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MATHEMATICS

ASSESSMENT TASK NO: 3

Date : 18/05/2015

TIME ALLOWED: *Reading time: 5 minutes*
Working time: 1 hour

INSTRUCTIONS: This paper consists of 7 questions
ANSWER Multiple choice questions on the multiple choice sheet
Start each question in a new booklet
Marks may not be given for untidy and poorly arranged work.
All relevant working should be shown.

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

OUTCOMES ASSESSED:

- H1 - seeks to apply mathematical techniques to problems in a wide range of practical contexts
- H2 - constructs arguments to prove and justify results
- H4 - expresses practical problems in mathematical terms based on simple given models
- H5 - applies appropriate techniques from the study of calculus to solve problems
- H6 - uses the derivative to determine the features of the graph of a function
- H7 - uses the features of a graph to deduce information about the derivative
- H8 - Uses techniques of integration to calculate areas and volumes
- H9 - communicates using mathematical language, notation, diagrams and graphs

- P1 - demonstrates confidence in using mathematics to obtain realistic solutions to problems
- P2 - provides reasoning to support conclusions which are appropriate to the context
- P3 - performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions
- P4 - chooses and applies appropriate arithmetic and algebraic techniques
- P5 - understands the concept of a function and the relationship between a function and its graph
- P6 - relates the derivative of a function to the slope of its graph
- P7 - determines the derivative of a function through routine application of the rules of differentiation
- P8 - understands and uses the language and notation of calculus

	MARKS
QUESTION 1-5 Multiple Choice	/5
QUESTION 6 Probability	/20
QUESTION 7 Logarithmic and Exponential Functions	/25
Total	/50

This assessment task constitutes 15% of the Higher School Certificate Course Assessment.

Section I

5 marks

Attempt Questions 1 to 5

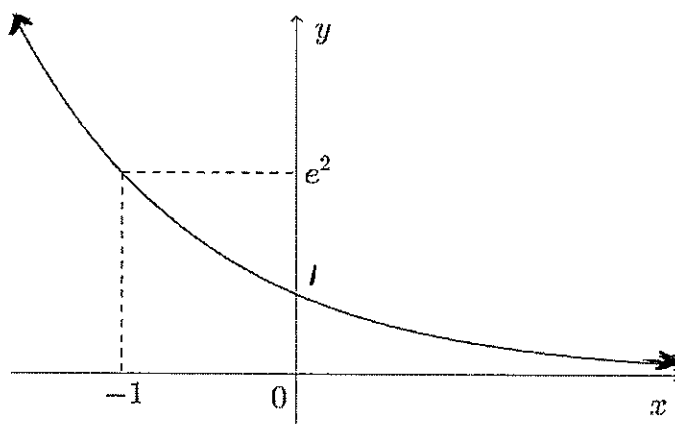
Allow about 5 minutes for this section

Use the multiple-choice answer sheet for Questions 1 to 5

1. A bag contains marbles numbered 1 to 50. A single marble is drawn out. What is the probability that the number drawn is a multiple of 3 or a multiple of 5?

- (A) $1/5$
- (B) $13/25$
- (C) $23/50$
- (D) $1/2$

2. The graph shows an exponential curve.



The equation of the graph could be:

- (A) $y = e^{2x}$
- (B) $y = e^{-x}$
- (C) $y = e^{2x}$
- (D) $y = e^{-2x}$

3. Jaz has 2 bags of apples.

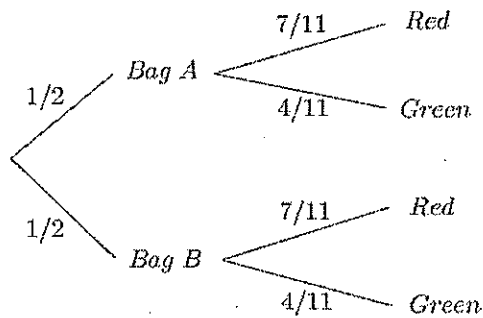
Bag A contains 4 red apples and 3 green apples.

