Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

2020

Year 12

Mathematics Advanced

Task 3
Monday, 25th May

Q	Marks
1	11
2	6
3	9
4	3
5	15
Total	**/44**

General Instructions:

- Reading time – 50 minutes
- Working time – 5 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations

Total Marks: 44

This question paper must not be removed from the examination room.

This assessment task constitutes 25% of the course.

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Question 1 (11 marks)

(a) Differentiate the following:

(i) $f(x) = x^3 - e^x$ **1**

(ii) $f(x) = x\sqrt{x}$ **2**

(ii) $f(x) = \tan(x^2)$ **2**

(iii) $y = \dfrac{\sin x}{x}$ **2**

(iv) $y = \log_3 x$ **1**

(v) $f(x) = \ln(e^x(x+2))$ **3**

Question 2 (6 marks)

(a) Given the curve $y = x^4 - 8x^2$. **3**
Find the stationary points and determine their nature.

(b) Determine the point of inflection of the curve $y = xe^x$. **3**

Question 3 (9 marks)

(a) Find the following.

(i) $\int 3 - x\, dx$ **2**

(ii) $\int \frac{2}{x}\, dx$ **1**

(iii) $\int_0^4 (x-1)^3\, dx$ **2**

(iv) $\int_{-2}^{0} e^{3x}\, dx$ **2**

(v) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{2} - \cos x \, dx$ **2**

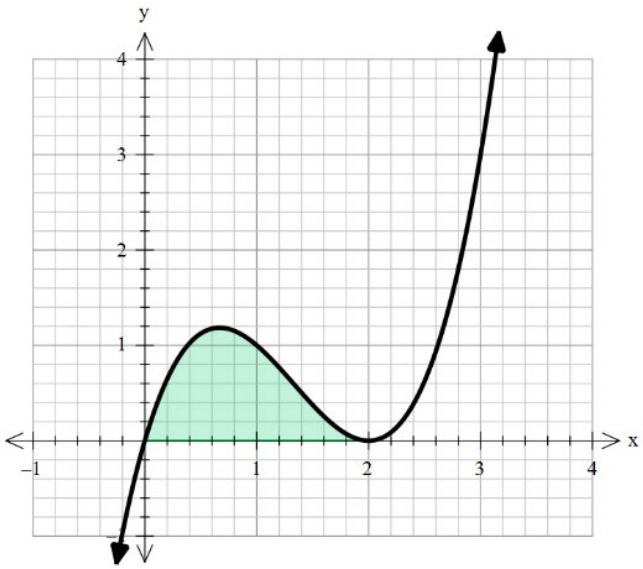
Question 4 (3 marks)

(a) (i) Differentiate $f(x) = \ln(x^2 + 16)$. **1**

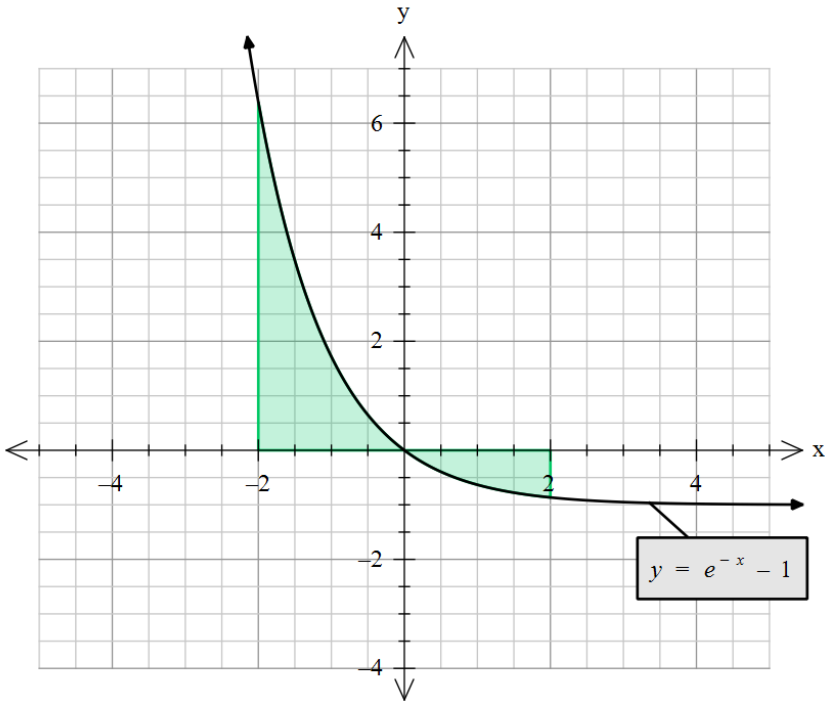
(ii) Hence or otherwise, find $\int_0^3 \frac{x}{x^2+16} \, dx$. **2**

