Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

2022 **YEAR 12**

Advanced Mathematics

Task 1 Validation Task

05/12/22

Q	Marks
1	/2
2	/2
3	/6
4	/2
5	/2
6	/5
7	/2
8	/6
9	/3
Total	/30

General Instructions:

- Reading time – 5 minutes
- Working time – 35 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 30

This question paper must not be removed from the examination room.

*This Validation task constitutes **60%** of the Assessment Task.*

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In Questions **1 – 9**, your response should include relevant mathematical reasoning and/or calculations.

Question 1 (2 marks)

If $\log_{10} 2 = 0.3010 \; and \; \log_{10} 3 = 0.4771$, find:

(a) $\log_{10} 6$ **1**

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.....

.....

(b) $\log_{10} \frac{2}{9}$ **1**

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Question 2 (2 marks)

Solve the following using change of base. Write your answer to three significant figures. **2**

$$5^x = 3$$

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Examination continues on next page

Question 3 (6 marks)

Solve for $\boldsymbol{x}$ if:

(a) $121^x = 11$ **1**

(b) $5^{x+3} = 25$ **1**

(c) $\log_2(x+1) = 3$ **2**

(d) $2\log_{10} x + 3 = \log_{10} x^5$ **2**

Question 4 (2 marks)

Solve the following for $\boldsymbol{x}$: **2**

$$81^{x^2} = 3^{-4x-1}.$$

Question 5 (2 marks)

Phillip has placed \$2 000 000 in an interest-bearing account. He invested at a rate of 5% p.a. Using the compound interest formula shown below, determine the number of years ($\boldsymbol{n}$) it will take for Phillip to at least double his money. **2**

$$\boldsymbol{A = P(1 + r)^n}$$

$$\boldsymbol{A}\ (\boldsymbol{Amount}) = Principal\ plus\ interest$$
$$\boldsymbol{P} = Principal$$
$$\boldsymbol{r} = Interest\ rate\ as\ a\ percentage$$
$$\boldsymbol{n} = time\ in\ years$$

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Question 6 (5 marks)

A logarithmic scale is used to measure the sound level with units in decibels.

$\boldsymbol{l} = 10 \log_{10} \frac{I}{I_o}$, where $\boldsymbol{l}$ is in decibels, I is the intensity of the sound in (watts/m^2) and I_o is the threshold of sound intensity detected by the human ear.

(a) What is the decibel rating of sound at the threshold of hearing (i.e. $I = I_o$). **1**