Bag B contains 3 red apples and 1 green apple.

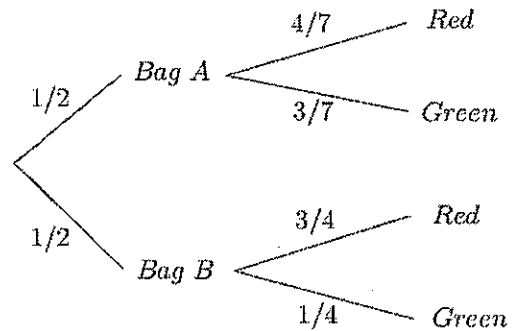
Jaz chooses an apple from one of the bags.

Which tree diagram could be used to determine the probability that Jaz chooses a red apple?

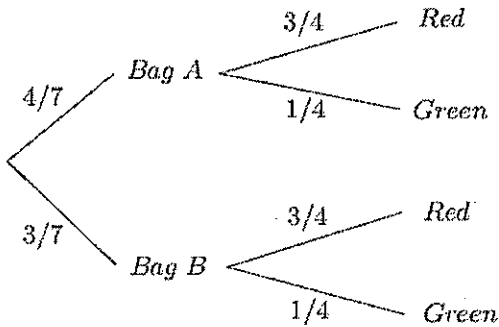
(A)



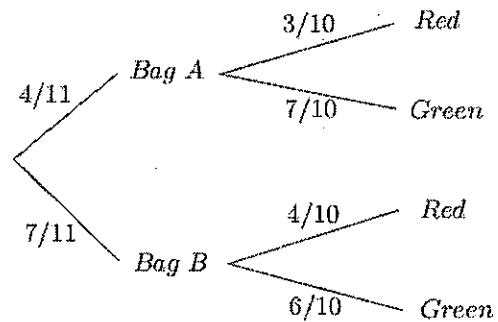
(B)



(C)



(D)



4. For the function $y = \ln(x - 2)$:
The domain is:

- (A) $x < 2$
 (B) $-2 < x < 2$
 (C) $x > 2$
 (D) all real x

5. 85% of the adults of a large town hold a driver's licence. If two adults are chosen at random, what is the probability that neither holds a licence?
- (A) 2.25%
 - (B) 7.5%
 - (C) 15%
 - (D) 30%

End of Section I

Section II

45 marks

For Questions 6 – 7 allow about 55 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 - Probability (20 marks)

Start a new writing booklet.

- (a) In a game, each round involves rolling two dice; each with faces marked 0, 1, 2, 3, 4, 5. The score for each round is calculated by multiplying the two numbers uppermost on the dice.
- | | | |
|-------|--|---|
| (i) | By drawing a diagram, or otherwise, list all of the possible outcomes. | 2 |
| (ii) | What is the probability of scoring zero on the first round? | 1 |
| (iii) | What is the probability of scoring 16 or more on the first round? | 1 |
| (iv) | What is the probability that the sum of the scores in the first two rounds to be less than 45? | 2 |
- (b) A woman knows that she is going to have triplets. What is the probability that she will have either 3 girls or 3 boys? 1
- (c) A box contains 5 good and 3 defective light bulbs. Two are chosen at random.
- | | | |
|------|---|---|
| (i) | What is the probability that the first one chosen is defective? | 1 |
| (ii) | By drawing a tree diagram, or otherwise, calculate the probability that two light bulbs chosen are: | |
| | α) both defective | 1 |
| | β) both good | 1 |
| | γ) one defective and one good | 1 |

