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Q	Marks
MC	/4
5	/7
6	/2
7	/6
8	/5
9	/5
10	/6
11	/7
12	/3
Total	/45

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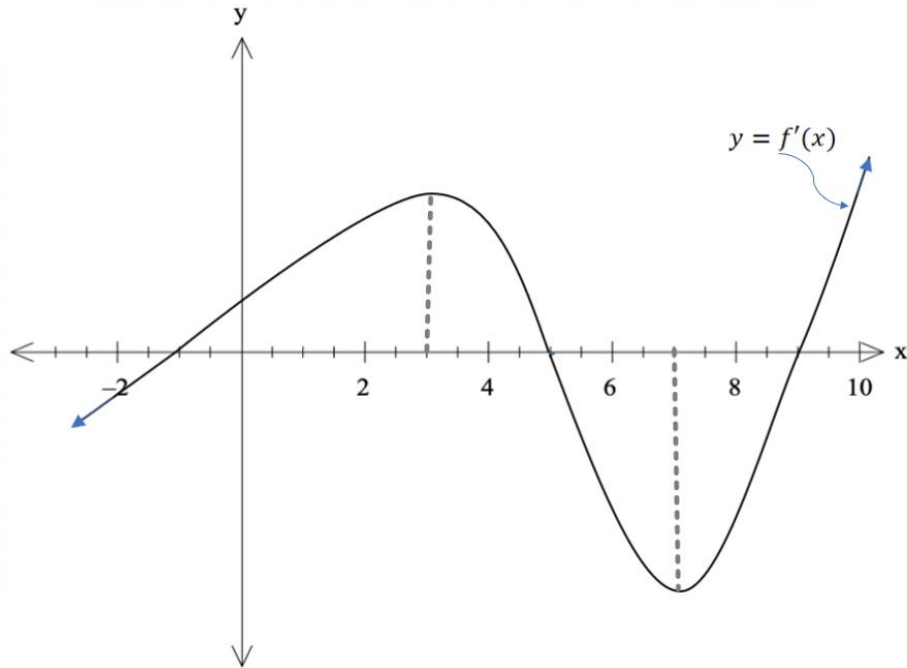
Section I

4 marks

Allow about 5 minutes for this section

Use the multiple-choice sheet for Question 1–4

- 1 The graph of the derivative $y = f'(x)$ is drawn below.



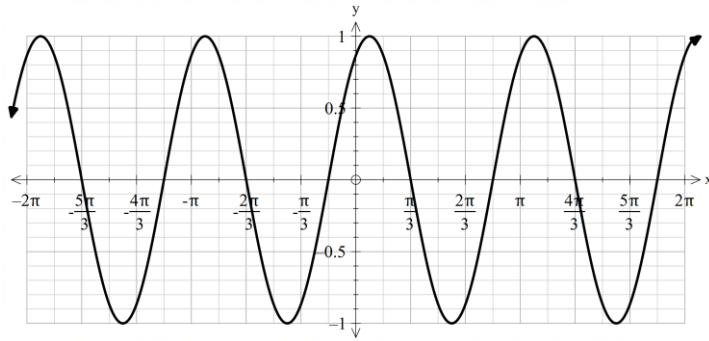
The maximum turning point for $f(x)$ occurs at

- (A) $x = -1$
- (B) $x = 3$
- (C) $x = 5$
- (D) $x = 7$

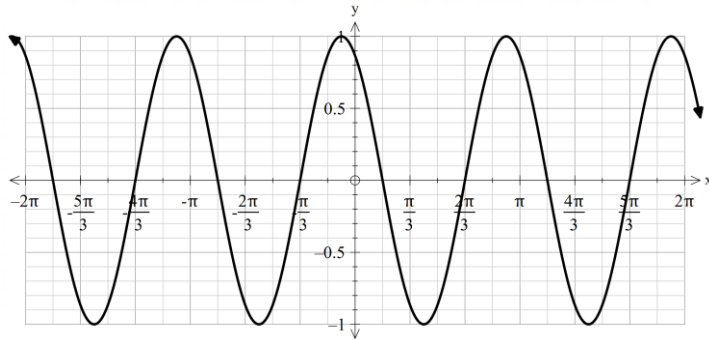
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The graph of the function $y = \sin\left(2x + \frac{\pi}{3}\right)$ is best represented by

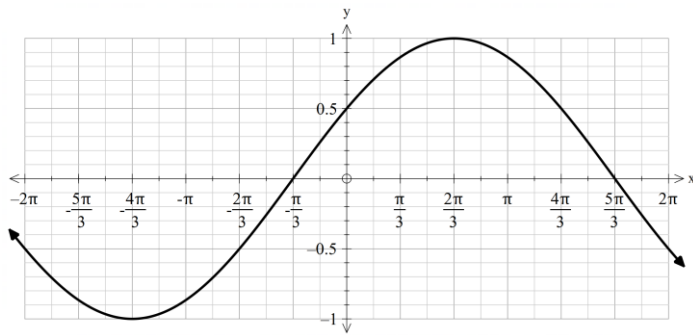
(A)



