct Centre (1) Correct Equation (1) Correct Diagram (1) Correct Labels (1) Logical Reason (1)

6 (d)	(i) The latus rectum is a horizontal line going through the focus. (i.e. $y = a$) The intersection: $x^2 = 4a(a)$ $\Rightarrow x = \pm 2a \Rightarrow$ The points of intersections are $(2a, a)$ and $(-2a, a)$ (ii) $x^2 = 4ay \Rightarrow y = x^2/4a$ $y' = x/2a$ The gradient of the tangent at $(2a, a)$: $2a/2a = 1$ The gradient of the tangent at $(-2a, a)$: $-2a/2a = -1$ Since the gradients of the tangents multiply to -1, therefore the two gradients intersect at right angles.	Correct Solution (1) Correct Gradient Function (1) Correct Solution (1)
7 (a)		1 mark for correct shape 1 mark for correct intercepts and turning point
7 (b)	**(i)** $y = 2x^3 - 3x^2$ $y' = 6x^2 - 6x$ $6x^2 - 6x = 0$ $6x(x-1) = 0$ $x = 0$ and $x = 1$	3 marks for all points and the nature of each 2 marks for finding 1 stationary point and progress towards finding the nature 1 mark for finding the derivative.

When $x=0$, $y=0$

When $x=1$

$y=2-3=1$

Stationary points at $(0,0)$ and $(1,-1)$.

Test their nature using the second derivative.

$y''=12x-6$

When $x=0$, $y''=-6$

When $x=1$, $y''=6$

$(0,0)$ is a maximum turning point and $(1,-1)$ is a minimum turning point.

(ii)

1 mark for finding the second derivative

$y''=12x-6$

$12x-6=0$

1 mark for checking change in concavity.

$x=\frac{1}{2}$

When $x=\frac{1}{2}$, $y=-\frac{1}{2}$

Test for change in concavity

x	0	$-\frac{1}{2}$	1
y''	- 6	0	6

Concavity changes, therefore there is a point of inflexion at $(0.5,-0.5)$.

(iii)

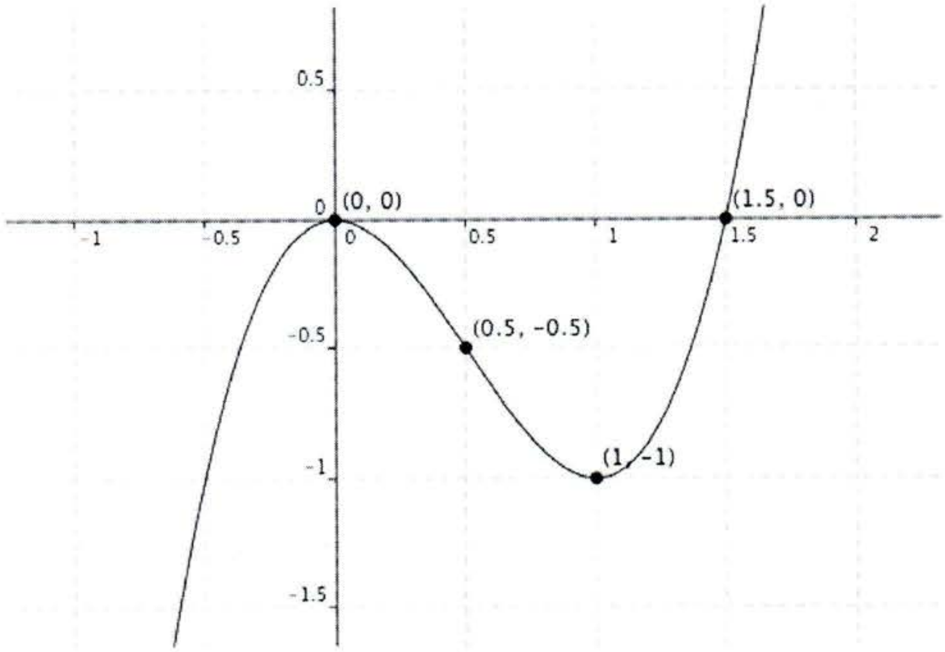
For x-intercepts, when y=0

$2x^3 - 3x^2 = 0$

$x^2(2x-3) = 0$

$x = 0$ and $x = \frac{3}{2}$

Intercepts at $(0,0)$ and $(1.5,0)$



1 mark for correct shape

1 mark for labelling stationary points

1 mark for labelling point of inflexion

(iv)

Curve is concave up when second derivative is greater than 0.

