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Centre Number

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Student Number

**2021** YEAR 12

# Advanced Mathematics

## Task 2

Monday 1<sup>st</sup> March, 2021

| Q            | Marks |
|--------------|-------|
| MC           |       |
| 5            |       |
| 6            |       |
| 7            |       |
| 8            |       |
| 9            |       |
| 10           |       |
| 11           |       |
| 12           |       |
| <b>Total</b> |       |

### General Instructions:

- Reading time – 5 minutes
- Working time – 55 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations

**Total Marks:**  
**34**

### Section I - 4 marks

- Allow about 7 minutes for this section

### Section II - 30 marks

- Allow about 48 minutes for this section

***This question paper must not be removed from the examination room.***

*This assessment task constitutes 20% of the course.*

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## Section I

**4 marks**

**Allow about 7 minutes for this section**

Use the multiple-choice sheet for Question 1–4

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**1** Find the value of  $\log_4 93$ , correct to 3 significant figures.

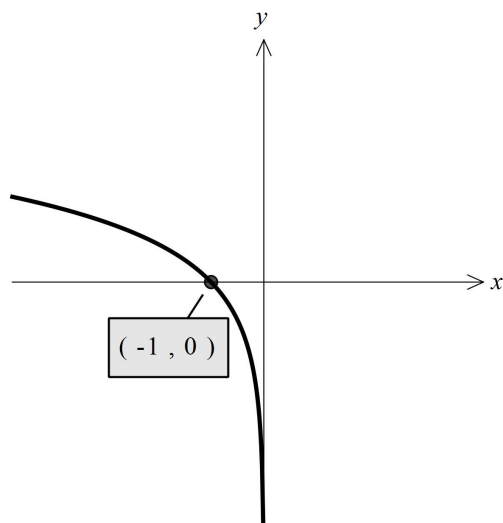
- (A) 1.97
- (B) 1.968
- (C) 3.27
- (D) 3.270

**2** Find  $\frac{d}{dx} \cos(x + 1)$

- (A)  $\cos(x + 1)$
- (B)  $\sin(x + 1)$
- (C)  $-\sin(x + 1)$
- (D)  $-\cos x \sin(x + 1)$

**Examination continues on next page**

- 3 Differentiate  $e^x \sin x$  with respect to  $x$ .
- (A)  $e^x \cos x$
- (B)  $-e^x \cos x$
- (C)  $e^x(\sin x + \cos x)$
- (D)  $e^x(\sin x - \cos x)$
- 4 Which equation corresponds to the graph below?



- (A)  $y = -\ln(x)$
- (B)  $y = \ln(-x)$
- (C)  $y = e^{-x} - 1$
- (D)  $y = -e^{-x}$

**End of Section I**

## Section II

30 marks

Allow about 48 minutes for this section

In Questions 5 – 12, your response should include relevant mathematical reasoning and/or calculations.

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### Question 5 (5 marks)

Differentiate the following, with respect to  $x$ , and fully simplify:

(a)  $f(x) = 6x - 7 + e^{-2x}$  1

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(b)  $y = 3^{-x}$  1

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(c)  $y = \log_e(\sec x)$  3

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Examination continues on next page

**Question 6** (4 marks)

Consider the stem and leaf plot below with 17 scores.

|   |          |
|---|----------|
| 0 | 58       |
| 1 | 04668999 |
| 2 | 00026    |
| 3 | 00       |

- (a) Give the 5-point summary.

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- (b) State the value of any outliers. Justify your answer with calculations.

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**Question 7** (4 marks)

Consider the function  $f(x) = 3 - e^{-x}$

- (a) Evaluate  $\lim_{x \rightarrow \infty} f(x)$

**1**

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- (b) Sketch the graph of  $f(x)$ , clearly stating the exact value of all intercepts.

