Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

2021 **YEAR 12**

Advanced Mathematics

Task 1 Validation Task

9/12/21

Q	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

General Instructions:

- Reading time – 5 minutes
- Working time – 35 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 28

This question paper must not be removed from the examination room.

This Validation task constitutes 60% of the Assignment.

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In Questions 1 – 9, your response should include relevant mathematical reasoning and/or calculations.

Question 1 (2 marks)

Solve $3^x = 25$, giving your answer to 2 significant figures. **2**

Question 2 (2 marks)

Solve $3\log_2(10) + \log_2(x) = 6$. Express answer in simplest form. **2**

Question 3 (3 marks)

Solve $2^{x^2} = 8^{\frac{2}{3}x+1}$. **3**

Examination continues on next page

Question 4 (3 marks)

Differentiate the following, simplifying where necessary.

a) $y = 2e^{4x-3}$ **1**

b) $y = \left(2 - e^{-x^2}\right)^6$ **2**

Question 5 (2 marks)

If $\log_x(6) = a$ and $\log_x(3) = b$, find $\log_x(8)$ in terms of a and b. **2**

Question 6 (5 marks)

Radioactive decay is a process which causes a substance to break down over time. The half-life of a substance refers to the amount of time it takes for the substance to decay to half its original mass.

Consider the formula for the radioactive decay of the substance carbon-14,

$$C = C_0 e^{-kt},$$

where C_0 and k are constants, t is time in years, and C is the amount of carbon-14 measured in grams.

a) Find the value of C_0 if there is initially 1000 grams of carbon-14. **1**

b) The half-life of carbon-14 is known to be 5730 years. **3**

Use this to show that $k = \frac{\ln(2)}{5730}$.

c) Hence, find the amount of carbon-14, to the nearest gram, after 10,000 years. **1**