Question 6 continues on the following page

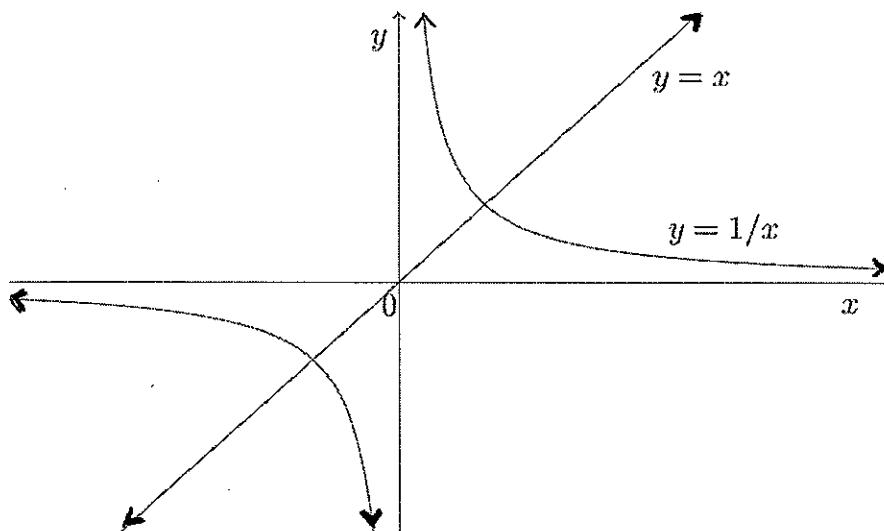
- (d) Each week Helen and Jacky take part in a raffle at their respective work places. The probability of Helen winning a prize is $\frac{1}{10}$ while the probability of Jacky winning a prize in her raffle is $\frac{1}{15}$. What is the probability that, during the next 3 weeks, at least one of them wins a prize? 2
- (e) In a game of chess between two players Michael and Vicky, who are about equal in ability. The player with the white pieces having the first move has a probability of winning of 0.5 while the player with the black pieces has a probability of winning of 0.3.
- (i) What is the probability of the game ending in a draw? 1
- Michael and Vicky play each other twice in a chess competition, each player having the white pieces once. In the competition the player who wins a game scores 1 point, the player who loses a game scores 0 points and a draw scores $\frac{1}{2}$ point for each player. What is the probability, after playing 2 games that:
- (ii) Michael scores 2 points? 1
- (iii) Michael scores less than $1\frac{1}{2}$ points? 2
- (f) In the jackpot lottery the probability of the jackpot prize being won in any draw is approximately 1 in 50.
- (i) What is the probability that the jackpot prize will not be won in each of 3 consecutive draws of the lottery? 1
- (ii) How many consecutive draws must be made for it to be 99% certain that a jackpot prize will have been won? 2

End of Question 6

Question 7 - Logarithmic and Exponential Functions (25 marks)

Start a new writing booklet.

