Killara HIGH SCHOOL

Student Number

2015

MID COURSE EXAMINATION

Mathematics

***

General Instructions

- Reading time – 5 minutes
- Working tine 3 hours
- Write using blue or black pen
  Black pen is preferred
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-16 show relevant mathematical reasoning and/or calculations
- Start a new booklet for each question

Total Marks – 100

Section I - Pages 2 - 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - Pages 6 - 12

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hour and 45 minutes for this section

Question	Marks
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
15	/15
16	/15

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 30% of the Higher School Certificate Course Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1 – 10 (Detach from paper)

---

1 What is 5.03976 correct to 4 significant figures?

(A) 5.039

(B) 5.0397

(C) 5.040

(D) 5.0398

2 The function $y = 3 + (x-2)^2$ is:

(A) An EVEN function

(B) An ODD function

(C) Symmetric about $x = 2$

(D) Symmetric about $y = 3$

3 The solution to $(2x-5)(6-x) \geq 0$ is:

(A) $\{x : -2.5 \leq x \leq 6\}$

(B) $\{x : 2.5 \leq x \leq 6\}$

(C) $\{x : x \leq 2.5,\ x \geq -6\}$

(D) $\{x : x \leq -2.5,\ x \geq 6\}$

Section 1 continues on page 3

4 If $\int_0^a (4-2x)\,dx = 4$, the value of a is:

(A) -2

(B) 0

(C) 1

(D) 2

5 What is the value of $\int_{-2}^{2} \sqrt{4-x^2}\,dx$?

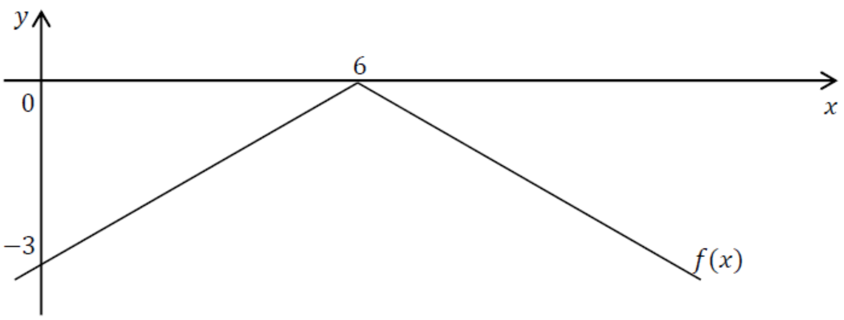
(A) $\frac{3\pi}{2}$

(B) 2π

(C) 3π

(D) 4π

6



The diagram shows the graph of $y = f(x)$. The equation of $y = f(x)$ is:

(A) $f(x) = \left|\frac{1}{2}x - 3\right|$

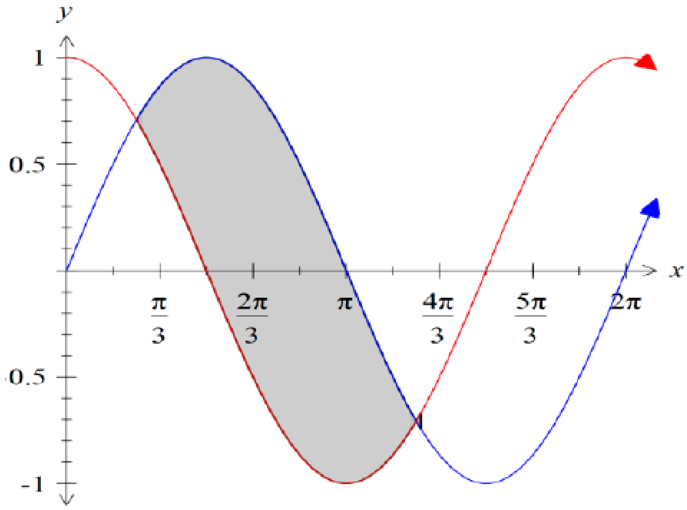
(B) $f(x) = -\left|2x - 3\right|$

(C) $f(x) = -\left|\frac{1}{2}x - 3\right|$

(D) $f(x) = -\left|\frac{1}{2}x + 3\right|$

Section 1 continues on page 4

7



Which of the following describes the area given in the graph above?

(A) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} (\sin x - \cos x)\, dx$

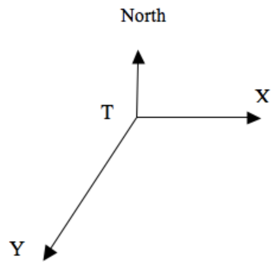
(B) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\, dx$

(C) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} (\cos x - \sin x)\, dx$

(D) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\cos x - \sin x)\, dx$

Section 1 continues on page 5

8 A point X is due East of a tower T. From another point Y, the bearing of the tower T is 061°. The size of angle XTY is equal to:



(A) **131°**

(B) **119°**

(C) **209°**

(D) **151°**

9 What is the correct expression for $\int \cos\frac{x}{3}dx$?

(A) $-\frac{1}{3}\sin\frac{x}{3}+C$

(B) $\frac{1}{3}\sin\frac{x}{3}+C$

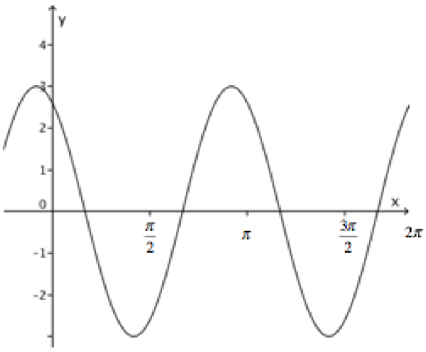
(C) $-3\sin\frac{x}{3}+C$

(D) $3\sin\frac{x}{3}+C$

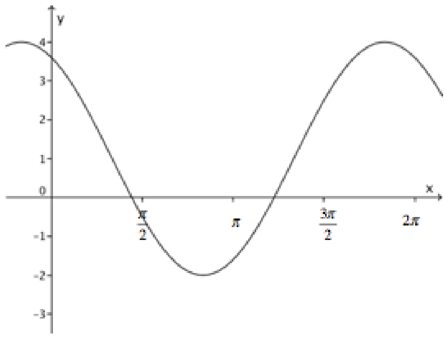
Section 1 continues on page 6

10 Which graph best represents $y = -3\sin\left(2x - \frac{\pi}{3}\right) + 1$

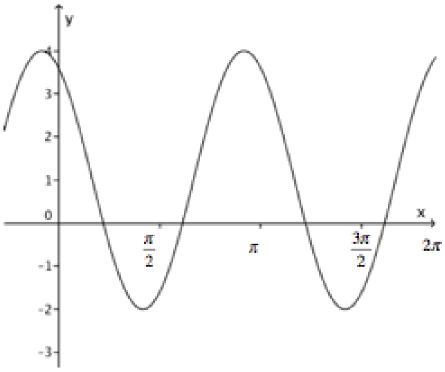
(A)



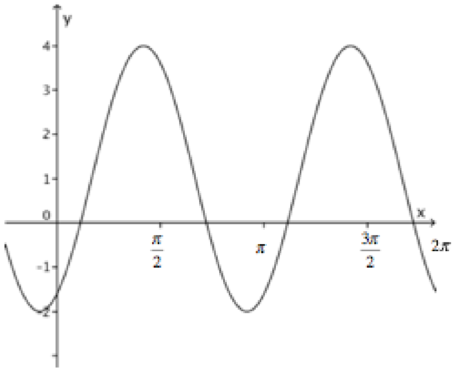
(B)



(C)



(D)



End of Section 1

Section II

90marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11(15 marks) Use a SEPARATE writing booklet

(a) Factorise completely $4x^3 - 32$ **2**

(b) Solve $2x^2 - 7x - 15 = 0$ **2**

(c) Solve $|3x - 7| \leq 12$ **2**

(d) Simplify $\frac{2}{x-1} - \frac{1}{x-3}$ **2**

(e) Find values of *a* and *b* such that $\frac{-2}{7-\sqrt{3}} = a + b\sqrt{3}$ **3**

(f) Find the exact value of $\sin\frac{4\pi}{3}$ **2**

(g) State the domain and range of $y = \frac{1}{3-x}$ **2**

End of Question 11

Question 12(15 marks) Use a SEPARATE writing booklet

(a) Differentiate the following:

(i) $(7-5x)^2$ **2**

(ii) $\dfrac{2x}{\cos 2x}$ **3**

(b) Find:

(i) $\int x^3+7x^2+4\ dx$ **2**

(ii) $\int \dfrac{x^3+1}{\sqrt{x}}dx$ **2**

(iii) $\int_0^3 2\sqrt{x}+x^3\ dx$ **2**

(c) Given the parabola $4y=x^2-12$, find the:

(i) Focal length **1**

(ii) Coordinates of the focus **1**

(d) Find the equation of the tangent to the curve $y=\dfrac{1}{2}\sin x$ at the point $(\pi,0)$. **2**

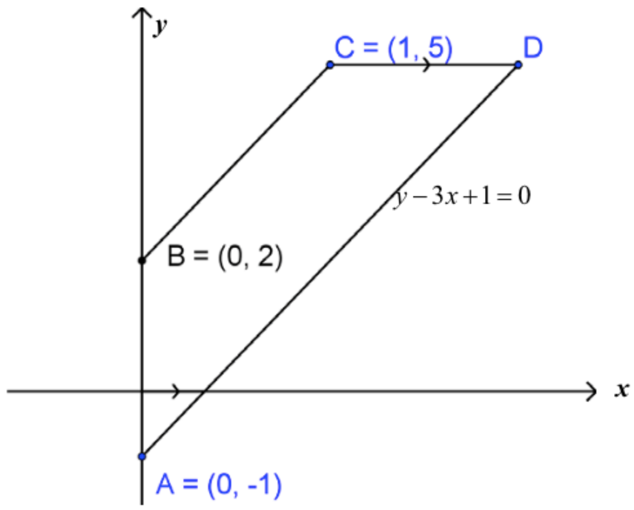
End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) The roots of the quadratic $px^2 - x + q = 0$ are -2 and 5. **2**

Find the values of p and q.