Question 5 (15 marks)

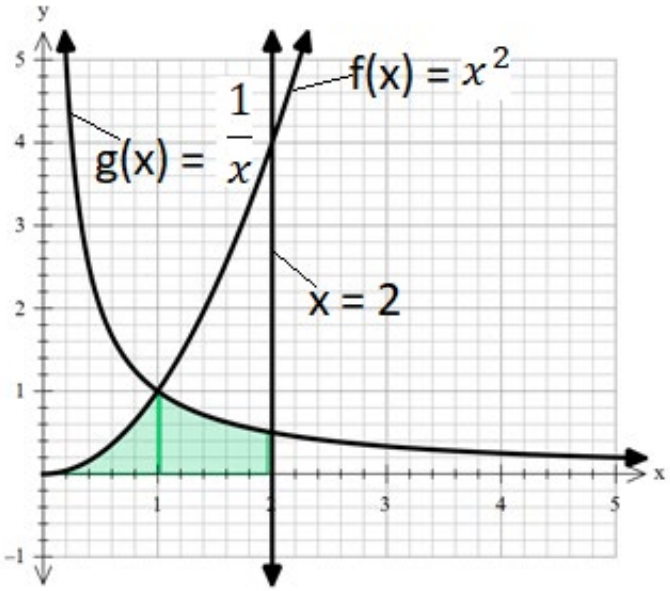
(a) Find the exact area of the region bounded by the curve $y = x^3 - 4x^2 + 4x$. **2**



(b) Find the exact area of the region bounded by the curve $y = e^{-x} - 1$. **2**



(c) Calculate the exact area bounded by $f(x) = x^2$, $g(x) = \frac{1}{x}$, $x > 0$, the x-axis and the line $x = 2$. **3**

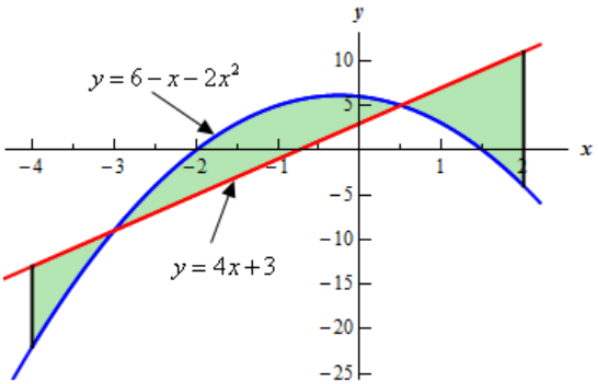


(d) (i) Complete the table of values. **1**

x	1	2	3	4
$\frac{1}{x^2+1}$				

(ii) Approximate $\int_1^4 \frac{1}{x^2+1}\,dx$ using the trapezoidal rule with 3 sub-intervals. Answer in 2 decimal places. **2**

(e) Determine the exact area of the region bounded by $y = 4x + 3, y = 6 - x - 2x^2$, $x = -4$ *and* $x = 2$. **5**



End of Exam

Extra writing space

If you use this space, clearly indicate which question you are answering.

