Killara HIGH SCHOOL

Student Number

2016
Mathematics

HSC Course Assessment Task 3
1st June 2016

General Instructions
- Reading time – 5 minutes
- Working time **60 minutes**
- Write using blue or black pen
  Black pen is preferred
- Board approved calculators may be used
- In Questions 4 - 16 show relevant mathematical reasoning and/or calculations
- Use a separate booklet for each question
- A reference sheet is provided at the back of this paper
- Diagrams are NOT to scale (unless indicated)

Total Marks – 42

Section I - Page 2
3 marks
- Attempt Questions 1 - 9
- Allow about5 minutes for this section

Section II - Pages 3 - 5
41 marks
- Attempt Questions 4 - 6
- Allow about 55 minutes for this section

Multiple Choice	**/ 3**
Q 4 (Logarithms and Exponentials)	**/ 21**
Q 5 (Probability)	**/ 7**
Q6 (Applications of Calculus to the Physical World)	**/ 11**
Total	**/ 42**

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 20% of the HSC *Course Assessment*

Section I

3 marks
Attempt Questions 1 - 3
Allow about 5 minutes for this section
Use the multiple-choice answer sheet for questions 1 -3 (Detach from paper)

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1. Given that $\log_a x = 0.3$ and $\log_a y = 0.2$, then the value of $\log_a x^2 y^3$ is

(A) $a^{0.36}$

(B) 1

(C) 1.2

(D) $a^{0.0036}$

2. If $y = e^x - e^{-x}$, then the gradient of the tangent at $x = 0$ is:

(A) 0

(B) 2

(C) -2

(D) e^2

3. At Killara High School the probability that a student takes a Mathematics course is 0.9 and the probability that a student takes a Science course is 0.8. The probability that a student takes a Mathematics or a Science course is 0.95.

What is the probability that a student does both Mathematics and Science?

(A) 0.1

(B) 0.2

(C) 0.75

(D) 1

Section II

39 marks
Attempt Questions 4 - 6
Allow about 55 minutes for this section

Answer each question in a separate booklet. Extra writing booklets are available.

In questions 4 – 6 your responses should include relevant mathematical reasoning and / or calculations.

Question 4 (21 marks) Use a SEPARATE writing booklet

(a) Differentiate with respect to x and fully simplify

i) $y = e^x \log_e x$ **2**

ii) $y = \cos(\log_e x)$ **2**

(b) Evaluate using exact values $\int_0^{e^2} \frac{4}{x}\,dx$ **3**

(c) Solve for x **3**

$$e^{2x} - 2e^x = 0$$

(d) Let $y = \log_e(x-1)$

i) Find the $x-$ intercept. **2**

ii) Sketch the graph of $y = \log_e(x-1)$. **2**

iii) Find the exact value of the area bounded by $y = \log_e(x-1)$, $x-axis$ and the line $x = 5$. **4**

iv) Find the exact volume of the solid generated when the area between $y = \log_e(x-1)$, $x-$axis and $y-$axis is rotated about the $y-$axis. **3**

Question 5 (7 marks) Use a SEPARATE writing booklet

a) Two standard six sided dice are tossed. Given that a 4 is showing on the upmost face of one of the dice , what is the probability that the sum of the upmost faces on both dice exceeds 8? **2**

b) In a game of chess between two players, X and Y, of about equal ability, the player with the white pieces, having the first move, has a probability of 0.5 of winning, and the probability that the player with the black pieces wins in 0.3.