(b)



NOT TO SCALE

In the diagram, *ABCD* is a quadrilateral. The equation of the line *AD* is $y - 3x + 1 = 0$

(i) Show that *ABCD* is a trapezium. **2**

(ii) The line *CD* is parallel to the *x*-axis. Find the coordinates of *D*. **1**

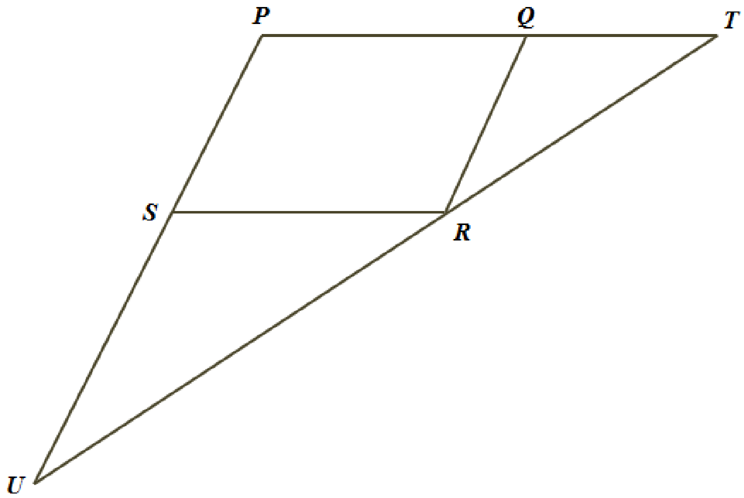
(iii) Find the length of *BC*. **1**

(iv) Show that the perpendicular distance from *B* to *AD* is $\frac{3}{\sqrt{10}}$ **2**

(v) Hence or otherwise, find the area of the trapezium *ABCD*. **2**

Question 13 continues on Page 10

(c)



$PQRS$ is a parallelogram. PQ is produced to T so that $TQ = QR$ and PS is produced to U so that $SU = PS$. The points U, R and T are collinear.

(i) Prove that $\Delta TQR \equiv \Delta RSU$. **3**

(ii) Hence, or otherwise, prove that $PQRS$ is a rhombus. **2**

End of Question 13

Question 14(15 marks) Use a SEPARATE writing booklet

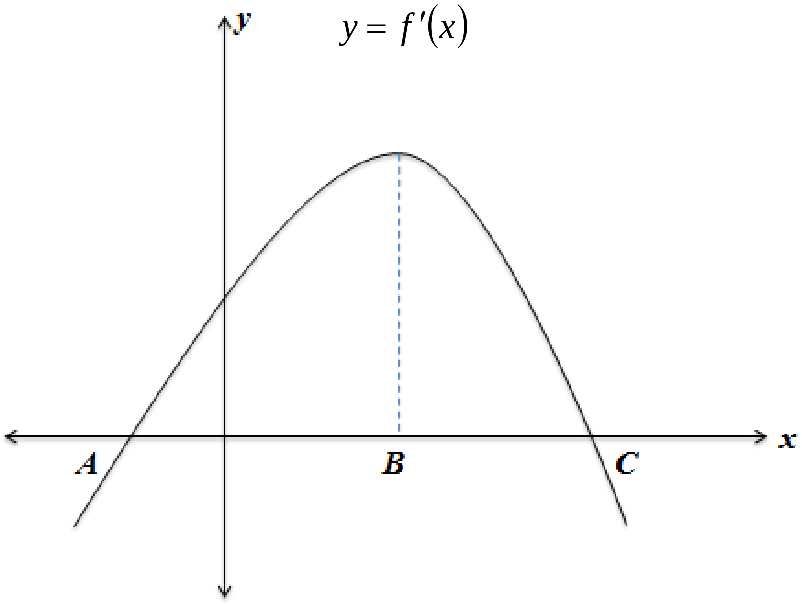
(a) Given the curve $f(x)=2x^3-12x^2$:

(i) Find the stationary points and establish their nature. **3**

(ii) Determine the points of inflection. **2**

(iii) Sketch the curve, in the domain $-1 \le x \le 6$, showing all the essential features. **3**

(b) The diagram shows the gradient function $y=f'(x)$ of a curve $y=f(x)$.



(i) Describe the shape of the curve $y=f(x)$ where $x=A$ and $x=B$. **2**

(ii) For what values of x is the function $y=f(x)$ decreasing? **1**

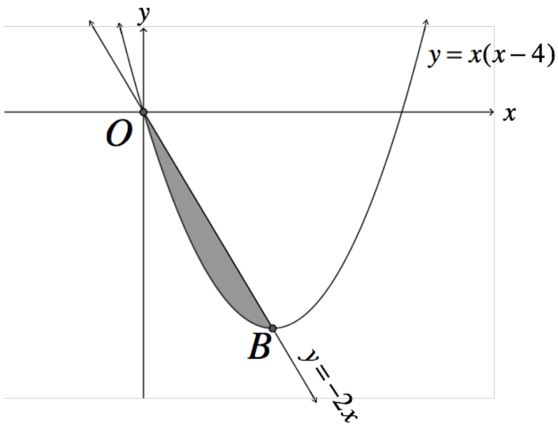
(iii) Sketch a possible curve $y=f(x)$, showing where $x=A$, B and C. **2**

(c) Evaluate $\int_{-1}^{1} 4^x dx$ using Simpson's rule with five function values. **2**

End of Question 14

Question 15(15 marks) Use a SEPARATE writing booklet

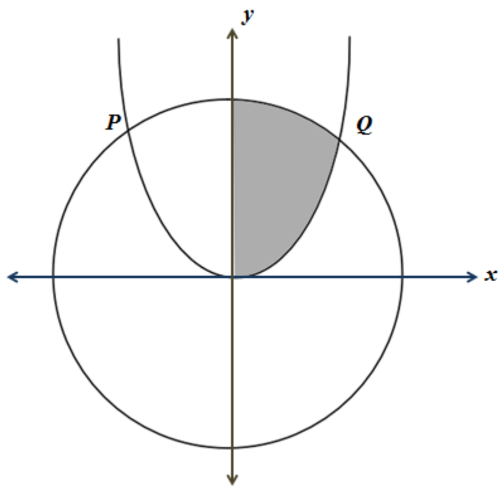
(a)



The graph of $y = x(x-4)$ and $y = -2x$ meet at $B(2, -4)$. **3**

Find the shaded area bounded by $y = x(x-4)$ and $y = -2x$

(b) The shaded region is bounded by the curve $y = x^2$ and the circle $x^2 + y^2 = 12$.