$y'' = 12x - 6$

$12x - 6 > 0$

$x > \frac{1}{2}$

1 mark for correct solution

7 (c)	Increasing when first derivative is greater than 0. $y = 2x^3 - 3x^2 - 12x + 8$ $y' = 6x^2 - 6x - 12$ $6x^2 - 6x - 12 > 0$ $6(x^2 - x - 2) > 0$ $6(x-2)(x+1) > 0$ $x < -1\ ,\ x > 2$	1 mark for finding the derivative and stating that it has to be greater than 0 1 mark for correct solution

Mr. R's Review of Locus Q6 in Exam

6a you should have noted $y = 2x$
$y = 2x - 8$ are
parallel and solved by drawing a sketch.
if you want to do algebraically
see alternate solution.

6b this is a parabola, you should realise
this immediately
there are 2 ways of solving
1 - as per answers (I've shown how to start)
2 - as of form, $y^2 = -4a(x-h)$ (see alternate solution)

6c I have provided an alternate solution
that uses methods I've shown in class.
Should realise straight away to use $m_1 \cdot m_2 = -1$

6d i If you remembered latus rectum is a
horizontal line through $y = a$ this was
was easy. sub $y = a$ into $x^2 = 4ay$ as per answers

ii need to show M_T for $x = 2a$, and M_T for $x = -2a$
multiply to give -1 (as per answers)

For Q7b i Many teachers at K.H.S want
max/min tested with y' with values
only 0.25 or 0.1 either side of point if you
have multiple turning points close together

Q7 was pretty standard stuff, redo if you
didn't get 12 or more...

6a Algebraically let R be (x, y)

$y = 2x$

$2x - y + 0 = 0$ general form

$$d_1 = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

$$d_1 = \frac{|2x-y|}{\sqrt{2^2+(-1)^2}}$$

$$d_1 = \frac{|2x-y|}{\sqrt{5}}$$

$y = 2x - 8$

$2x - y - 8 = 0$ general form

$$d_2 = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

$$d_2 = \frac{|2x-y-8|}{\sqrt{2^2+1^2}}$$

$$d_2 = \frac{|2x-y-8|}{\sqrt{5}}$$

as $d_1 = d_2$

$$\left(\frac{|2x-y|}{\sqrt{5}} = \frac{|2x-y-8|}{\sqrt{5}}\right) \times \sqrt{5}$$

$$\therefore |2x-y| = |2x-y-8|$$

$2x - y = 2x - y - 8$

$0 = -8$

$\therefore$ no solution

$-(2x-y) = 2x - y - 8$

$-2x + y = 2x - y - 8$

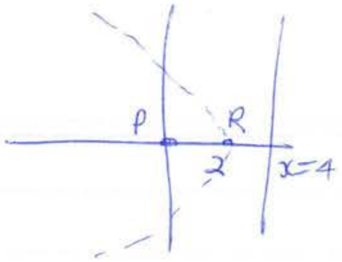
$\therefore 4x - 2y - 8 = 0$

$2y = 4x - 8$

$\underline{y = 2x - 4}$

$\therefore y = 2x - 4$ is the solution

6b



d_1 = distance from $R(x,y)$ to $P(0,0)$

$$d_1 = \sqrt{(x-0)^2 + (y-0)^2}$$

$$d_1 = \sqrt{x^2+y^2}$$

d_2 = distance from $R(x,y)$ to line $x=4$
treat as point $(4, y)$

$$d_2 = \sqrt{(x-4)^2 + (y-y)^2}$$

$$d_2 = \sqrt{(x-4)^2}$$

then follow solutions ...

alternate solution $(y-k)^2 = -4a(x-h)$ but $k=0$

$\therefore\ y^2 = -4a(x-h)$, $a=2$ $\because\ 2a = 4$

$y^2 = -4\times2(x-2)$ as vertex at $(2,0)$

$\therefore\ y^2 = -8(x-2)$

or $y^2 = -8x+16$ — 1 of these 2 forms is best ...

or $y^2+8x-16=0$

when asked for Locus
$y^2+8x-16=0$ is safest.

6c i $P(2,4)$, $R(x,y)$, $Q(-2,2)$

$M_{PR} = \frac{y-4}{x-2}$ $M_{RQ} = \frac{y-2}{x--2} = \frac{y-2}{x+2}$

$m_1 . m_2 = -1$

$m_{PR} . m_{RQ} = -1$

$\frac{y-4}{x-2} \cdot \frac{y-2}{x+2} = -1$

$\frac{(y-4)(y-2)}{(x-2)(x+2)} = -1$

$\frac{y^2-6y+8}{x^2-4} = -1$

$y^2-6y+8 = -x^2+4$

$\underline{x^2+y^2-6y+4=0}$ is the Locus of R

complete the square on y

$$\left.\begin{aligned} x^2 + y^2-6y &= -4 \\ x^2 + y^2-6y \underline{+9} &= -4 \underline{+9} \\ x^2 + (y-3)^2 &= 5 \end{aligned}\right.$$

$\therefore$ circle centre $(0,3)$, radius $= \sqrt{5}$