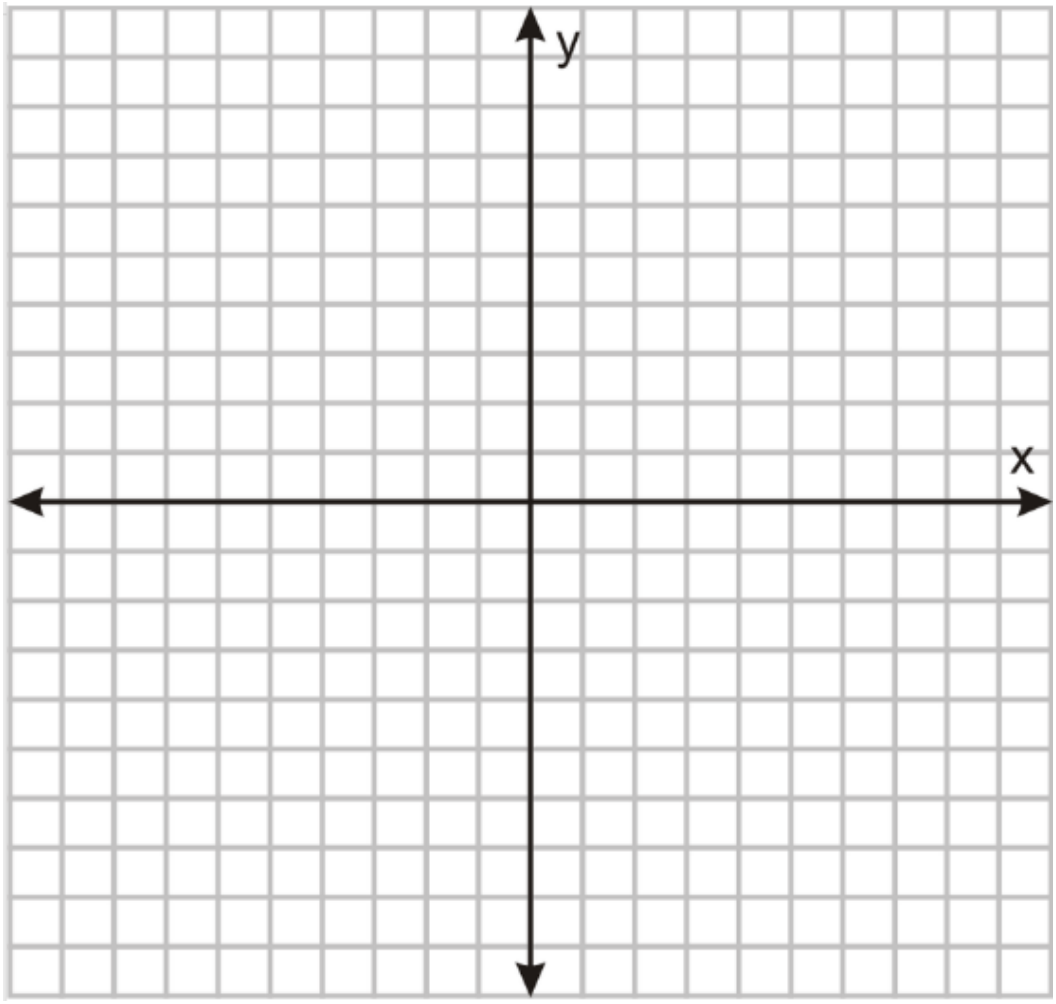
(b) Find the sound level of a conversation, which is 1 million times more intense than the threshold of human hearing. **1**

(c) How much greater is the noise intensity of an iPod playing at a peak volume of 115dB (I_1) compared with a half volume level of 85 dB (I_2)? **3**

Question 7 (2 marks)

Sketch the graph of the following exponential function in the box below. Label any intercepts and the asymptote. **2**

$$y = 3e^{-x} + 1$$



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Question 8 (6 marks)

Differentiate the following, simplifying where necessary.