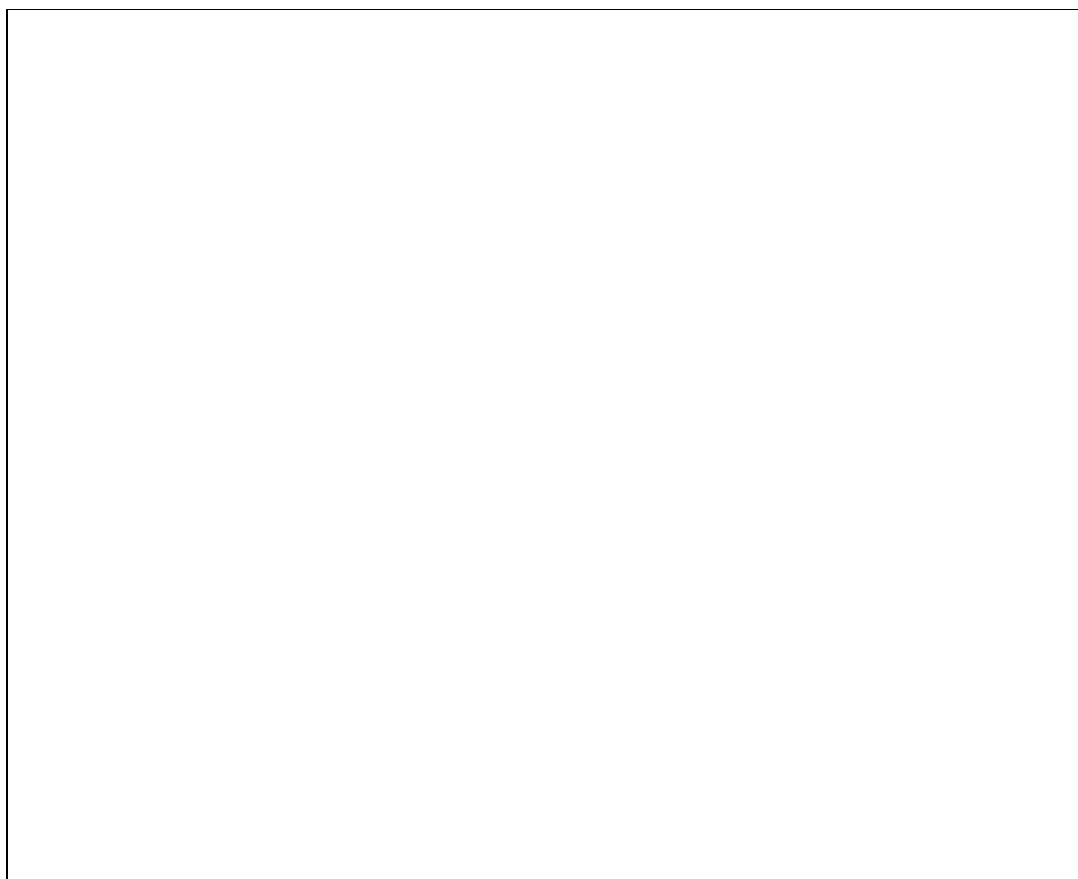
**3**

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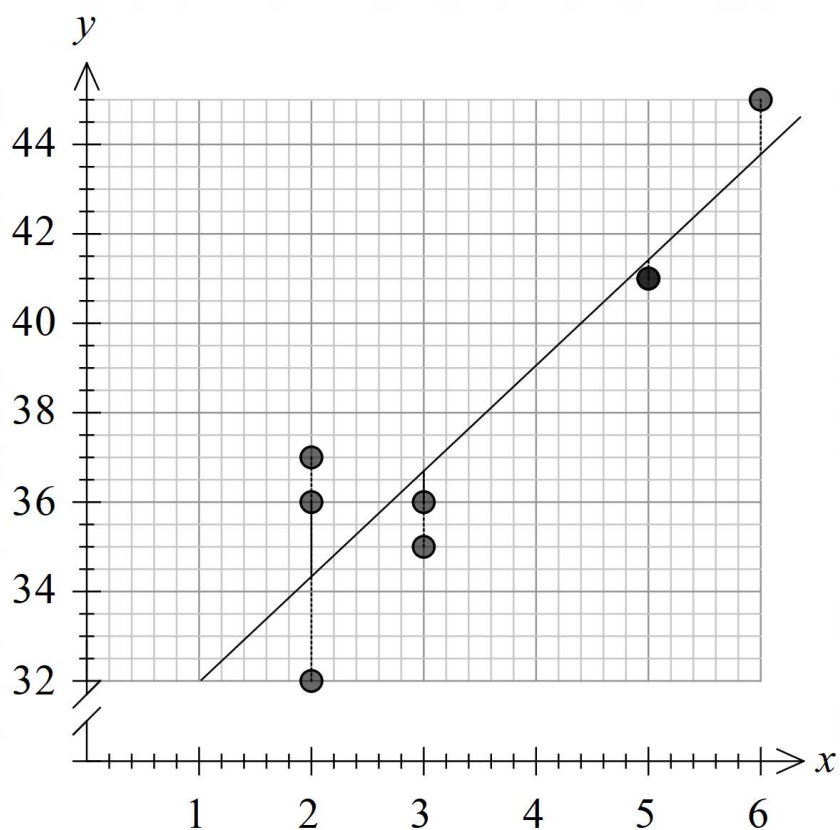
**Examination continues on next page**

**Question 8** (4 marks)

A set of data was collected comparing the age of cars to their stopping distance during a brake test.

The collected data is shown in the table and graphed in the scatterplot shown.

|                       |    |    |    |    |    |    |    |    |
|-----------------------|----|----|----|----|----|----|----|----|
| Age of car (years)    | 2  | 2  | 5  | 3  | 6  | 2  | 5  | 3  |
| Stopping Distance (m) | 36 | 37 | 41 | 35 | 45 | 32 | 42 | 36 |



- (a) Find the Pearson correlation coefficient,  $r$ , correct to 3 decimal places.

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- (b) Find the equation of the least-squares line of best fit, giving the coefficients to 1 decimal place. **1**

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- (c) Use your result from part (b) to predict the stopping distance of car that is 4 years old. Comment on the reliability of your prediction. **2**

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**Examination continues on next page**

**Question 9** (2 marks)

If  $\log_{10} x = 0.6$  and  $\log_{10} y = 0.2$ , evaluate  $\log_{10} \left( \frac{x}{\sqrt{y}} \right)$ .

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**Question 10** (2 marks)

Given that  $f(x) = \tan^3 x$ , calculate  $f' \left( \frac{\pi}{4} \right)$ .

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**Question 11** (4 marks)