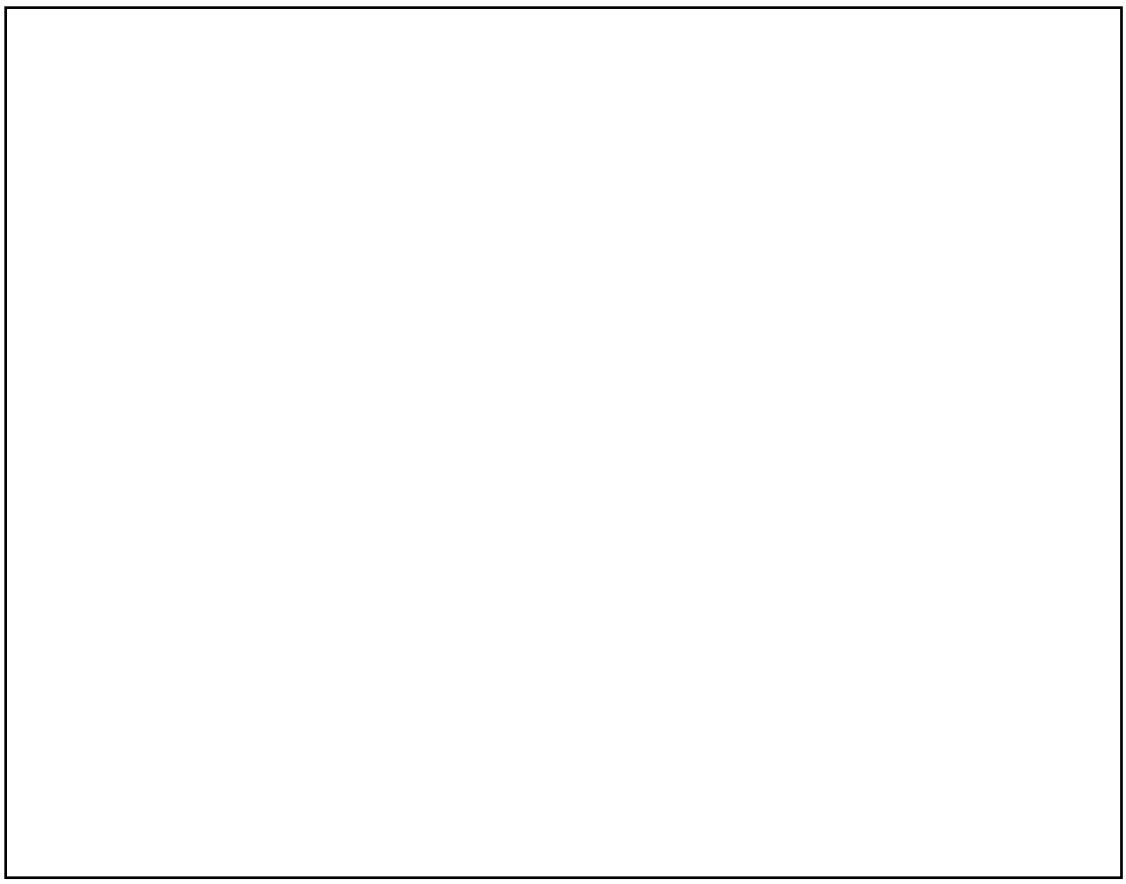
Examination continues on next page

Question 7 (3 marks)

(a) Sketch the graph of the following exponential function in the box below. **3**
Label x and y intercepts and the asymptote.

$$y = e^{-x} - 2$$

Note: The box suggests the size of the expected graph



Question 8 (6 marks)

Sound intensity level (L) measured in decibels (dB) is a logarithmic measure given as:

$$L = 10 \log_{10} \left(\frac{I}{I_0} \right)$$

The reference level of the intensity of sound (I_0) is 10^{-12} watts/m^2.

(a) The table below shows the average sound intensity levels (L) and the intensity of the sound (I) of four instruments of a symphony orchestra. **2**

Musical Instrument	Average sound intensity level, L (to nearest dB)	Intensity of sound, I (watts/m^2) to 7 decimal places
Cello	87	(i)
Violin	93	0.0019953
Trombone	100	0.0100000
Timpani and bass drum rolls	(ii)	0.0501187

Determine the missing values of (i) and (ii).

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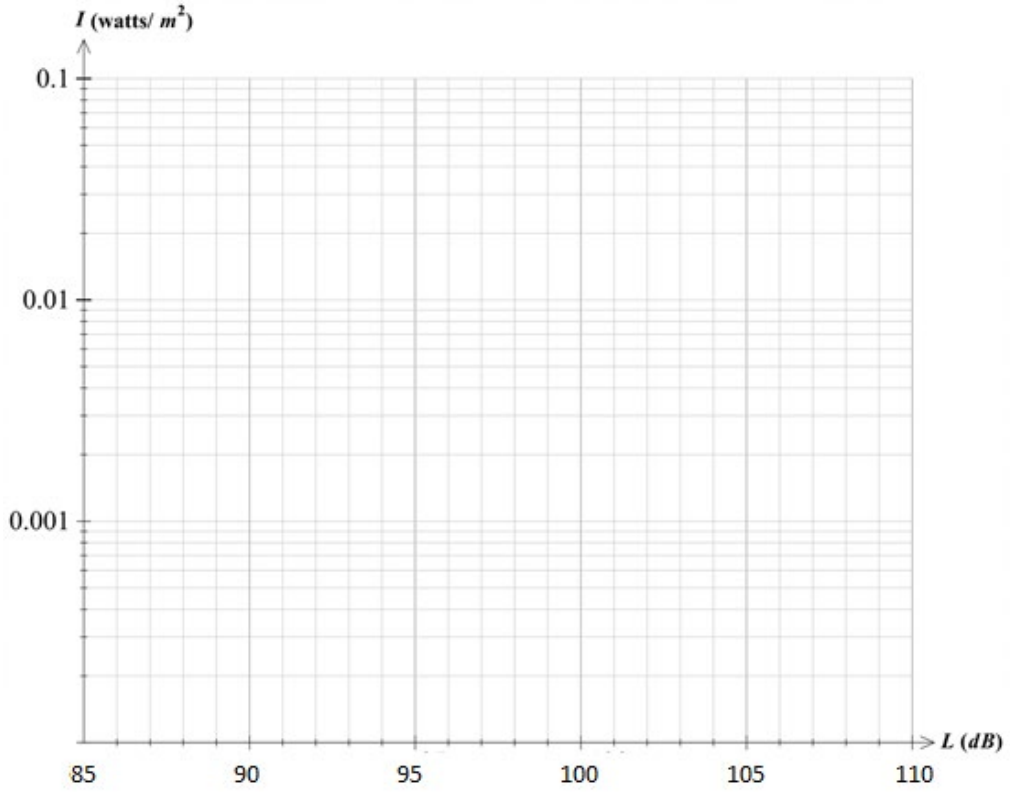
(b) Plot the intensity of sound (I) against the average sound intensity levels (L) for the **four instruments listed in part (a)** on the semi-logarithm graph below. Connect the points with a line. **2**