(a) $y = (x^2 + x + 1)e^{2x}$ **2**

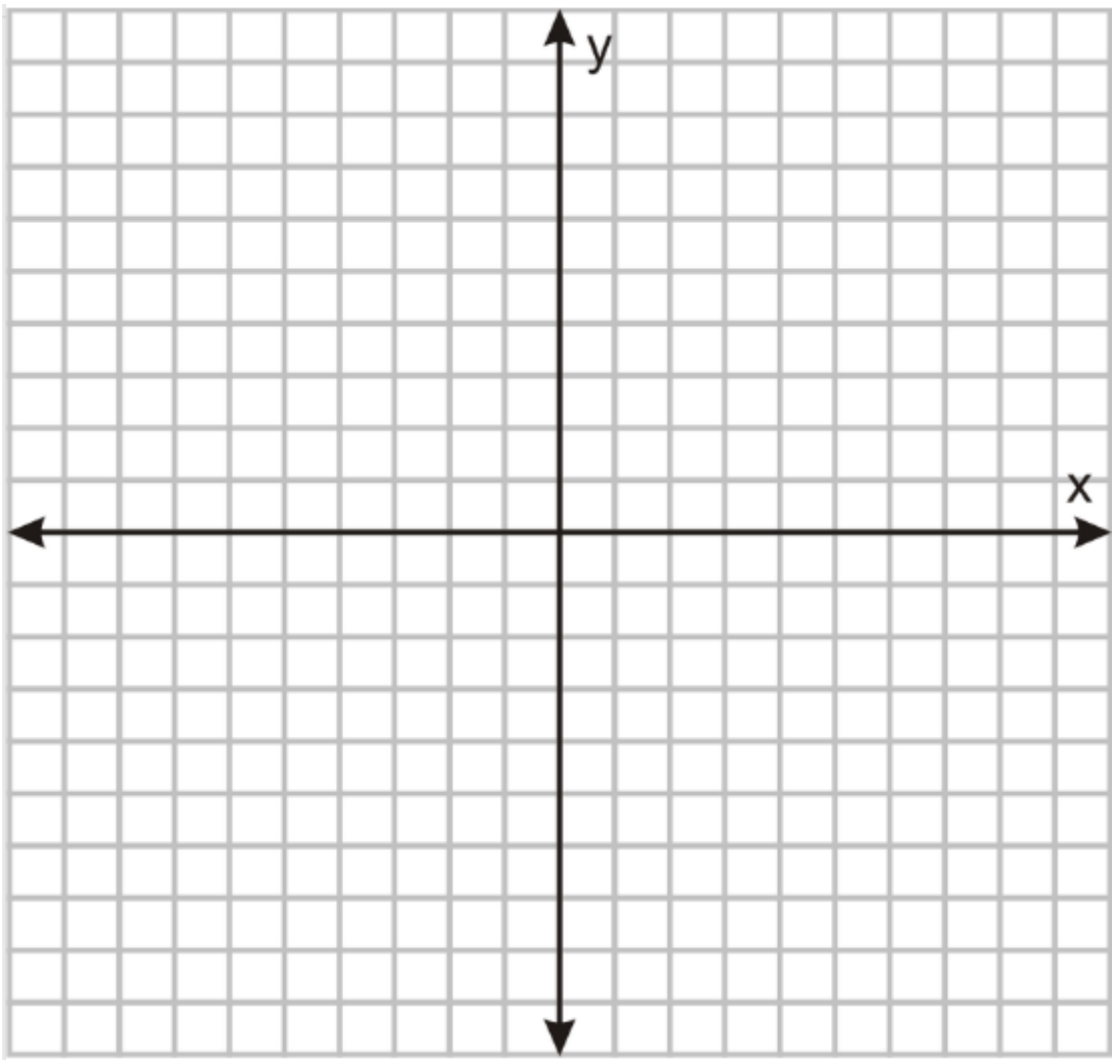
(b) $y = e^{x-\frac{1}{x}}$ **2**

(c) $y = \dfrac{3 + e^x}{e^{4x}}$ **2**

Question 9 (3 marks)

Sketch the following graph, clearly indicating the vertical asymptote and labelling all critical points. **3**

$$-y = \log_e(x - 2) + 1$$



END OF EXAMINATION

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Extra writing space

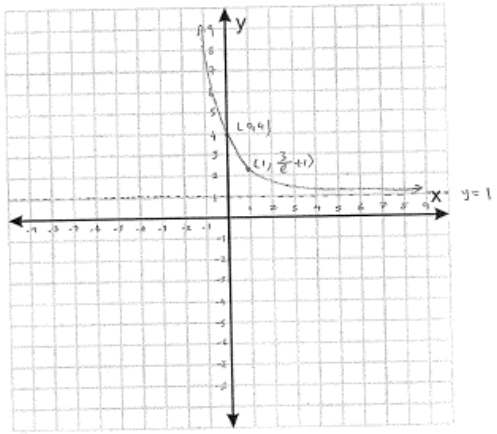
If you use this space, clearly indicate which question you are answering.

1.	Solutions and Marking Schemes	Markers Feedback
1(a)	**Question 1** (2 marks) If $\log_{10} 2 = 0.3010$ *and* $\log_{10} 3 = 0.4771$, find: (a) $\log_{10} 6$ 1 $= \log_{10} 2 + \log_{10} 3$ $= 0.3010 + 0.4771$ $= 0.7781$ ① – mark for correct answer with working	
1(b)	(b) $\log_{10} \frac{2}{9}$ 1 $= \log_{10} 2 - 2\log_{10} 3$ $= 0.3010 - 2 \times 0.4771$ $= -0.6532$ ① – mark for correct answer with working	Some confusion with power rule.
2	**Question 2 (2 marks)** Solve the following using change of base. Write your answer to three significant figures. 2 $5^x = 3$ $x \ln 5 = \ln 3$ → This is not using change of base $x = \frac{\ln 3}{\ln 5}$ → They must show this line for one mark. $x = 0.683$ – ① mark for correct answer	Must show change of base in your working, as stated in the question.