Super Mine World 64 is a video game known for "speedrunning", where players aim to complete the game in as short a run time as possible. A list of the world record fastest times are listed in the table below to the nearest second.

4

|      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|
| 840s | 848s | 859s | 912s | 924s | 931s | 937s | 939s | 951s |
|------|------|------|------|------|------|------|------|------|

When a new score was added to this list, the sample mean and interquartile range both decreased.

Using interval notation, state the interval in which this new score must lie.

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**Question 12** (5 marks)

The energy,  $E$  Megajoules, radiated by an earthquake is given by the formula

$$E = k \times 10^{1.5R}$$

where  $R$  is the magnitude of the earthquake on the Richter scale and  $k$  is a constant. An earthquake of magnitude 3 radiates 2000 Megajoules of energy.

- (a) Show that  $\log_{10} E = \log_{10} 2 + 1.5(R - 1)$  **3**

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- (b) Find the magnitude of an earthquake which radiates 5 million Megajoules of energy correct to 2 decimal places. **2**

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**End of examination**

### Extra writing space

**If you use this space, clearly indicate which question you are answering.**

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## Section I

4 marks

Allow about 7 minutes for this section

Use the multiple-choice sheet for Question 1–4

---

- 1 Find the value of  $\log_4 93$ , correct to 3 significant figures.