(i) By solving the two equations simultaneously, show that the points P and Q both have a y-coordinate of 3. **2**

(ii) Show that the volume generated when the shaded region is rotated about the y-axis is $\pi\left(16\sqrt{3} - \frac{45}{2}\right)$ cubic units. **3**

Question 15 continues on page 13

Question 15 (continued)

(c) Find the equation of the line which passes through the point of intersection of the lines $2y = x + 8$ and $6y = 4x + 27$ and has a gradient of $-\frac{3}{2}$. **3**

(d) (i) Show that $\frac{d}{dx}\left(\sqrt{1+x^2}\right) = \frac{x}{\sqrt{1+x^2}}$ **1**

(ii) Hence, evaluate $\int_0^1 \frac{x}{2\sqrt{1+x^2}}dx$ **3**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet

(a) $\cos\alpha = -p$ and $\tan\alpha > 0$, find $\sec(90+\alpha)$. **2**

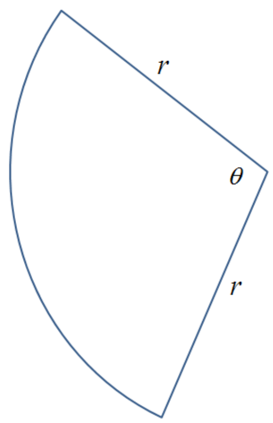
(b) (i) Show that $x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ are the solutions of the equation $1 + 2\cos 2x = 0$ for $0 \le x \le \pi$. **1**

(ii) Draw the graph of $y = 1 + 2\cos 2x$ for $0 \le x \le \pi$. **3**

(iii) Find the area between $y = 1 + 2\cos 2x$ and the x-axis for $\dfrac{\pi}{3} \le x \le \dfrac{2\pi}{3}$. **2**

(c) A landscaping company is designing a garden bed in a local park in the shape of a sector with radius r and angle θ.

They need 375 metres of garden edging material to use in the perimeter of the garden bed.



(i) Show that the area of A of the garden bed is $\dfrac{r}{2}(375 - 2r)$. **2**

(ii) Find the greatest area of the garden bed which can be made using 375 metres of edging material. **3**

(iii) After inspecting the location for the garden bed , the designers calculate that the angle of the sector shaped garden bed will be less than $110°$. **2**

Can they still build the garden bed with the maximum area found in part (ii)? Justify your answer.

End of Paper

BLANK PAGE

Killara HIGH SCHOOL

Student Number

Mathematics

Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 10. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ✖ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ✖ B ✖ C ○ D ○

Start Here →

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

BLANK PAGE

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2015 Half-yearly solutions

Multiple choice

1. A	2. C	3. B	4. D	5. B
6. C	7. B	8. D	9. D	10. C

Free response

Question 11

a)	$4x^3-32$ $4(x^3-8)$ $4(x-2)(x^2+2x+4)$	1 mark: takes 4 common factor out 1 mark: correctly factorises the difference of cubes	
b)	$2x^2-7x-15=0$ $(2x+3)(x-5)=0$ $x=\frac{-3}{2},\quad x=5$	1 mark: correctly factorises 1 mark: correctly gives the solution from the factored form.	
c)	$\lvert 3x-7\rvert \leq 12$ $3x-7\leq 12 \qquad -(3x-7)\leq 12$ $3x\leq 19 \qquad 7-3x\leq 12$ $x\leq\frac{19}{3} \qquad -3x\leq 5$ $x\geq\frac{5}{3}$	1 mark for each correct answer with correct working.	
d)	$\frac{2}{x-1}-\frac{1}{x-3}$ $\frac{2(x-3)-(x-1)}{(x-1)(x-3)}\quad 1mark$ $\frac{2x-6-x+1}{(x-1)(x-3)}$ $\frac{x-5}{(x-1)(x-3)}\quad 1mark$	1 mark: makes it a single fraction with common denominator 1 mark: correctly simplifies	
e)	$\frac{-2}{7-\sqrt{3}}=\frac{-2\times(7+\sqrt{3})}{(7-\sqrt{3})(7+\sqrt{3})}$ $=\frac{-14-2\sqrt{3}}{46}$ $=-\frac{7}{23}-\frac{1}{23}\sqrt{3}$	2 marks correct values of *a* and *b* using suitable working 1 mark – identifying the need to multiply by the conjugate and showing appropriate working	

	$\therefore a=-\frac{7}{23} \qquad b=-\frac{1}{23}$		
f)	$\sin\frac{4\pi}{3}$ $\sin\left(\pi+\frac{\pi}{3}\right)$ $-\sin\left(\frac{\pi}{3}\right)$ $=\frac{-\sqrt{3}}{2}$	2 mark: correct answer from correct working 1 mark: correct bald answer given	
g)	$y=\frac{1}{3-x}$ Domain x all real except $x\neq 3$ Range y all real except $y\neq 0$	2 marks both range and domain correct 1 mark only one of range or domain correct.	