The two players, X and Y, play each other twice in a chess competition, each player having the white pieces once. In the competition, a player who wins a games scores 1 point, a draw score as $\frac{1}{2}$ a point to each player and a player who loses scores 0 points.

i) What is the probability that the game ends in a draw (i.e. no player wins) **1**

ii) What is the probability that, as a result of these two games X scores 2 points? **2**

iii) What is the probability that, as a result of these two games X scores $1\frac{1}{2}$ points? **2**

Question 6 (11 marks) Use a SEPARATE writing booklet

a) The rate of flow (R litres per minute) of water through a filter is given by

$$R = 6t - \frac{t^2}{3}$$

i) At what times is no water flowing through the filter **2**

ii) Find the volume of the water that flows through the filter in the first 4 minutes **3**

b) In a chemical reaction the amount (M kilograms) of undissolved solid after t hours is given by

$$M = Ae^{-kt}.$$

After 24 hours 15% of the initial mass of solid remains undissolved.

i) The value of k **2**

ii) What percentage of solid has dissolved after 5 hours (give your answer correct to one decimal place) **2**

iii) How long it takes for 50% of the solid to be dissolved. (give your answer correct to the nearest minute) **2**

End of paper ☺

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Student Number

Killara HIGH SCHOOL

HSC Mathematics

Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 3. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

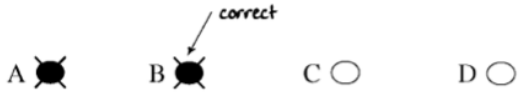
Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ✕ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.



Start Here →

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

HSC Mathematics Task 3 2016 Marking Scheme

Question 4

(a) Differentiate with respect to x and fully simplify

i) $y = e^x \log_e x$ **2**

1 mark – correctly identifies need to use product rule **1 mark –** finds the correct derivative from product rule	
$y = e^x \log_e x$ $u = e^x \quad v = \log_e x$ $u' = e^x \quad v' = \frac{1}{x}$ $\therefore \frac{dy}{dx} = uv' + vu'$ $\therefore \frac{dy}{dx} = \frac{e^x}{x} + e^x \log_e x$	*Generally very well done*

ii) $y = \cos(\log_e x)$ **2**

1 mark – gives $\sin(\log_e x)$ as the solution or a minor error **2 mark –** finds the correct derivative from chain rule	
$y = \cos(\log_e x)$ $\frac{dy}{dx} = -\sin(\log_e x) \times \frac{1}{x} = -\frac{\sin(\log_e x)}{x}$	*Generally very well done*

(b)

Evaluate using exact values $\int_1^{e^2} \frac{4}{x} dx$ **3**

1 mark – finds correct primitive **1 mark –** substitutes limits correctly **1 mark** – finds correct exact value	
$\int_1^{e^2} \frac{4}{x} dx = 4\left[\ln x\right]_1^{e^2}$ $= 4\left[\ln e^2 - \ln 1\right]$ $= 4\left[2 - 0\right]$ $= 8$	*Some students got mixed up with the constant – but genrally well done* *May did not simplify to the whole number*

(c) Solve for x **3**

$$e^{2x} - 2e^{x} = 0$$

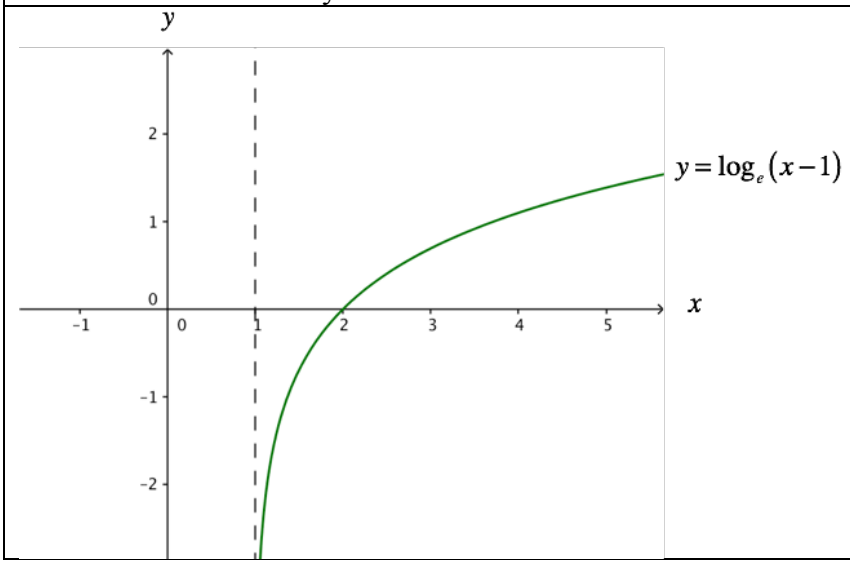
1 mark – factorises as a quadratic **1 mark** – discounts $e^x = 0$ with a reason **1 mark** – correct solution $x = \log_e 2$	
$e^{2x} - 2e^{x} = 0$ $\therefore e^{x}\left(e^{x} - 2\right) = 0$ $\therefore e^{x} = 0$ no solution since $e^{x} > 0$ OR $\therefore e^{x} = 2$ $x = \log_e 2$	*Many students did not create a quadratic* *Many students did not say why the solution $e^x = 0$ and lost a mark*