(c) Using the graph determine the average sound intensity level(dB) of the sound from a musical instrument that has an intensity of 0.006 watts/m^2. **1**

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(d) Explain why a semi-logarithm grid is useful for this data. **1**

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Question 9 (2 marks)

Given $\log_c(y) = \log_c(A) + tk\log_c(b)$, where A, b, c, k are (constant) positive real values with $b \neq 1$ and $c \neq 1$, and t and y are variables, show that $y = Ab^{kt}$. **2**

Examination continues on next page

Extra writing space

If you use this space, clearly indicate which question you are answering.

In Questions 1 – 9, your response should include relevant mathematical reasoning and/or calculations.

Question 1 (2 marks)

Solve $3^x = 25$, giving your answer to 2 significant figures. **2**

$x = \log_3(25)$ ① mark

$x = 2.9$ ① mark (2 sig. fig.)

Question 2 (2 marks)

Solve $3\log_2(10) + \log_2(x) = 6$. Express answer in simplest form. **2**

$\log_2(10^3) + \log_2(x) = 6$

$\log_2(1000x) = 6$ ① mark: correctly simplifying surds

$1000x = 2^6$

$x = \frac{2^6}{1000}$

$x = \frac{8}{125}$ ① mark: fully simplified

Question 3 (3 marks)

Solve $2^{x^2} = 8^{\frac{2}{3}x+1}$. **3**

$2^{x^2} = \left(2^3\right)^{\frac{2}{3}x+1}$

$2^{x^2} = 2^{2x+3}$ ① mark: correctly converting to same base

$\Rightarrow x^2 = 2x+3$

$x^2 - 2x - 3 = 0$ ① mark: recognising quadratic

$(x-3)(x+1) = 0$

$x = 3, -1$ ① mark: correct solutions

Examination continues on next page

Question 4 (3 marks)

Differentiate the following, simplifying where necessary.

a) $y = 2e^{4x-3}$ **1**

$\frac{dy}{dx} = 8e^{4x-3}$ ① mark

b) $y = (2 - e^{-x^2})^6$ **2**

$\frac{dy}{dx} = 6(2-e^{-x^2})^5 \times 2xe^{-x^2}$ ① mark

$= 12xe^{-x^2}(2-e^{-x^2})^5$ ① mark: fully simplified

Question 5 (2 marks)

If $\log_x(6) = a$ and $\log_x(3) = b$, find $\log_x(8)$ in terms of a and b. **2**

$\log_x(8) = \log_x(2^3)$

$= 3\log_x(2)$ ① mark: recognising 8 is a power of 2 and simplifying

$= 3[\log_x(6) - \log_x(3)]$

$= 3(a-b)$ ① mark: correct answer

Question 6 (5 marks)

Radioactive decay is a process which causes a substance to break down over time. The half-life of a substance refers to the amount of time it takes for the substance to decay to half its original mass.

Consider the formula for the radioactive decay of the substance carbon-14,

$$C = C_0 e^{-kt},$$

where C_0 and k are constants, t is time in years, and C is the amount of carbon-14 measured in grams.

a) Find the value of C_0 if there is initially 1000 grams of carbon-14. **1**

$1000 = C_0 e^{-k \times 0}$

$C_0 = 1000$

① mark: correct answer

b) The half-life of carbon-14 is known to be 5730 years. **3**

Use this to show that $k = \frac{\ln(2)}{5730}$.

$500 = 1000 e^{-5730k}$ ① mark: correct value of C and t

$e^{-5730k} = \frac{1}{2}$

$-5730k = \ln\left(\frac{1}{2}\right)$

$k = \frac{\ln\left(\frac{1}{2}\right)}{-5730}$ ① mark: correctly solving for k

$k = \frac{\ln(2^{-1})}{-5730}$

$k = \frac{\ln(2)}{5730}$, as req. ① mark: recognising $\frac{1}{2} = 2^{-1}$ and simplifying

c) Hence, find the amount of carbon-14, to the nearest gram, after 10,000 years. **1**

$C = 1000 \times e^{-\frac{\ln(2)}{5730} \times 10,000}$

$= 298\,g$ ① mark: answer, correctly rounding

Examination continues on next page

Question 7 (3 marks)