Question 12

a)(i)	$y=(7-5x)^2$ $\frac{dy}{dx}=2(7-5x)\times -5$ $=-10(7-5x)$	2 mark: correctly differentiates 1 mark: gives $2(7-5x)$, forgets to multiply by the derivative of the bracket	
a)(ii)	$y=\frac{2x}{\cos 2x}$ $u=2x \qquad u'=2$ $v=\cos 2x \quad v'=-2\sin 2x$ $\frac{dy}{dx}=\frac{2\cos 2x--4x\sin 2x}{\cos^2 2x}$ $=\frac{2\cos 2x+4x\sin 2x}{\cos^2 2x}$	2 marks: correct answer 1 mark: attempts to use quotient rule from incorrect u' or v'	
b)(i)	$\int x^3+7x^2+4\,dx$ $\frac{x^4}{4}+\frac{7x^3}{3}+4x+C$	2 marks: correct answer 1 mark: attempts to integrate, but +C required	
b)(ii)	$\int \frac{x^3+1}{\sqrt{x}}\,dx$ $\int x^{\frac{5}{2}}+x^{\frac{-1}{2}}\,dx$ $\frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{1}{2}}+C$	1 mark: writes the integrand in the correct radical form 1 mark: correctly integrates	
b)(iii)	$\int_0^3 2\sqrt{x}+x^3\,dx$ $\int_0^3 2x^{\frac{1}{2}}+x^3\,dx$ $\left[\frac{4}{3}x^{\frac{3}{2}}+\frac{x^4}{4}\right]_0^3$ $\left(\frac{4}{3}.3\sqrt{3}+\frac{81}{4}\right)-0$ $4\sqrt{3}+\frac{81}{4}$	1 mark: correctly integrates 1 mark: correct evaluation. (must give exact solution)	

c)(i)	$4y = x^2 - 12$ $x^2 = 4y + 12$ $x^2 = 4(y+3)$ $4a = 4$ $a = 1$ Focal length = 1	1 mark: gives correct focal length from correct working	
c)(ii)	Vertex $(0,-3)$ Hence, focus = $(0,-3+1)$ = $(0,-2)$	1 mark: uses their focal length and vertex to calculate the focus	
d)	$y = \frac{1}{2}\sin x$ at the point $(\pi, 0)$ $\frac{dy}{dx} = \frac{1}{2}\cos x$ At $x = \pi$, $\frac{dy}{dx} = \frac{1}{2}\cos\pi = -\frac{1}{2}$ Equation of the tangent is $y - 0 = -\frac{1}{2}(x - \pi)$ $2y = -x + \pi$ $x + 2y - \pi = 0$	2 marks: correct equation of the tangent from correct working 1 mark: calculates the correct gradient and attempts to find the equation of the tangent using point-gradient formula.	

Question 13

13a)	$px^2 - x + q = 0 \quad \alpha = -2$ and $\beta = 5$ $\therefore -2 + 5 = \frac{1}{p}$ $\therefore p = \frac{1}{3}$ and $-10 = \frac{q}{\frac{1}{3}}$ $\therefore q = -\frac{10}{3}$	2 marks correct values of p and q showing full working 1 mark correct value one of p or q showing full working	
13bi)	$m_{BC} = \frac{5-2}{1-0} = 3$ line AD has equation $y - 3x + 1 = 0$ $\therefore y = 3x - 1$ $\therefore m_{AD} = 3$ and $m_{AB} =$ undefined (vertical) and $m_{CD} = 0$ (horizontal) $\therefore ABCD$ is a trapezium as it has one pair of parallel sides	2 marks Concludes shape is a trapezium by comparing all 4 gradients 1 mark only compares gradients of parallel sides	
13bii)	Point D is the intersection of $y = 5$ and $y - 3x + 1 = 0$ $\therefore 5 - 3x + 1 = 0$ $\therefore x = 2$ and $y = 5$ (horzontal line) $D\ (2,5)$	1 mark Finds coordinates of D using any correct method	
13biii)	$l_{BC} = \sqrt{(1-0)^2 + (5-2)^2}$ $= \sqrt{10}$	1 mark correctly evaluates the length of BC	
13biii)	$B\ (0,2) \quad -3x + y + 1 = 0$ $\perp_{dist} = \frac{\lvert(-3\times 0) + (1\times 2) + 1\rvert}{\sqrt{(-3)^2 + 1^2}}$ $= \frac{3}{\sqrt{10}}$	2 marks correct substitution into perpendicular distance formula 1 mark – minor error	
13biv)	$l_{AD} = \sqrt{(2-0)^2 + (5-(-1))^2}$ $= \sqrt{40} = 2\sqrt{10}$ $A_{ABCD} = \frac{1}{2}(l_{AD} + l_{BC}) \times \frac{3}{\sqrt{10}}$ $= \frac{3\sqrt{10}}{2} \times \frac{3}{\sqrt{10}}$ $= \frac{9}{2}$ units ^{2}	2 marks any correct method for finding area of trapezium using their point D 1 mark significant progress towards finding area including correct formula for area of a trapezium	