(d) Let $y = \log_e(x-1)$

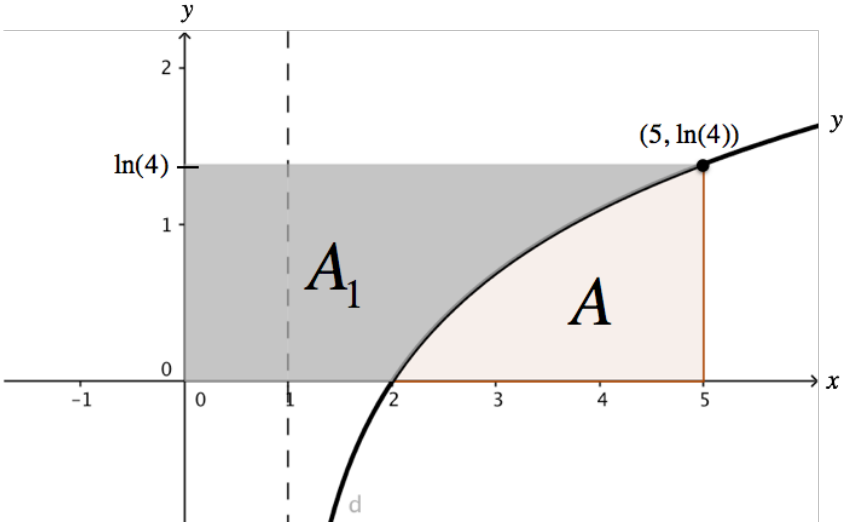
i) Find the $x-$ intercept. **2**

1 mark – correct x intercept **1 mark** – sufficient working	
$y = \log_e(x-1)$ $\therefore 0 = \log_e(x-1)$ $\therefore x = 2$ x intercept is $(2, 0)$	*Genearlly very well done*

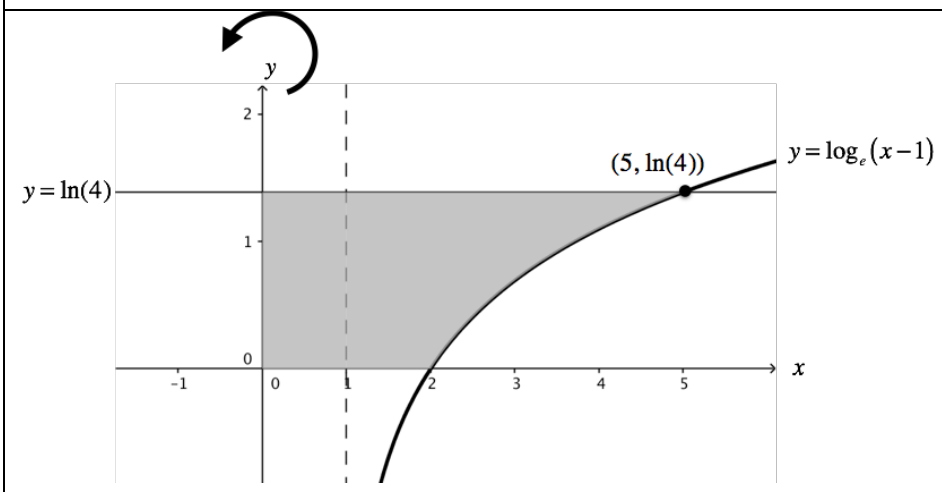
ii) Sketch the graph of $y = \log_e(x-1)$ **2**