Solutions with Marking Scheme

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Total Marks: 43

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Question 1 (11 marks)

(a) Differentiate the following:

(i) $f(x) = x^3 - e^x$ **1**

$f'(x) = 3x^2 - e^x$ ✓

(ii) $f(x) = x\sqrt{x}$ **2**

$f(x) = x^{\frac{3}{2}}$ ✓

$f'(x) = \frac{3}{2}x^{\frac{1}{2}}$

$= \frac{3\sqrt{x}}{2}$ ✓

(ii) $f(x) = \tan(x^2)$ **2**

$f'(x) = 2x\sec^2(x^2)$

✓ ✓

1 mistake:
1 mark off.

(iii) $y = \frac{\sin x}{x}$ **2**

$u = \sin x \quad v = x$
$u' = \cos x \quad v' = 1$ ✓

$y' = \frac{u'v - uv'}{v^2}$

$= \frac{x\cos x - \sin x}{x^2}$ ✓

1 mistake:
1 mark off

(iv) $y = \log_3 x$ **1**

$y' = \dfrac{1}{\log_e 3 \times x}$ ✓

(v) $f(x) = \ln(e^x(x+2))$ **3**

$f(x) = \ln e^x + \ln(x+2)$ ✓ 1 mistake:

$f'(x) = x + \ln(x+2)$ ✓ 1 mark off.

$f'(x) = 1 + \dfrac{1}{x+2}$ ✓

$= \dfrac{x+2+1}{x+2}$

$= \dfrac{x+3}{x+2}$ ✓

Question 2 (6 marks)

(a) Given the curve $y = x^4 - 8x^2$. **3**
Find the stationary points and determine their nature.

1 mistake;
1 mark off

✓ $y' = 4x^3 - 16x$

✓ $y' = 0, \quad 4x(x^2 - 4) = 0$

$\therefore$ stationary points at $x = \pm 2, x = 0$

✓ $y'' = 12x^2 - 16$

✓ $y''(0) = -16 < 0$

$y''(2) = 32 > 0, \quad y''(-2) = 32 > 0$

$\therefore$ Maximum point at $x = 0$

minimum points at $x = -2$ and $x = +2$

(b) Determine the point of inflection of the curve $y = xe^x$. **3**

1 mistake: 1 mark off

$y' = e^x + xe^x$ ✓

$y'' = e^x + e^x + xe^x$

$= 2e^x + xe^x$

$y'' = 0$. $e^x(2+x) = 0$ ✓

$\therefore e^x \neq 0$. $2 + x = 0$.

~~$\therefore$ when $x = -2$~~

$\therefore$ The point of inflection at $x = -2$.

x	-3	-2	-1
y''	$-$	0	$+$
Concavity	$\cap$	$\cdot$	$\cup$

✓

$y(-2) = -2e^{-2} = \frac{-2}{e^2}$

$\therefore$ The point inflection $\left(-2, \frac{-2}{e^2}\right)$

Question 3 (9 marks)

(a) Find the following.

(i) $\int 3 - x\,dx$ **2**

1 mark off not missing "+C".

$= 3x - \frac{x^2}{2} + C$ ✓

✓

(ii) $\int \frac{2}{x}\,dx$ **1**

$= 2\ln x + C$ ✓

No mark taking off for not having "+C".