(A) 1.97

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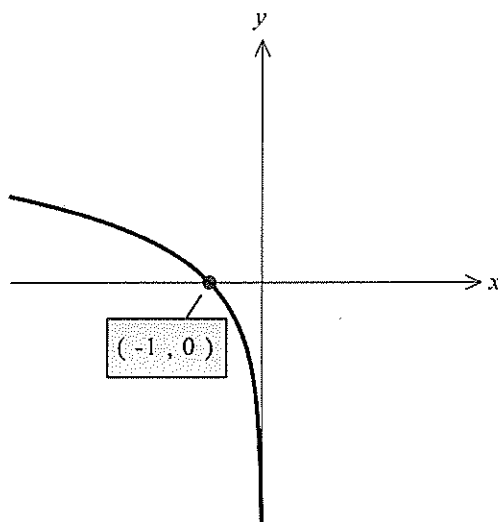
(A)  $e^x \cos x$

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4 Which equation corresponds to the graph below?



(A)  $y = -\ln(x)$

(B)  $y = \ln(-x)$

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(D)  $y = -e^{-x}$

End of Section I

## Section II

30 marks

Allow about 48 minutes for this section

In Questions 5 – 12, your response should include relevant mathematical reasoning and/or calculations.

### Question 5 (5 marks)

Differentiate the following, with respect to  $x$ , and fully simplify:

(a)  $f(x) = 6x - 7 + e^{-2x}$  1

$$f'(x) = 6 - 2e^{-2x}$$

(b)  $y = 3^{-x}$  1

$$y' = -\ln 3 \times 3^{-x}$$

(c)  $y = \log_e(\sec x)$  3

$$\text{Let } u = \sec x$$

$$= (\cos x)^{-1}$$

$$u' = -(-\sin x)(\cos x)^{-1}$$

$$= \frac{\sin x}{\cos^2 x}$$

1 mark for derivative  
of  $\sec x$

$$y = \log_e u$$

$$y' = \frac{u'}{u}$$

$$= \frac{\frac{\sin x}{\cos^2 x} \times \cos x}{\frac{1}{\cos x}}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

1 mark for  
simplifying

1 mark for correctly  
applying

$$\frac{d \ln f(x)}{dx} = \frac{f'(x)}{f(x)}$$

Examination continues on next page



**Question 6** (4 marks)

Consider the stem and leaf plot below with 17 scores.

|   |          |        |
|---|----------|--------|
| 0 | 58       | Median |
| 1 | 04668999 |        |
| 2 | 00026    |        |
| 3 | 00       |        |

- (a) Give the 5-point summary.

$$\frac{n+1}{2} = \frac{17+1}{2} = 9^{\text{th}} \text{ score}$$

2

$$x_{\text{m.h.}} = 5$$

$$Q_1 = 15$$

$$Q_2 = 19$$

$$Q_3 = 21$$

$$Q_4 = 30$$

-1 mark for any mistakes

- (b) State the value of any outliers. Justify your answer with calculations.

2

$$IQR = 21 - 15 = 6$$

$\therefore 5$  is an outlier as  $5 < 6$

$$1.5 \times IQR = 9$$

$$\text{Upper fence} = 21 + 9 = 30$$

$$\text{Lower fence} = 15 - 9 = 6$$

1 mark for IQR

1 mark for stating outlier is less than the lower fence = 6

Do NOT write in this area.

Question 7 (4 marks)

Consider the function  $f(x) = 3 - e^{-x}$

- (a) Evaluate  $\lim_{x \rightarrow \infty} f(x)$

1

$$= 3 - 0$$

$$= 3$$

- (b) Sketch the graph of  $f(x)$ , clearly stating the exact value of all intercepts.