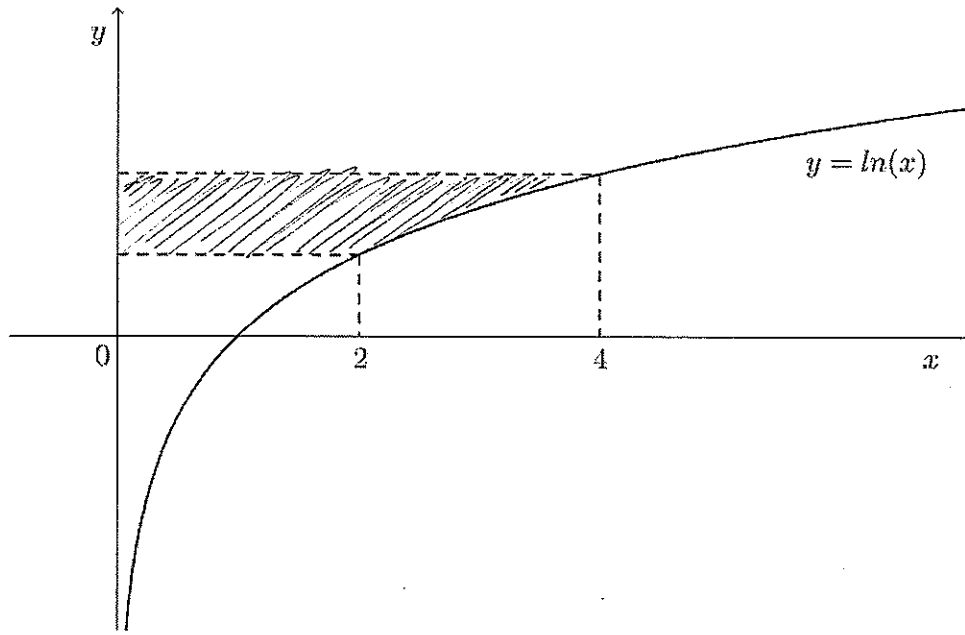
- (a) Find the primitive function of: $\frac{x}{x^2+1}$ 1
- (b) Find the value of: $\log_5 0.008$ 1
- (c) Find the value of: $\log_2 10 - \log_2 5$ 1
- (d) Solve for x : $4^x = 32$ 1
- (e) Solve for x : $\ln(7x - 12) = 2 \ln x$ 2
- (f) Find: $\frac{d}{dx} x^3 \log_e 2x$ 2
- (g) Given that $y = \frac{e^{2x}}{2x+1}$, show that $\frac{dy}{dx} = \frac{4xe^{2x}}{(2x+1)^2}$ 2
- (h) Find the equation of the tangent to the curve $y = 3x + e^{-x}$ at the point where $x = 0$. 2
- (i) Use Simpson's Rule with three function values to find an approximation for: 3
- $$\int_2^6 \frac{x}{\log_e x} dx$$
- Give your answer correct to one decimal place.
- (j) The following diagram shows the graphs of $y = x$ and $y = \frac{1}{x}$. Find the exact area between the curves from $x = 1$ to $x = 3$. 3



Question 7 continues on the following page

- (k) A solid of revolution is formed by rotating the curve $y = 2(1 + e^{2x})$ between $x = 0$ and $x = \frac{1}{2}$ about the x -axis. Show that the volume of this solid is: $\pi(e^2 + 4e - 3) \text{ units}^3$ 4

- (l) The diagram shows the curve $y = \ln x$, and a shaded area bounded by the curve, the y -axis and the y values corresponding to $x = 2$ and $x = 4$. 3



Find the exact value of this area.

End of paper



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Multiple Choice Answer Sheet

Completely fill the response oval representing the most correct answer.

1. A ☐ B ☐ C ☐ D ☐
2. A ☐ B ☐ C ☐ D ☐
3. A ☐ B ☐ C ☐ D ☐
4. A ☐ B ☐ C ☐ D ☐
5. A ☐ B ☐ C ☐ D ☐

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

TASK 3, TERM II 2015
SOLUTIONSSECTION 1 - MULTIPLE CHOICE
(5 marks)

1) $P(5) = \frac{10}{50}$

$P(3) = \frac{16}{50}$

$P(3 \cap 5) = \frac{3}{50}$

$$\therefore P(\text{mult. 3 or 5}) = \frac{10 + 16 - 3}{50}$$

$$= \frac{23}{50} \quad (C)$$

2) $(-1, e^2)$ satisfies.

test $y = e^{-2x}$
 $e^2 = e^{-2(-1)}$
 $e^2 = e^2 \quad \checkmark$

$\therefore y = e^{-2x} \quad (D)$

3) Bag A $P(R) = \frac{4}{7}$ $P(G) = \frac{3}{7}$
 Bag B $P(R) = \frac{3}{4}$ $P(G) = \frac{1}{4}$

 $\therefore$ diagram (B)

4) $y = \ln(x-2)$
 $x-2 > 0$
 $\therefore x > 2 \quad (C)$

5) $P(\overline{L} \overline{L})$
 $= 0.15 \times 0.15$
 $= 0.0225$
 $= 2.25\% \quad (A)$

MARKING SCHEME

1 mark each = 5 marks

SECTION II

2

QUESTION 6 - PROBABILITY (20 marks)

a) i)