c)(i)	$\angle RSP = \angle PQR = \alpha$ (opposite angles of a parallelogram are equal) $\therefore \angle RSU = \angle TQR = 180 - \alpha$ (supplementary angles add to $180°$) In $\Delta RQT = \Delta USR$, $\angle TQR = \angle RSU$ 1. $QR = SU$ ($QR = PS$ (opposite sides of a parallelogram are equal. $PS = SU$ (given) 2. $PU \parallel QR$, opposite sides of a parallelogram are parallel) $\angle QRT = \angle SUR$ ($PU \parallel QR$, corresponding angles in parallel lines are equal) 3. $\therefore \Delta TQR \equiv \Delta RSU$ (angle- angle- side) using 1., 2., 3.	3 marks: correct proof given 2 marks: proves $\therefore \angle RSU = \angle TQR$ and $\angle QRT = \angle SUR$ with correct reasoning 1 mark: anyone of 1., 2. or 3. proved with correct reasoning	
c)(ii)	$\therefore SR = QT$ corresponding sides of congruent triangles are equal **1 mark** *QT* = *PQ* (given) $\therefore PQ = SR$ Now, *PS* = *QR*, *PQ* = *SR* (all sides are equal, *PQRS is a rhombus* **1 mark**		

Question 14

<table>
<tr>
<td>a)(i)</td>
<td>

$f(x)=2x^3-12x^2$

For stationary points, $f'(x)=0$

$6x^2-24x=0$

$6x(x-4)=0$

$x=0,\quad x=4$

$x=0,\qquad x=4$

$y=0\qquad y=-64$ **1 mark**

$(0,0)\qquad(4,-64)$

Determining nature:

$f''(x)=12x-24$

$x=0,\quad f''(x)=-24<0$

$\therefore$ maximum turning point at $(0,0)$ **1 mark**

$x=4,\quad f''(x)=24>0$

$\therefore$ minimum turning point at $(4,-64)$**1 mark**

</td>
<td>

1 mark: finds the two stationary points

1 mark each: for determining the nature of each of the stationary points

</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>

For points of inflection,

$f''(x)=0$

$12x-24=0$

$x=2$

$x=2,\quad y=-32$

Concavity test:

<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>$f''(x)$</td><td>-12</td><td>0</td><td>12</td></tr>
<tr><td></td><td>$\frown$</td><td></td><td>$\smile$</td></tr>
</table>

Change in concavity.

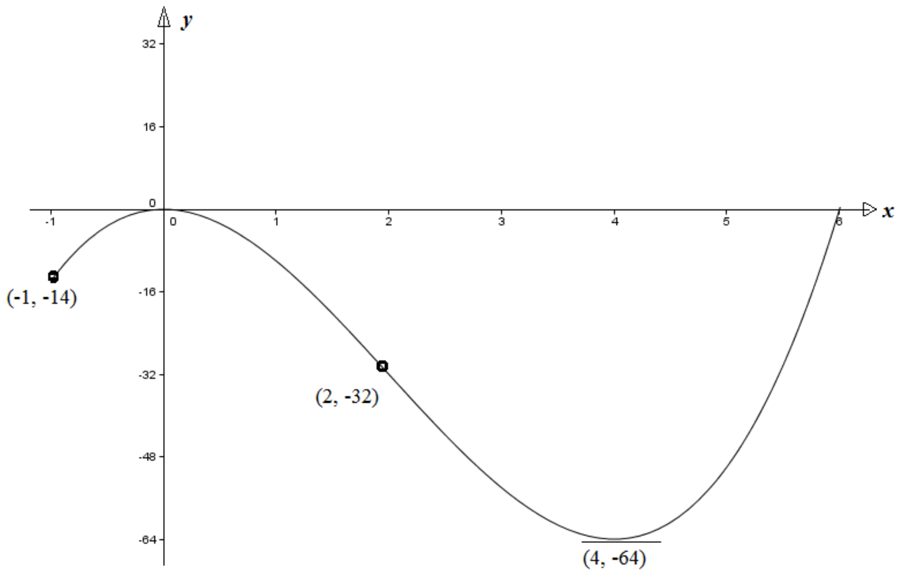
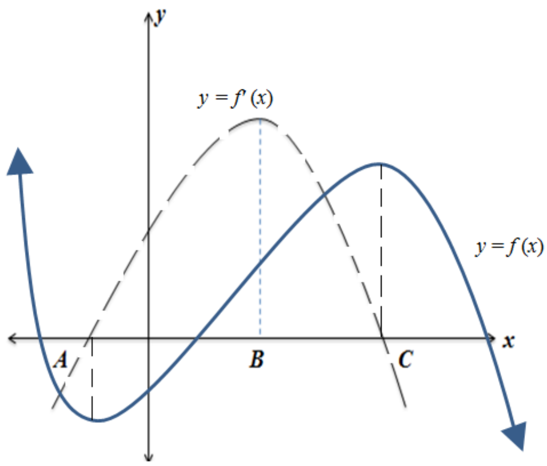
$\therefore$ point of inflection at $(2,-32)$