3

When  $x = 0$

$$f(0) = 3 - e^0$$

$$= 3 - 1$$

$$= 2$$

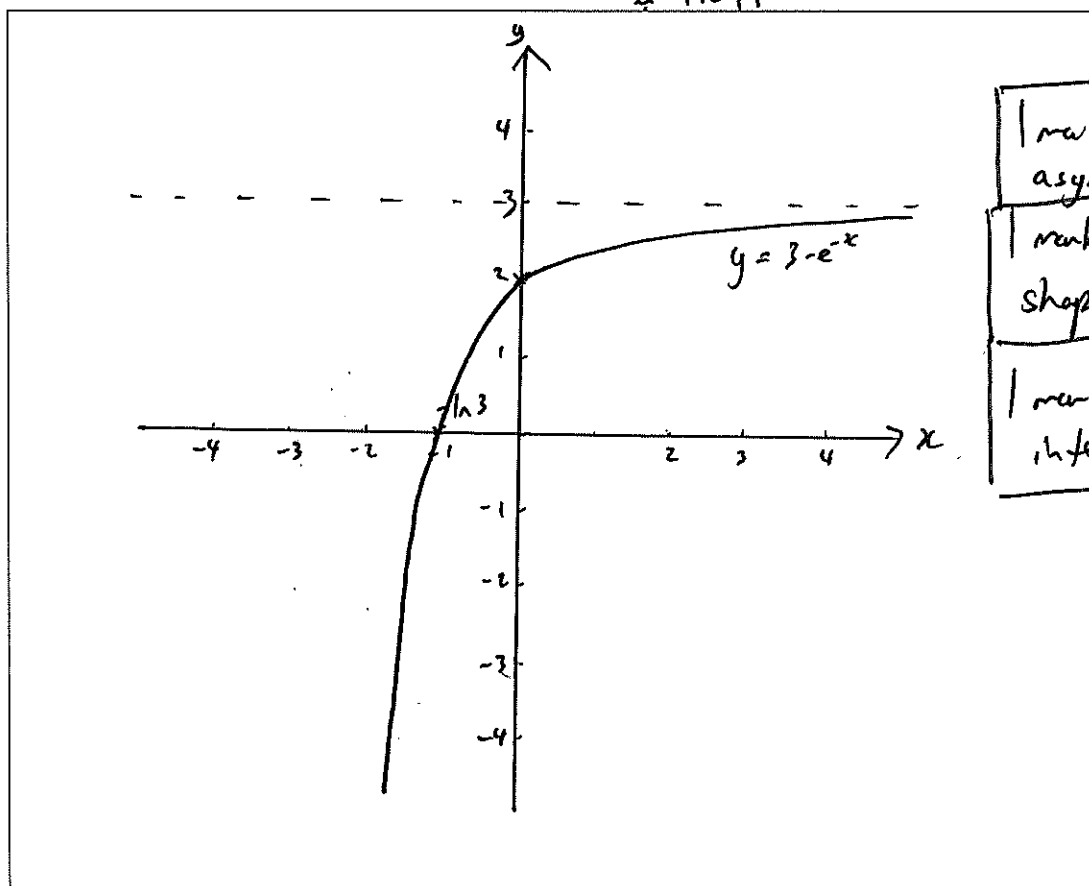
When  $f(x) = 0$

$$3 - e^{-x} = 0$$

$$3 = e^{-x}$$

$$\ln 3 = -x$$

$$x = -\ln 3 \text{ or } x = \ln \frac{1}{3}$$



1 mark for asymptote

1 mark for shape

1 mark for both correct intercepts

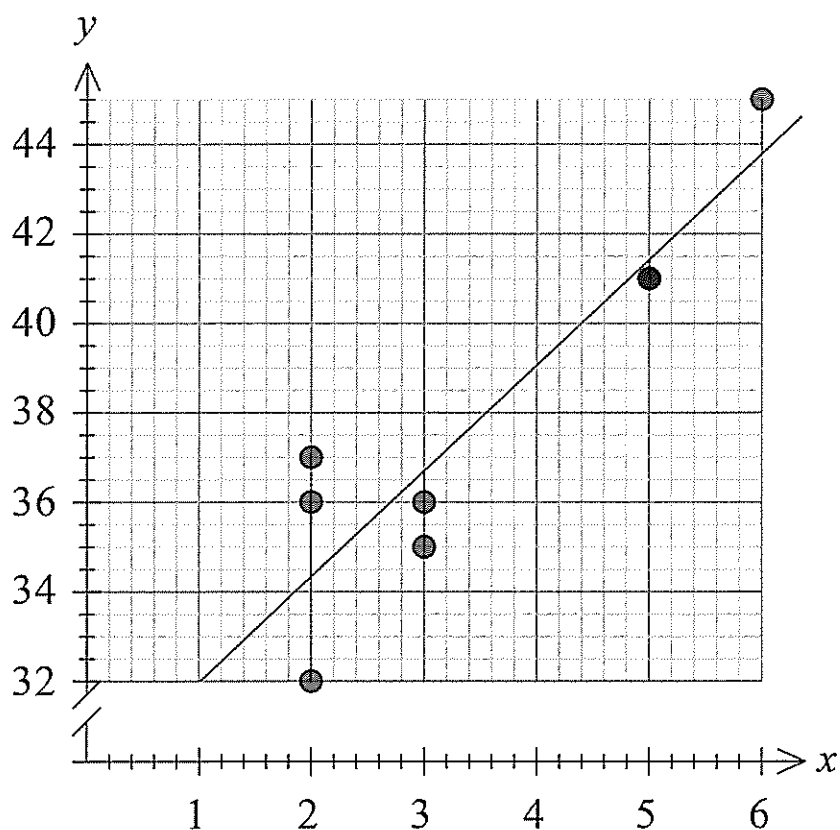
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The collected data is shown in the table and graphed in the scatterplot shown.

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|-----------------------|----|----|----|----|----|----|----|----|
| Age of car (years)    | 2  | 2  | 5  | 3  | 6  | 2  | 5  | 3  |
| Stopping Distance (m) | 36 | 37 | 41 | 35 | 45 | 32 | 42 | 36 |



- (a) Find the Pearson correlation coefficient,  $r$ , correct to 3 decimal places.

1

$r = 0.917$   
Using calculator

-1 mark for incorrect rounding

- (b) Find the equation of the least-squares line of best fit, giving the coefficients to 1 decimal place. 1

$$A = 29.4$$

$$y = 29.4 + 2.4x$$

$$B = 2.4$$

or

Using calculator