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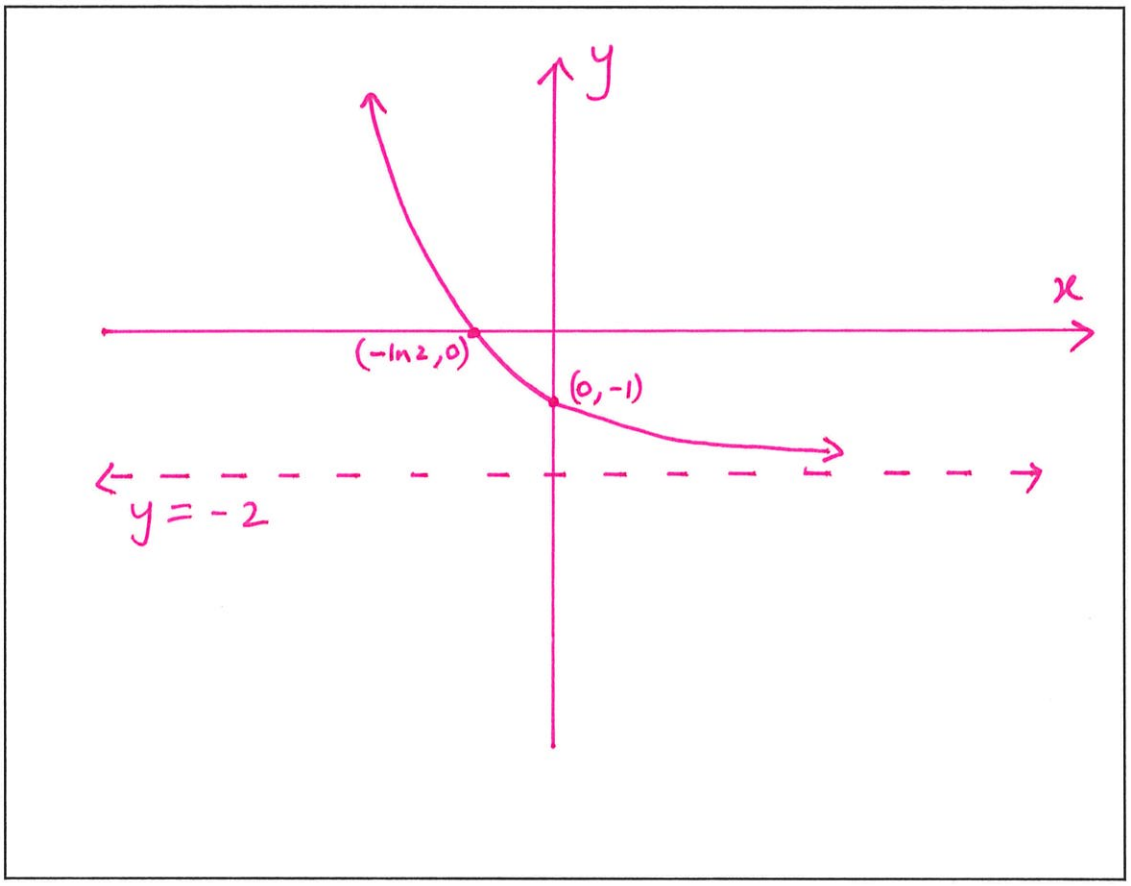
$$y = e^{-x} - 2$$

Note: The box suggests the size of the expected graph



Asymptote: as $x \to \infty$, $y \to -2 \Rightarrow y = -2$ is the asymptote

x-int: set $y = 0$; $e^{-x} - 2 = 0$

$e^{-x} = 2$

$-x = \ln(2)$

$x = -\ln(2)$

y-int: set $x = 0$: $y = e^0 - 2$

$= -1$

① mark: correct asymptote and correctly drawn with a dotted line and labelled

① mark: correct intercepts ~~exist~~ and correctly labell

① mark: correct ~~y-int and correctly labell~~ shape of curve

Question 8 (6 marks)

Sound intensity level (L) measured in decibels (dB) is a logarithmic measure given as:

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Violin	93	0.0019953
Trombone	100	0.0100000
Timpani and bass drum rolls	(ii)	0.0501187

Determine the missing values of (i) and (ii).