</td>
<td>

1 mark: sets $f''(x)=0$ and finds the possible point of inflection

1 mark: performs concavity test to establish point of inflection

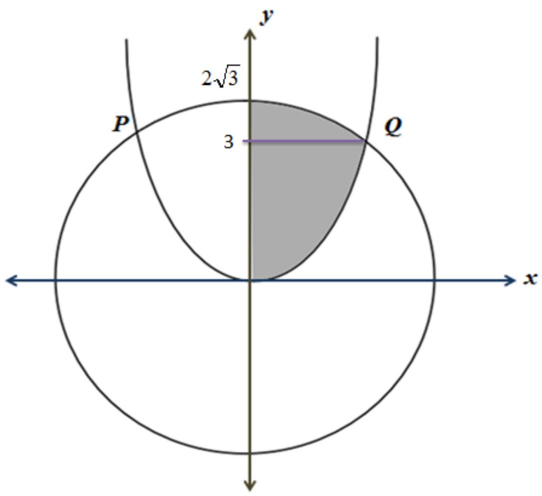
</td>
<td></td>
</tr>
</table>

(iii)		1 mark: correct shape 1 mark: Correct end points clearly shown and labelled 1 mark: their stationary points and point of inflection labelled.
b)(i)	Minimum turning point at *A* and point of inflection at *B*. 	1 mark for each
(ii)	The function is decreasing when $f'(x) < 0$. $\therefore \quad x < A$ and $x > C$	1 mark (need both regions to get the mark)
b)(iii)		1 mark: correct shape 1 mark: positions of *A*, *B* and *C* clearly marked and correct nature for each of the points demonstrated

<table>
<tr>
<td></td>
<td>$\int_{-1}^{1} 4^x dx$ using Simpsons rule with 5 function values
<table>
<tr><td>x</td><td>-1</td><td>-0.5</td><td>0</td><td>0.5</td><td>1</td></tr>
<tr><td>y</td><td>0.25</td><td>0.5</td><td>1</td><td>2</td><td>4</td></tr>
</table>
$$\int_{-1}^{1} 4^x\,dx \approx \frac{0.5}{3}\left(0.25+4\times 0.5+2\times 1+4\times 2+4\right)$$
$$= \frac{65}{24}$$
</td>
<td>2 marks – correctly applies Simpsons rule with 5 function values

1 mark – has table of values but small calculation error or correctly states Simpsons rule but error in table of values</td>
</tr>
</table>

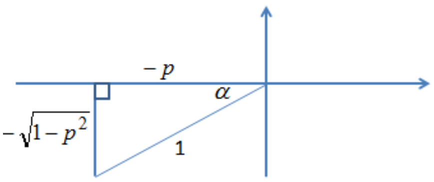
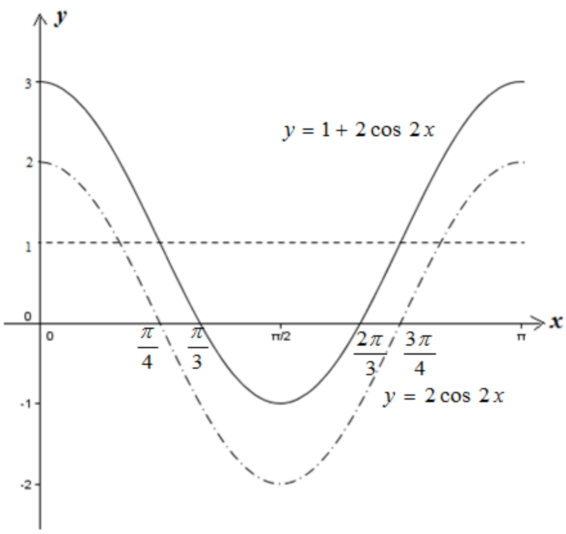
Question 15

a)	$A=\int_0^2\left((-2x)-(x^2-4x)\right)dx$ $=\int_0^2\left(2x-x^2\right)dx$ $=\left[x^2-\frac{x^3}{3}\right]_0^2$ $=\left[4-\frac{8}{3}\right]-0$ $=\frac{4}{3}\text{ units}^2$	1 mark1 correctly identifies the integral to be evaluated between the two curves - must have functions in correct order 1 mark finds correct primitive for their integral 1 mark correctly substitutes bounds for their integral (if student just takes absolute value at end then they lose 1 mark)
b)(i)	$y=x^2$ and $x^2+y^2=12$.  $x^2+y^2=12.\quad y=x^2$ Solving simultaneously, $x^2+x^4-12=0$ $x^4+x^2-12=0$ $(x^2+4)(x^2-3)=0$ **1 mark** $x^2+4=0$, No solution $x^2-3=0\qquad \therefore x=\pm\sqrt{3}$ Then $y=(\sqrt{3})^2=3$ **1 mark**	1 mark: writes the simultaneous equation and factorises correctly. 1 mark: correctly solves for *x* and hence proves the result (MUST reject the $x^2+4=0$)