Stopping distance =  $29.4 + 2.4 \times \text{Age of car}$

- (c) Use your result from part (b) to predict the stopping distance of car that is 4 years old. Comment on the reliability of your prediction. 2

$$A + 4B = 39.2 \text{ (1 d.p.)}$$

Using calculator

1 mark (must not have used rounded values from (b))

Prediction is reliable as it is interpolating using the trend.

1 mark for reasoning "reliable" and "interpolation"

DO NOT write in this area.

Examination continues on next page

Question 9 (2 marks)

If  $\log_{10} x = 0.6$  and  $\log_{10} y = 0.2$ , evaluate  $\log_{10} \left( \frac{x}{\sqrt{y}} \right)$ .

2

$$\begin{aligned} \log_{10} \left( \frac{x}{\sqrt{y}} \right) &= \log_{10} x - \log_{10} \sqrt{y} \\ &= \log_{10} x - \log_{10} y^{\frac{1}{2}} \\ &= \log_{10} x - \frac{1}{2} \log_{10} y \\ &= 0.6 - \frac{1}{2} \times 0.2 \\ &= 0.5 \end{aligned}$$

1 mark for separating two logs

1 mark for final result

Question 10 (2 marks)

Given that  $f(x) = \tan^3 x$ , calculate  $f' \left( \frac{\pi}{4} \right)$ .