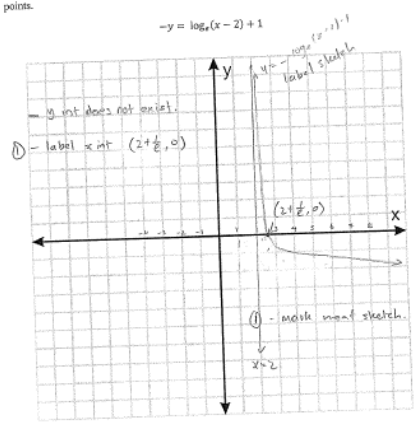
3(a)	Solve for x if: (a) $121^x = 11$ **1** $11^{2x} = 11^1$ or $x = \frac{\log_e 11}{\log_e 121}$ $2x = 1$ $x = \frac{1}{2}$ $x = 0.5$ ① - mark for answer.	Some put $x = 2$. Double check your work.
3(b)	(b) $5^{x+3} = 25$ **1** $5^{x+3} = 5^2$ $x + 3 = 2$ $x = -1$ ① - mark for correct answer.	
3(c)	(c) $\log_2(x+1) = 3$ **2** $x + 1 = 2^3$ — ① mark for w/u. $x + 1 = 8$ $x = 7$ — ① mark for correct answer (ECF applies.)	
3(d)	(d) $2\log_{10} x + 3 = \log_{10} x^5$ **2** $2\log_{10} x + 3 = 5\log_{10} x$ $3 = 3\log_{10} x$ $\log_{10} x = 1$ — ① mark for w/u $x = 10^1$ $x = 10$ — ① mark for correct answer	Some added brackets which changed the question to $2(x+3) = x^5$

4	Solve the following for x: 2 $81^{x^2} = 3^{-4x-1}$. $3^{4x^2} = 3^{-4x-1}$ $4x^2 = -4x - 1$ $4x^2 + 4x + 1 = 0$ → (1) - mark. $(2x+1)^2 = 0$ $\therefore x = -\frac{1}{2}$ (1) - mark for correct answer.	Some were stuck on factorising the non-monic quadratic.
5	Phillip has placed \$2 000 000 in an interest-bearing account. He invested at a rate of 5% p.a. Using the compound interest formula shown below, determine the number of years (n) it will take for Phillip to at least double his money. 2 $A = P(1+r)^n$ A (**Amount**) = *Principal plus interest* P = *Principal* r = *Interest rate as a percentage* n = *time in years* $4000000 = 2000000(1+5\%)^n$ $2 = 1.05^n$ $\log 2 = n \log 1.05$ $n = \frac{\log 2}{\log 1.05}$ $n = 14.20669905...$ - (1) mark for n/o. $n = 15$ years - (1) mark for Rounding to the next year.	Poorly done. - Many assumed that $5\% = 5$ - Many did not know how to set up the initial equation - Many did not know to round up to an integer, as interest is only compounded at the **end of a year**.