X	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25

MARKING SCHEME

2 marks - all outcomes shown in table or listed

1 mark at least $\frac{1}{2}$ outcomes e.g. tree diagram

1 mark for $\frac{11}{36}$

1 mark for $\frac{4}{36}$ or $\frac{1}{9}$

2 marks for 0.996 or $\frac{1291}{1296}$

1 mark significant attempt at 'complement'

$$ii) P(0) = \frac{11}{36}$$

$$iii) P(\geq 16) = \frac{4}{36} = \frac{1}{9}$$

$$iv) P(< 45) = 1 - P(\geq 45)$$

$$= 1 - P((20, 25) \text{ or } (25, 20) \text{ or } (25, 25))$$

$$= 1 - \left(\frac{2}{36} \times \frac{1}{36} + \frac{1}{36} \times \frac{2}{36} + \frac{1}{36} \times \frac{1}{36} \right)$$

$$= 1 - \frac{5}{36^2} = 1 - \frac{5}{1296} = \frac{1291}{1296}$$

$$= 0.9964$$

$$b) P(3G \text{ or } 3B)$$

$$= \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right)$$

$$= \frac{1}{8} + \frac{1}{8}$$

$$= \frac{2}{8}$$

$$= \frac{1}{4}$$

1 mark for $\frac{1}{4}$ or $\frac{2}{8}$

c)

$\frac{5}{8}$ G	$\frac{4}{7}$ G
$\frac{3}{8}$ D	$\frac{3}{7}$ D
	$\frac{5}{7}$ G
	$\frac{2}{7}$ D

i) $P(D) = \frac{3}{8}$

1 mark $\frac{3}{8}$

$$ii) \alpha) P(DD) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$$

$$\beta) P(GG) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$$

$$\gamma) P(DG \text{ or } GD) = \frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{3}{7}$$

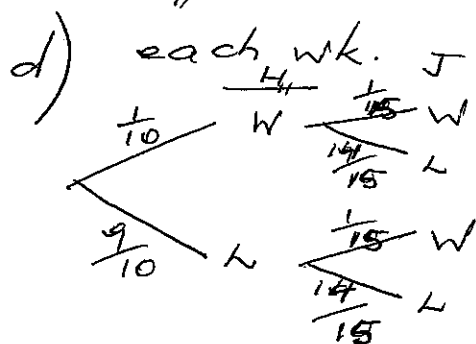
$$= \frac{30}{56} = \frac{15}{28}$$

1 mark $\frac{6}{56}$ or $\frac{3}{28}$

1 mark $\frac{20}{56}$ or $\frac{5}{14}$

1 mark $\frac{30}{56}$ or $\frac{15}{28}$

QUES 6 cont



$$\therefore P(LW) = \frac{6}{10} \times \frac{4}{5} = \frac{126}{150} = \frac{21}{25}$$