i) $87 = 10 \log_{10}\left(\frac{I}{I_0}\right)$

$\log_{10}\left(\frac{I}{I_0}\right) = \frac{87}{10}$

$\frac{I}{I_0} = 10^{8.7}$

$I = 10^{8.7} \times 10^{-12}$

$= 0.0005012$

correct working and ① mark: correct to 7 decimals

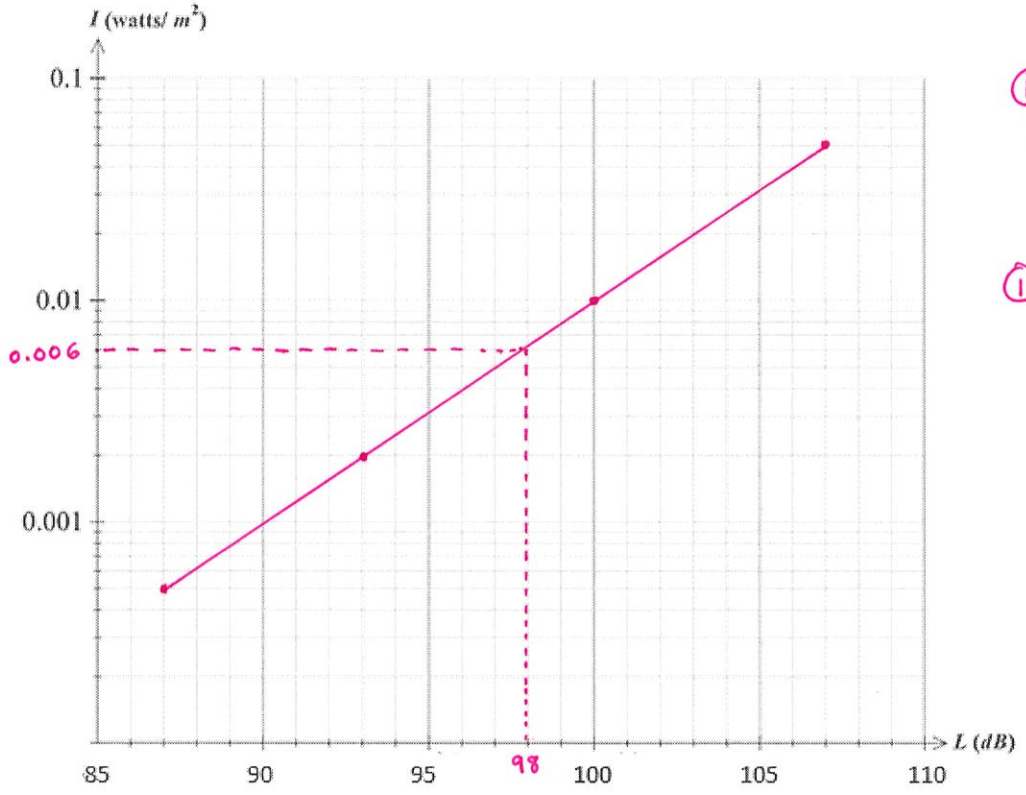
ii) $L = 10 \log_{10}\left(\frac{0.0501187}{10^{-12}}\right)$

$= 107$

① mark: correct to nearest dB

Examination continues on next page

(b) Plot the intensity of sound (I) against the average sound intensity levels (L) for the **four instruments listed in part (a)** on the semi-logarithm graph below. Connect the points with a line. **2**



① mark: correctly plotted points

① mark: lines drawn through the points

(c) Using the graph determine the average sound intensity level(dB) of the sound from a musical instrument that has an intensity of 0.006 watts/m^2. **1**

98 dB

① mark: correct answer with dotted lines shown on graph

(d) Explain why a semi-logarithm grid is useful for this data. **1**

It makes it easier to read the values from the graph compared to a linear scale on both axes, especially for very small or large values.

① mark: equivalent logical and sensible explanation

Question 9 (2 marks)

Given $\log_c(y) = \log_c(A) + tk\log_c(b)$, where A, b, c, k are (constant) positive real values with $b \neq 1$ and $c \neq 1$, and t and y are variables, show that $y = Ab^{kt}$. **2**

$\log_c(y) = \log_c(A) + \log_c(b^{kt})$

$\log_c(y) = \log_c(Ab^{kt})$ ① mark: correctly applies log laws

By taking exponential of both sides with base c $\Rightarrow y = Ab^{kt}$, as req. ① mark: correct answer with logical working

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Examination continues on next page