6(a)	A logarithmic scale is used to measure the sound level with units in decibels. $l = 10 \log_{10} \frac{I}{I_o}$, where l is in decibels, I is the intensity of the sound in (watts/m^2) and I_o is the threshold of sound intensity detected by the human ear. (a) What is the decibel rating of sound at the threshold of hearing (i.e. $I = I_o$). **1** at $I = I_o$ | $10 \log_{10} \frac{I_o}{I_o}$ $= 10 \log_{10} 1$ $= 10 \times 0$ $= 0$ — (1) mark w/o . . .	
6(b)	(b) Find the sound level of a conversation, which is 1 million times more intense than the threshold of human hearing. **1** $I = 10^6 \cdot I_o$ $10 \log_{10} \left(\frac{10^6 \cdot I_o}{I_o} \right) = 2$ $= 60$ dB (1) - mark for substituting $I = I_o$ and an answer of 60dB	
6(c)	(c) How much greater is the noise intensity of an iPod playing at a peak volume of 115dB (I_1) compared with a half volume level of 85 dB (I_2)? **3** $L_1 = 115 = 10 \log_{10} \left(\frac{I_1}{I_o} \right)$ $L_2 = 42.5 = 10 \log_{10} \left(\frac{I_2}{I_o} \right)$ $11.5 = \log_{10} \left(\frac{I_1}{I_o} \right)$ $4.25 = \log_{10} \left(\frac{I_2}{I_o} \right)$ $10^{11.5} = \frac{I_1}{I_o}$ $10^{4.25} = \frac{I_2}{I_o}$ $I_o = \frac{I_1}{10^{11.5}}$ — (1) mark $I_o = \frac{I_2}{10^{4.25}}$ — (1) mark $\frac{I_2}{I_1} = \frac{10^{11.5}}{10^{4.25}} = 10^{6.25}$ — (1) mark answer. HOw much greater is the noise intensity of an iPod playing at a peak volume of 115dB (I_1) compared with a half volume level of 85 dB (I_2)? L	

7	Sketch the graph of the following exponential function in the box below. Label any intercepts and the asymptote. **2** $y = 3e^{-x} + 1$  There are no x intercepts. ① - mark for labelling the y intercept ① - mark for a neat sketch.	
8(a)	Differentiate the following, simplifying where necessary. **2** (a) $y = (x^2 + x + 1)e^{2x}$ u v $\frac{dy}{dx} = vu' + uv'$ $= e^{2x} \times (2x+1) + (x^2+x+1) \times 2e^{2x}$ $= (2x^2 + 2x + 2)e^{2x} + (2x+1)e^{2x}$ $= e^{2x}(2x^2 + 4x + 3)$ — ① mark for simplifying correctly. u/v. $u = x^2 + x + 1$ $u' = 2x + 1$ ① - mark for u/v. $v = e^{2x}$ $v' = 2e^{2x}$	

8(b)	(b) $y = e^{x-\frac{1}{x}}$ Let $u = x - \frac{1}{x}$ 2 $\frac{dy}{dx} = e^{x-\frac{1}{x}} \cdot \left(1 + \frac{1}{x^2}\right)$ $\frac{du}{dx} = 1 + \frac{1}{x^2}$ $= \left(1 + \frac{1}{x^2}\right) e^{x-\frac{1}{x}}$ $y = e^u$ ① answer ① answer $\frac{dy}{du} = e^u = e^{x-\frac{1}{x}}$	
8(c)	(c) $y = \frac{3+e^x}{e^{4x}}$ $\frac{u}{v}$ 2 $u = 3 + e^x$ $u' = e^x$ $\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$ $v = e^{4x}$ $v' = 4e^{4x}$ $= \frac{e^{4x} \cdot e^x - (3+e^x) \cdot (4e^{4x})}{(e^{4x})^2}$ $= \frac{e^{5x} - 12e^{4x} - 4e^{5x}}{e^{5x}}$ $= \frac{-12e^{4x} + 3e^{5x}}{e^{5x}}$ — ① correct numerator — ① correct denominator or $y = 3e^{-4x} + e^{-3x}$ $y' = -12e^{-4x} - 3e^{-3x}$ $= -\frac{12}{e^{4x}} - \frac{3}{e^{3x}}$	

9	Sketch the following graph, clearly indicating the vertical asymptote and labelling all critical points. 3 $y = \log_e(x-2) + 1$  - y int does not exist. ① - label x int $(2\frac{1}{e}, 0)$ ① - asymptote and sketch. $y = -\log_e(x-2) - 1$	