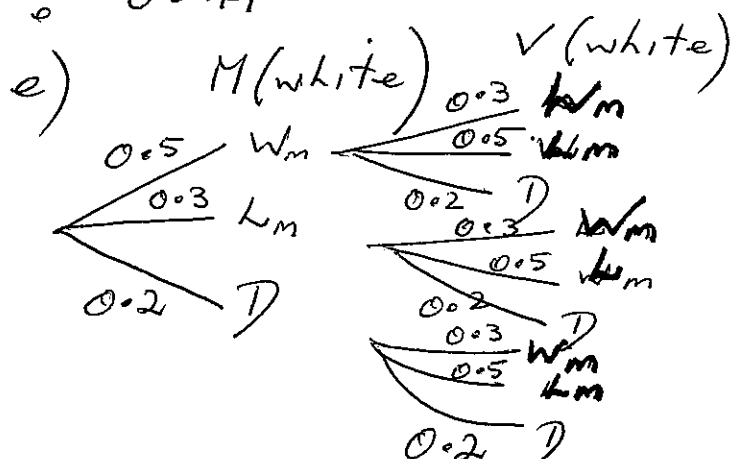
$P(\text{at least 1 wins})$

$$= 1 - \text{neither winning}$$

$$= 1 - \frac{126}{150} \times \frac{126}{150} \times \frac{126}{150}$$

$$= 0.407296$$

$$\approx 0.41$$



points m	V
2	0
1	1
1 1/2	1/2
0	2
0	2
1/2	1 1/2
1 1/2	1/2
1 1/2	1/2

i) $P(\text{draw}) = 0.2$

ii) $P(M = 2 \text{ pts}) = 0.5 \times 0.3 = 0.15$

iii) $P(M, < 1 \frac{1}{2}) = 1 - P(M, 1 \frac{1}{2} \text{ or } 2)$
 $= 1 - (0.15 + 0.10 + 0.6)$
 $= 1 - 0.85$
 $= 0.15$

f) i) $P(\overline{W} \overline{W} \overline{W}) = \left(\frac{49}{50}\right)^3 = 0.941192$
 ii) $P(J) = 1 - \left(\frac{49}{50}\right)^n$
 $0.99 = 1 - \left(\frac{49}{50}\right)^n$
 $\left(\frac{49}{50}\right)^n = 0.01$

MARKING SCHEME

2 marks for 0.41

1 mark for finding

$$P(\overline{W} \overline{W}) = \frac{21}{25} \text{ each week}$$

or similar.

i) 1 mark for 0.2

ii) 1 mark for 0.15

iii) 2 marks for 0.69

1 mark for significant progress either using 'complement' or listing outcomes with probability tree.

f) i) 1 mark 0.941192
 ii) 2 marks

QUESTION 6 continued..

$$\therefore \log_{0.98} 0.01 = n.$$

$$n = 227.9$$

$\therefore$ approx. 228 draws for at least one jackpot.

MARKING SCHEME (14)

f)ii)

2 marks for 228 draw

1 mark for significant progress, show working, using logs, or 'trial and error'

0 mark for bald answer of 228 draw.

QUESTION 7 - LOGS, EXP. FNS (25 marks)

5

$$a) \int \frac{x}{x^2+1} dx = \frac{1}{2} \log(x^2+1) + C$$

MARKING SCHEME

a) 1 mark for $\frac{1}{2} \log(x^2+1) + C$
no penalty for no 'c'
0 marks for $\log(x^2+1)$

$$b) \log_5 0.008 = x$$

$$\therefore 5^x = 0.008$$

$$5^x = \frac{1}{125}$$

$$5^x = 5^{-3}$$

$$\therefore x = -3$$

b) 1 mark for $x = -3$.

$$c) \log_2 10 - \log_2 5$$

$$= \log_2 \frac{10}{5}$$

$$= \log_2 2$$

$$= 1$$

c) 1 mark for value is 1

$$d) 4^x = 32$$

$$2^{2x} = 2^5$$

$$\therefore 2x = 5$$

$$x = \frac{5}{2}$$

d) 1 mark for $x = \frac{5}{2}$ or 2.5

$$e) \ln(Tx-12) = 2 \ln x$$

$$\ln(Tx-12) = \ln x^2$$

$$\therefore x^2 = Tx - 12$$

$$x^2 - Tx + 12 = 0$$

$$(x-3)(x-4) = 0$$

$$\therefore x = 3 \text{ or } 4$$

e) 2 marks for both $x=3$ and $x=4$
1 mark for either $x=3$ or $x=4$
1 mark for using log laws to get $x^2 - Tx - 12 = 0$

QUESTION 7 cont..