(iii) $\int_0^4 (x-1)^3\,dx$ **2**

$= \left[\frac{(x-1)^4}{4}\right]_0^4$ ✓

$= \frac{81}{4} - \frac{(-1)^4}{4}$

$= \frac{80}{4} = 20$ ✓

(iv) $\int_{-2}^0 e^{3x}\,dx$ **2**

$= \left[\frac{e^{3x}}{3}\right]_{-2}^0$ ✓

$= \frac{1}{3} - \frac{e^{-6}}{3}$

$= \frac{1-e^{-6}}{3}$ ✓

OR

$= \frac{e^6 - 1}{3e^6}$ ✓

OR

≈ 0.3325 ✓

(v) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{2} - \cos x \, dx$ **2**

$= \left[-\frac{\cos x}{2} - \sin x \right]_0^{\frac{\pi}{2}}$ ✓

$= -\left[\frac{\cos x}{2} + \sin x \right]_0^{\frac{\pi}{2}}$

$= -\left[(0+1) - \left(\frac{1}{2} - 0\right) \right]$

$= -\frac{1}{2}$ ✓

Question 4 (3 marks)

(a) Evaluate the following:

(i) ~~Find the derivative of~~ Differentiate $f(x) = \ln(x^2 + 16)$. **1**

$f'(x) = \frac{2x}{x^2+16}$ ✓

(ii) Hence or otherwise, find $\int_0^{\ln 4^3} \frac{x}{x^2+16} \, dx$. **2**

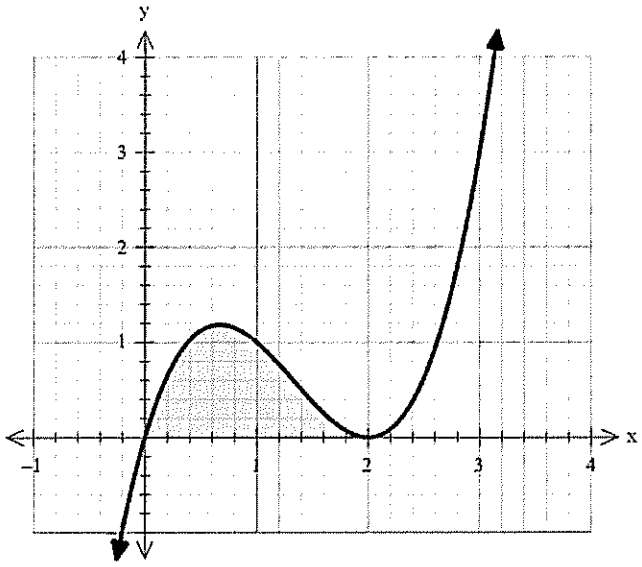
$= \left[\frac{\ln(x^2+16)}{2} \right]_0^3$ ✓

$= \frac{\ln 5^2}{2} - \frac{\ln 4^2}{2} = \ln 5 - \ln 4 \approx 0.22$

✓ OR ✓

Question 5 (15 marks)

(a) Find the exact area of the region bounded by the curve $y = x^3 - 4x^2 + 4x$. **2**



$$\int_0^2 x^3 - 4x^2 + 4x \, dx$$

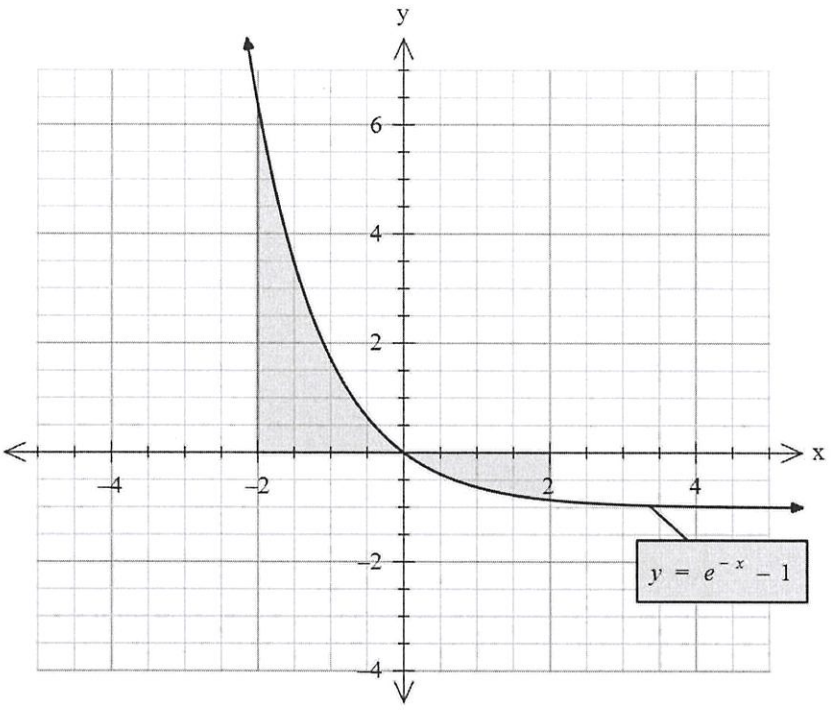
$$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2 \right]_0^2 \quad \checkmark$$