(ii)	$V = \pi \int x^2 dy$ $y = x^2$ $y^2 = 12 - x^2$ $= \pi \int_0^3 y\,dy + \pi \int_3^{2\sqrt{3}} 12 - y^2 dy$ $= \pi \left[\frac{y^2}{2}\right]_0^3 + \pi \left[12y - \frac{y^3}{3}\right]_3^{2\sqrt{3}}$ $= \pi \left[\frac{9}{2} - 0\right] + \pi\left[24\sqrt{3} - 8\sqrt{3} - 36 + 9\right]$ $= \pi\left(16\sqrt{3} - \frac{45}{2}\right)$ unit^3.	3 marks: correct solution from correct working. 2 marks: gives the correct expression for volume and makes significant progress towards the answer 1 mark: calculates one of the volumes correctly (possibly $\frac{9}{2}\pi$
c)(i)	$x - 2y + 8 = 0$ $4x - 6y + 27 = 0$ Equation of the line that passes through the point of intersection is of the form $x - 2y + 8 + k(4x - 6y + 27) = 0$ **1 mark** $x(1 + 4k) - y(2 + 6k) + 8 + 27k = 0$ Gradient of this line is $m = \frac{1 + 4k}{2 + 6k} = -\frac{3}{2}$ $2 + 8k = -6 - 18k$ $26k = -8$ $k = \frac{-8}{26} = \frac{-4}{13}$ **1 mark** Hence, the required line is $x - 2y + 8 + \frac{-4}{13}(4x - 6y + 27) = 0$ $13x - 26y + 104 - 16x + 24y - 274 = 0$ $-3x - 2y - 170 = 0$ $3x + 2y + 170 = 0$ **1 mark**	1 mark: writes the general form of the line that passes through the point of intersection of two other lines. 1 mark: Equates the gradient of the line to $-\frac{3}{2}$ and evaluates k. 1 mark: substitutes the value of k into the general form and simplifies to give the equation

d(i)	$\frac{d}{dx}\left(\sqrt{1+x^2}\right)=\frac{1}{2\sqrt{1+x^2}}\times 2x = \frac{x}{\sqrt{1+x^2}}$	1 mark: correctly differentiates, showing all the working
d)(ii)	From (i), using the anti- derivative, $\int \frac{x}{\sqrt{1+x^2}}dx=\sqrt{1+x^2}+C$ **1 mark** $\int \frac{x}{2\sqrt{1+x^2}}dx=\frac{1}{2}\sqrt{1+x^2}+C$ $\int_0^1 \frac{x}{2\sqrt{1+x^2}}dx = \frac{1}{2}\left[\sqrt{1+x^2}\right]_0^1$ **1 mark** $= \frac{1}{2}\left(\sqrt{2}-1\right)$ **1 mark**	1 mark: writes the anti-derivative statement 1 mark: gives the correct expression for evaluating the integral 1 mark: correct solution

Question 16

a)	 $\sec(90+\alpha) = \dfrac{1}{\cos(90+\alpha)}$ $= \dfrac{1}{-\sin\alpha}$ **1 mark** $= \dfrac{1}{--\sqrt{1-p^2}}$ $= \dfrac{1}{\sqrt{1-p^2}}$ **1 mark**	2 marks: correct answer from correct working 1 mark: • Draws the correct diagram to show the dimensions OR • Writes $\sec(90+\alpha) = \dfrac{1}{-\sin\alpha}$
b)(i)	$1+2\cos 2x = 0$ $\cos 2x = -\dfrac{1}{2}$ $2x = \dfrac{2\pi}{3}$ or $\dfrac{4\pi}{3}$ $\therefore x = \dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$	1 mark: solves the equation or shows by substitution
(ii)		1 mark: correct shape 1 mark: correct intercepts and shows how $y = 2\cos 2x$ transports to $y = 1+2\cos 2x$ at $\dfrac{\pi}{4}$ and $\dfrac{3\pi}{4}$ at $y=1$ 1 mark: an even scale in terms of $\dfrac{\pi}{4}$ and $\dfrac{\pi}{3}$ are drawn on a rules axis

(iii)	Area = $\left\| \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} 1+2\cos 2x\,dx \right\|$ = $\left\| \left[x+\sin 2x\right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \right\|$ $\left\| \left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)-\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right) \right\|$ = $\left\| \left(\frac{\pi}{3}-\sqrt{3}\right) \right\|$ $\sqrt{3}-\frac{\pi}{3}$ sq. units	2 marks: correct answer from correct working 1 mark: incorrect evaluation or not using the absolute for area. (Take a mark off, if inconsistent with absolute sign from line to line.)
c)(i)	Perimeter $P = r\theta + 2r$ $r\theta + 2r$ = 375 **1 mark** $\theta = \frac{375-2r}{r}$ $A = \frac{1}{2}r^2\theta$ $= \frac{1}{2}r^2 . \frac{375-2r}{r}$ $= \frac{r}{2}(375-2r)$	1 mark: gives the correct expression for perimeter 1 mark: correctly derives the result showing all the working
(ii)	From (i), $A = \frac{375}{2}r - r^2$ $\frac{dA}{dr} = \frac{375}{2} - 2r$ = 0 $2r = \frac{375}{2}$ $r = \frac{375}{4}$ **1 mark** $\frac{d^2A}{dr^2} = -2 < 0$ $\therefore$ Area is maximum when $r = \frac{375}{4}$ **1 mark** Maximum area $= \frac{1}{2} \times \frac{375}{4}\left(375 - 2 \times \frac{375}{4}\right)$ **1 mark** = $8789.0625\ m^2$	1 mark: finds the stationary point correctly 1 mark: proves that the area is maximum 1 mark: finds the maximum area

(iii)	From (i), $\theta = \frac{375}{r} - 2$ $= \frac{375}{\left(\frac{375}{4}\right)} - 2$ = 2 radians $= 2 \times \frac{180}{\pi}$ $= 114°35'$ $110° < 114°35'$ No, the built area will be less	2 mark: correct answer with correct working. (MUST state $110° < 114°35'$) 1 mark: calculates θ and makes some reasonable comment based on their value of θ.