1 mark – x intercept and asymptote clearly and correctly indicated **1 mark** - correct shape of graph lose 1 mark if not evenly marked scale	*Many did not label the asymptote or have the correct one* *Many had no even scale or their graph did not work on the scale they had given*
	

iii) Find the exact value of the area bounded by $y = \log_e(x-1)$, $x-axis$ and the line $x = 5$. **4**

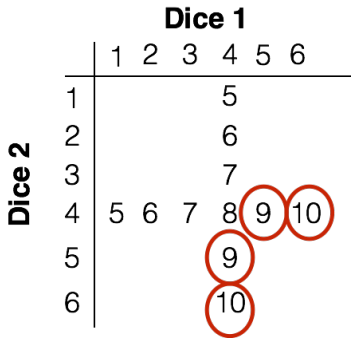
1 mark – breaking into rectangle with two separate areas **1 mark** - making x the subject $\quad \text{if } y = \log_e(x-1)$ $\therefore x = e^y + 1$ **1 mark** – finding correct primitive and expression for the area **1 mark** - finding exact value of area	
 if $y = \log_e(x-1)$ $\therefore x = e^y + 1$ $\int_2^5 \log_e(x-1)\,dx = A_{\text{rectangle}} - A_1$ $= (\ln(4)\times 5) - \int_0^{\ln 4} e^y + 1\,dy$ $= 5\ln(4) - \left[e^y + y\right]_0^{\ln 4}$ $= 5\ln 4 - \left[(4+\ln 4) - (1-0)\right]$ $= (4\ln 4 - 3)\text{ units}^2$	*Not very well done* *Many tried to integrate log* *Some found A_1 but did not subtract from rectangle*

iv) Find the exact volume of the solid generated when the area between $y = \log_e(x-1)$, the line $y = \log_e 4$, the $x-$axis and the $y-$axis is rotated about the $y-$axis. **3**

1 mark: $V = \pi \int_0^{\ln 4} \left(e^y + 1\right)^2 \, dy$ **1 mark:** correct primitive for their integral (unless trivial) **1 mark:** correct value for their integral	
 $V = \pi \int_0^{\ln 4} x^2 \, dy$ $= \pi \int_0^{\ln 4} \left(e^y + 1\right)^2 \, dy$ $= \pi \int_0^{\ln 4} \left(e^{2y} + 2e^y + 1\right) dy$ $= \pi \left[\frac{1}{2}e^{2y} + 2e^y + y\right]_0^{\ln 4}$ $= \pi \left[\left(\frac{e^{2\ln 4}}{2} + 2e^{\ln 4} + \ln 4\right) - \left(\frac{1}{2} + 2 + 0\right)\right]$ $= \pi \left[\left(\frac{16}{2} + 2 \times 4 + \ln 4\right) - \left(\frac{5}{2}\right)\right]$ $= \pi \left(\frac{27}{2} + \ln 4\right)$	*Many did not square or change the subject* *Many did not square correctly* *Generally poorly done*

Question 5

a) Two standard six sided dice are tossed. Given that a 4 is showing on the upmost face of one of the dice, what is the probability that the sum of the upmost faces on both dice exceeds 8? **2**

1 mark – lists the sample space in any format **1 mark** – finds the correct probability	
The sample space for a 4 showing on one upmost face  $\therefore P(\text{exceeds } 8) = \frac{4}{11}$	

b) In a basketball shootout, a player gets one attempt at scoring a 3 point shot and three attempts and three attempts at scoring from a one point shot. From past statistics a players has a probability of 0.2 of scoring a three point shot and a probability of 0.75 of scoring a one-point shot.