$$= \left[\frac{16}{4} - \frac{32}{3} + 8 \right] - 0$$

$$= 12 - \frac{32}{3}$$

$$= \frac{4}{3} \quad \checkmark$$

(b) Find the exact area of the region bounded by the curve $y = e^{-x} - 1$. **2**



1 mark off:
1 mistake

$\int_{-2}^{2} e^{-x} - 1 \, dx$

$= \int_{-2}^{0} e^{-x} - 1 \, dx + \left| \int_{0}^{2} e^{-x} - 1 \, dx \right|$ ✓

$= [-e^{-x} - x]_{-2}^{0} + \left| [-e^{-x} - x]_{0}^{2} \right|$

$= -[e^{-x} + x]_{-2}^{0} + [e^{-x} + x]_{0}^{2}$

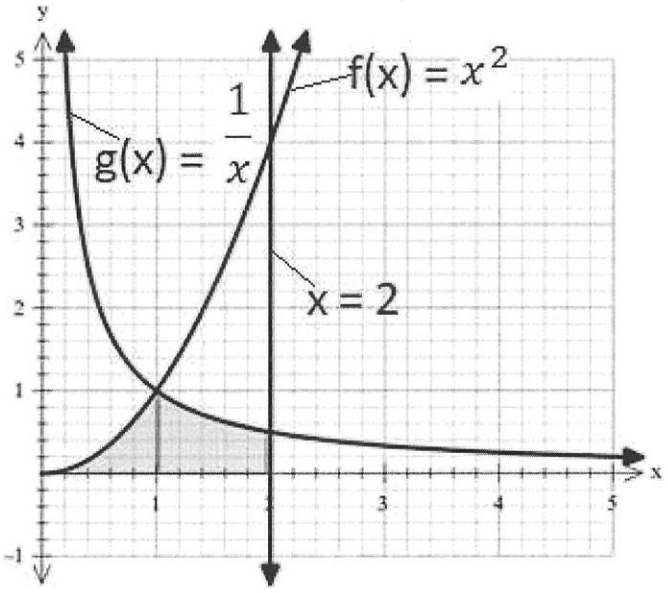
$= -[1 - (e^{2} - 2)] + [(e^{-2} + 2) - (1)]$

$= -(1 - e^{2} + 2) + (e^{-2} + 1)$

$= e^{2} - 3 + e^{-2} + 1$

$= -2 + e^{2} + e^{-2}$ ✓

(c) Calculate the exact area bounded by $f(x) = x^2$, $g(x) = \frac{1}{x}$, $x > 0$, the x-axis and the line $x = 2$. **3**



Total area

$= \int_0^1 x^2\,dx + \int_1^2 \frac{1}{x}\,dx$ ✓

$= \left[\frac{x^3}{3}\right]_0^1 + \left[\log_e x\right]_1^2$ ✓

$= \frac{1}{3} + (\ln 2 - \ln 1)$

$= \frac{1}{3} + \ln 2$

OR

$= \frac{1}{3} + \log_e 2$ ✓

OR

$= \frac{1 + 3\log_e 2}{3}$

1 mark off; 1 mistake.