2

$$f'(x) = 3 \tan^2 x \times \sec^2 x$$

1 mark for derivative

$$\begin{aligned} f' \left( \frac{\pi}{4} \right) &= 3 \times 1^2 \times (\sqrt{2})^2 \\ &= 6 \end{aligned}$$

1 mark for final result

Do NOT write in this area.

Question 11 (4 marks)

Super Mine World 64 is a video game known for "speedrunning", where players aim to complete the game in as short a run time as possible. A list of the world record fastest times are listed in the table below to the nearest second.

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| 840s | 848s | 859s | 912s | 924s | 931s | 937s | 939s | 951s |
|------|------|------|------|------|------|------|------|------|

When a new score was added to this list, the sample mean and interquartile range both decreased.

Using interval notation, state the interval in which this new score must lie.

$$\bar{x} = 904.5$$

$$Q_1 = \frac{848 + 859}{2}$$

$$= 853.5$$

$$Q_3 = \frac{937 + 939}{2}$$

$$= 938$$

$$IQR = 84.5$$

Let  $y$  be the new score.

As  $y \leq 904$  and the list is now 10 scores,

the new  $Q_3$  becomes 937

1 mark for recognising new  $Q_3 = 937$

$$937 - 84.5 = 852.5$$

1 mark for  $Q_3 - IQR$

$$\therefore 852.5 \leq y \leq 904.5$$

However, the list is quoted to the nearest second.

1 mark for  $\bar{x}$  and IQR

$$\therefore 853 \leq y \leq 904$$

or

$$[853, 904]$$

1 mark for either interval notation and rounded integers

Examination continues on next page

Question 12 (5 marks)

The energy,  $E$  Megajoules, radiated by an earthquake is given by the formula

$$E = k \times 10^{1.5R}$$

where  $R$  is the magnitude of the earthquake on the Richter scale and  $k$  is a constant. An earthquake of magnitude 3 radiates 2000 Megajoules of energy.

- (a) Show that  $\log_{10} E = \log_{10} 2 + 1.5(R - 1)$

3

$$\begin{aligned}
 2000 &= k \times 10^{1.5 \times 3} \\
 k &= \frac{2000}{10^{4.5}} \quad \boxed{\text{1 mark for finding } k \text{ in any form}} \\
 &= 2 \times 10^3 \times 10^{-4.5} \\
 &= 2 \times 10^{-1.5} \\
 E &= 2 \times 10^{-1.5} \times 10^{1.5R} \\
 &= 2 \times 10^{1.5R - 1.5} \\
 &= 2 \times 10^{1.5(R-1)} \\
 \log_{10} E &= \log_{10} (2 \times 10^{1.5(R-1)}) \quad \boxed{\text{1 mark for correctly taking } \log_{10} \text{ of both sides}} \\
 &= \log_{10} 2 + \log_{10} 10^{1.5(R-1)} \\
 &= \log_{10} 2 + 1.5(R-1) \quad \boxed{\text{1 mark for final result with correct working}}
 \end{aligned}$$

- (b) Find the magnitude of an earthquake which radiates 5 million Megajoules of energy correct to 2 decimal places.

2

$$\begin{aligned}
 \log_{10} E &= \log_{10} 2 + 1.5(R-1) \\
 \log_{10} E - \log_{10} 2 &= 1.5(R-1) \\
 \log_{10} \left( \frac{E}{2} \right) &= 1.5(R-1) \\
 R-1 &= \frac{\log_{10} \left( \frac{E}{2} \right)}{1.5} \\
 R &= \frac{\log_{10} \left( \frac{E}{2} \right)}{1.5} + 1
 \end{aligned}$$

When  $E = 5000000$

$$\begin{aligned}
 R &= \frac{\log_{10} \left( \frac{5000000}{2} \right)}{1.5} + 1 \\
 &= 5.27 \text{ (2 d.p.)}
 \end{aligned}$$

$\boxed{\text{1 mark for substitution}}$   
 $\boxed{\text{1 mark for correct final result.}}$

End of examination