i) What is the probability that a player scores 0 points **1**

1 mark: correct probability with working	
$P(0 \text{ points}) = 0.8 \times (0.25)^3 = 0.0125$	

ii) What is the probability that a player scores all three one point shots but misses the three point shot? **1**

1 mark: correct probability with working	
$P(\text{score three 1 points but miss 3 point}) = 0.8 \times (0.75)^3 = 0.3375$	

iii) What is the probability that, a player scores at least 2 points? **3**

1 mark: recognises complimentary event or lists possible outcomes **1 mark:** correctly calculates the probability for 1 point **1 mark:** obtains correct probability	
$P(\text{at least 2 points}) = 1 - \left(P(0 \text{ points}) + P(1 \text{ point})\right)$ $= 1 - \left(0.0125 + \left(0.8 \times 0.75 \times (0.25)^2 \times 3\right)\right)$ $= 0.959$	

Question 6

a) The rate of flow (R litres per minute) of water through a filter is given by

$$R = 6t - \frac{t^2}{3}$$

i) At what times is no water flowing through the filter **2**

1 mark: equates derivative to 0 **1 mark:** correctly finds both times	
we need $R = \frac{dV}{dt} = 6t - \frac{t^2}{3} = 0$ $\therefore 18t - t^2 = 0$ $\therefore t(18 - t) = 0$ $\therefore t = 0$ minutes or $t = 18$ minutes	

ii) Find the volume of the water that flows through the filter in the first 4 minutes **3**

1 mark: establishes correct integral **1 mark:** finds correct primitive function **1 mark:** finds correct value and gives units	
Since $\frac{dV}{dt} > 0$ for $0 \le t \le 4$ $V(\text{water first } 4 \text{ min}) = \int_0^4 \left(6t - \frac{t^3}{9}\right) dt$ $= \left[3t^2 - \frac{t^3}{9}\right]_0^4$ $= \left[3(4)^2 - \frac{4^3}{9}\right] - [0]$ $= \frac{368}{9}$ litres	

b) In a chemical reaction the amount (M kilograms) of undissolved solid after t hours is given by

$$M = Ae^{-kt}.$$

After 24 hours 15% of the initial mass of solid remains undissolved.

i) The value of k **2**

1 mark: substituting into expression to obtain $0.15A = Ae^{-24k}$ **1 mark:** finds correct value of k (any number of decimal places) with working	
When $t = 24$, $M = 0.15A$ $\therefore 0.15A = Ae^{-24k}$ $\therefore 0.15 = e^{-24k}$ $\therefore \log_e(0.15) = -24k$ $\therefore k = -\dfrac{\log_e(0.15)}{24}$ $\therefore k = 0.07904666604$	

ii) What percentage of solid has dissolved after 5 hours (give your answer correct to one decimal place) **2**

1 mark: finding $\therefore M = 0.6735228677A$ with working **1 mark:** correctly concluding the amount of the original solid that has dissolved	
When $t = 5$, $k = -\dfrac{\log_e(0.15)}{24}$, $M = ?$ $\therefore M = Ae^{-5\times\left(\frac{\log_e(0.15)}{24}\right)}$ $\therefore M = 0.6735228677A$ **(this is the amount remaining)** $\therefore 32.65\%$ of the orginal solid has disolved	

iii) How long it takes for 50% of the solid to be dissolved. (give your answer correct to the nearest minute) **2**

1 mark: finding $\therefore \log_e(0.5) = -\left(\dfrac{\log_e(0.15)}{24}\right)t$ **1 mark:** correctly identifying the correct time (to the nearest minute)	
When $t = ?$, $k = -\dfrac{\log_e(0.15)}{24}$, $M = 0.5A$ $\therefore 0.5A = Ae^{-\left(\frac{\log_e(0.15)}{24}\right)t}$ $\therefore \log_e(0.5) = -\left(\dfrac{\log_e(0.15)}{24}\right)t$ $\therefore t = 8$ hours 46 minutes	