(d) (i) Complete the table of values. 1

x	1	2	3	4
$\frac{1}{x^2+1}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{17}$ ✓

(ii) Approximate $\int_1^4 \frac{1}{x^2+1}\,dx$ using the trapezoidal rule with 3 sub-intervals. Answer in 2 decimal places. 2

$\int_1^4 \frac{1}{x^2+1}\,dx$

$\approx \frac{1}{2} \times \left[f(1) + 2f(2) + 2f(3) + f(4) \right]$ ✓

OR

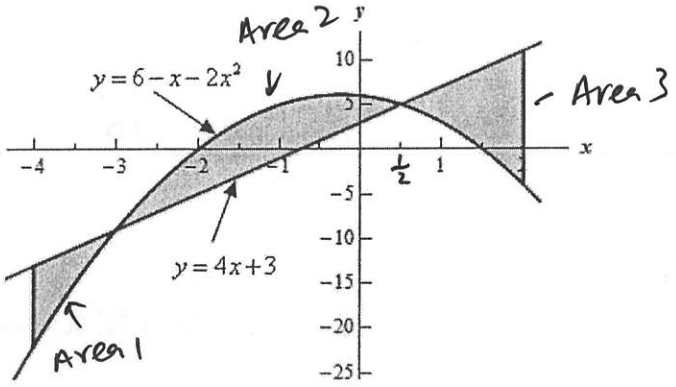
$\approx \frac{1}{2} \times \left[\frac{1}{2} + \frac{2}{5} + \frac{2}{10} + \frac{1}{17} \right]$ ✓

$= \frac{1}{2} \times \frac{197}{170}$

$\approx \frac{197}{340}$ ✓

exact

(e) Determine the area of the region bounded by $y = 4x + 3, y = 6 - x - 2x^2$, $x = -4$ *and* $x = 2$. **5**



each
- 1 mark for correct definite integral of each area.
- 1 mark for correct working
- 1 mark for correct answer.

Total Area 1

$$= \int_{-4}^{-3} 4x+3-(6-x-2x^2)\,dx \quad \checkmark$$

$$= \int_{-4}^{-3} 4x+3-6+x+2x^2\,dx$$

$$= \int_{-4}^{-3} 2x^2+5x-3\,dx$$

$$= \left[\frac{2x^3}{3}+\frac{5x^2}{2}-3x\right]_{-4}^{-3}$$

$$= \left(-18+\frac{45}{2}+9\right)-\left(\frac{2\times(-4)^3}{3}+40+12\right)$$

$$= \frac{27}{2}-\frac{28}{3} = \frac{25}{6}$$

Area 2

$$= \int_{-3}^{0.5} 6-x-2x^2-4x-3\,dx \quad \checkmark$$

$$= \int_{-3}^{0.5} -2x^2-5x+3\,dx$$

$$= \left[\frac{-2x^3}{3}-\frac{5x^2}{2}+3x\right]_{-3}^{0.5}$$

$$= \left(-\frac{1}{12}-\frac{5}{8}+1.5\right)-\left(+18-\frac{45}{2}-9\right)$$

$$= \frac{343}{24}$$

Area 3

$$= \int_{\frac{1}{2}}^{2} 2x^2+5x-3\,dx = \left[\frac{2x^3}{3}+\frac{5x^2}{2}-3x\right]_{\frac{1}{2}}^{2} \quad \checkmark$$

$$= \left(\frac{16}{3}+10-6\right)-\left(\frac{1}{12}+\frac{5}{8}-\frac{3}{2}\right)$$

$$= \frac{81}{8}$$

$\therefore$ Total area $= \frac{25}{6}+\frac{343}{24}+\frac{81}{8} = \frac{343}{12}$ $\checkmark$

End of Exam