$$f) \frac{d}{dx} x^3 \log_e 2x$$

$$= \log_e 2x \cdot 3x^2 + x^3 \cdot \frac{1}{2x} \cdot 2$$

$$= 3x^2 \log_e 2x + x^2$$

$$g) y = \frac{e^{2x}}{2x+1}$$

$$y' = \frac{(2x+1) \cdot 2e^{2x} - e^{2x} \cdot 2}{(2x+1)^2}$$

$$= \frac{4xe^{2x} + 2e^{2x} - 2e^{2x}}{(2x+1)^2}$$

$$= \frac{4xe^{2x}}{(2x+1)^2}$$

$$h) y = 3x + e^{-x} \quad x=0$$

$$x=0, y=1$$

$$y' = 3 - e^{-x}$$

$$f'(0) = 2$$

$$\therefore \text{tangent, } y - y_1 = m(x - x_1)$$

$$y - 1 = 2(x - 0)$$

$$y - 1 = 2x$$

$$0 = 2x - y + 1$$

MARKING SCHEME

6

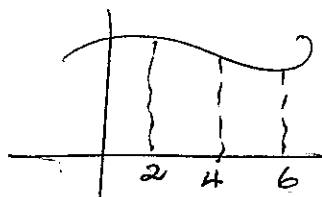
2 marks for correctly using product rule (with or without simplification)
1 mark for half of answer, i.e. $3x^2 \log_e 2x$ or x^2

g) 2 mark for correctly showing and arriving at answer, using quotient rule.
1 mark for correct use of ~~quotient~~ rule, without simplifying (showing this!)

h) 2 marks for correct equation of tangent.
1 mark for finding $f'(x)$ and $f'(0)$ correctly.
0 marks for just $f'(x)$.

QUESTION 7 continued...

$$i) \int_2^6 \frac{x}{\log_e x} dx.$$



$$\approx \frac{1}{3} [f(2) + 4f(4) + f(6)]$$

$$= \frac{2}{3} \left[\frac{2}{\log_e 2} + \frac{4 \times 4}{\log_e 4} + \frac{6}{\log_e 6} \right]$$

$$= 11.85040945$$

$$\approx 11.9 \text{ (1 d.p.)}$$

MARKING SCHEME

1

i)

3 marks for correct answer (no penalty for not to 1 d.p.)

2 marks for correct numerical expression

1 mark for correct Simpson's Rule statement

$$j) A = \int_1^3 x - \frac{1}{x} dx.$$

$$= \left[\frac{x^2}{2} - \log_e x \right]_1^3$$

$$= \frac{3^2}{2} - \log_e 3 - \left(\frac{1}{2} - \log_e 1 \right)$$

$$= \frac{9}{2} - \log_e 3 - \frac{1}{2} + 0$$

$$= (4 - \log_e 3) \text{ units}$$

j)

3 marks for correct answer in exact form (no penalty for evaluating as long as correct exact expression is shown)

2 marks for substituting into correct integral
1 mark for correct integral.

$$k) V = \pi \int_0^2 y^2 dx.$$

$$= \pi \int_0^2 [2(1+e^{2x})]^2 dx.$$

$$= 4\pi \int_0^2 (1 + 2e^{2x} + e^{4x}) dx$$

$$= 4\pi \left[x + \frac{2e^{2x}}{2} + \frac{e^{4x}}{4} \right]_0^2$$

$$= 4\pi \left[\frac{1}{2} + e^1 + \frac{e^2}{4} - \left(0 + 1 + \frac{1}{4} \right) \right]$$

$$= 4\pi \left[\frac{3}{4} + e + \frac{e^2}{4} \right]$$

$$= \pi [e^2 + 4e + 3] \text{ units}^3$$

k)

4 marks for complete correct / 'show' result.

3 marks for correct numerical substitution in integral

2 marks for correct integration

1 mark for correct expression to integrate
i.e. $\pi \int_0^2 [2(1+e^{2x})]^2 dx$

QUESTION 7 continued ..

$$\begin{aligned} \text{c) } A &= \int_{\ln 2}^{\ln 4} x \cdot dy & y &= \ln x \\ & & \therefore x &= e^y \\ &= \int_{\ln 2}^{\ln 4} e^y \cdot dy \\ &= \left[e^y \right]_{\ln 2}^{\ln 4} \\ &= e^{\ln 4} - e^{\ln 2} \\ &= 4 - 2 \\ &= 2 \text{ units}^2 \end{aligned}$$

MARKING SCHEME

8

1) 3 marks for correct answer
2 marks for correct integration
1 mark for correct expression to integrate with corresponding boundaries
i.e. $\int_{\ln 2}^{\ln 4} e^y \cdot dy$