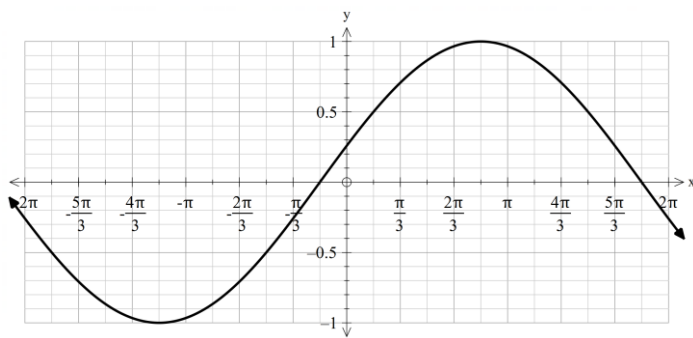
(B)



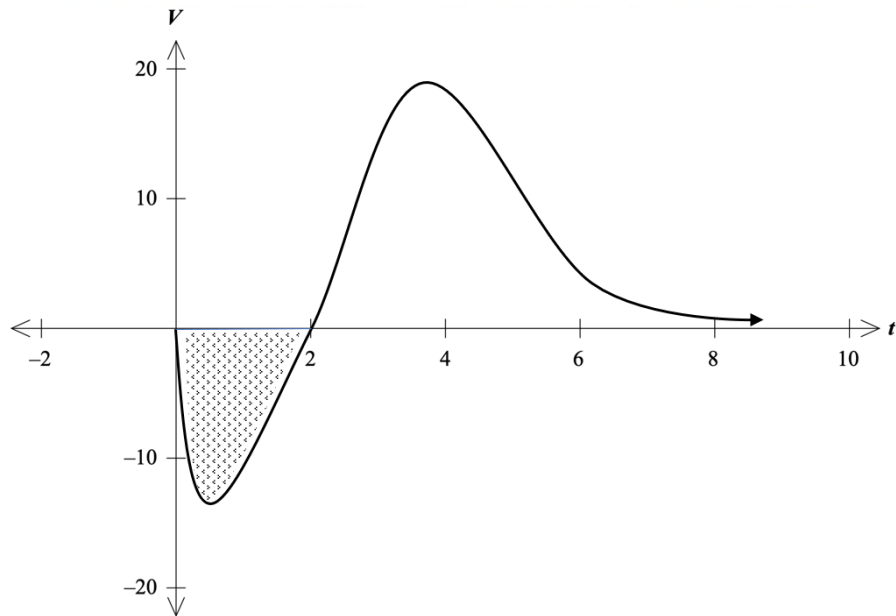
(C)



(D)



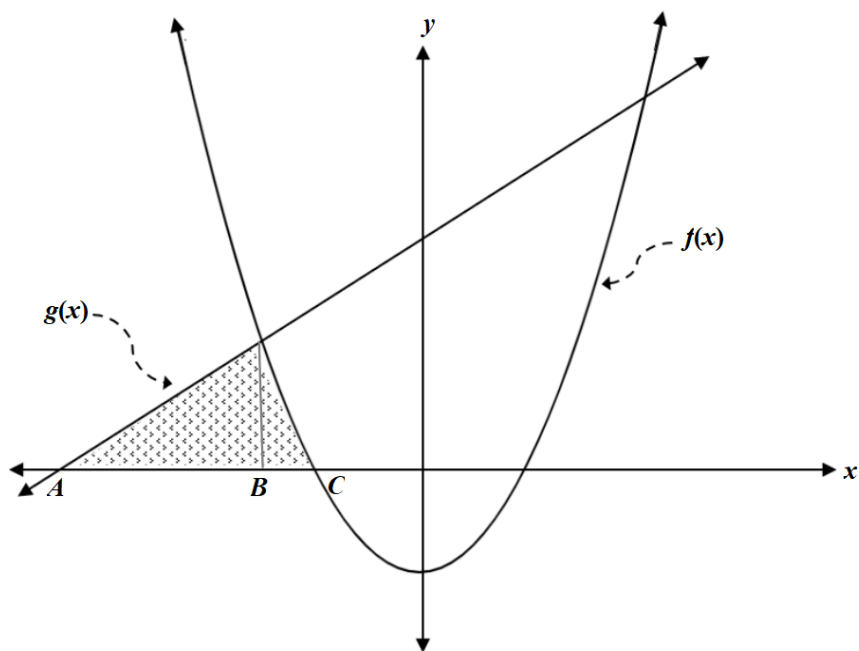
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The graph above represents the velocity (V m/s) of a particle after t seconds. The particle is moving in a straight line from rest.

Which of the following best describes $\int_0^2 V dt$?

- (A) The acceleration of the particle after 2 seconds.
- (B) The displacement of the particle after 2 seconds
- (C) The distance travelled by the particle in the first two seconds
- (D) The velocity of the particle in the first two seconds



The shaded region is best described by

(A) $\int_A^C g(x) + f(x) dx$

(B) $\int_A^C g(x) - f(x) dx$

(C) $\int_A^B g(x) dx + \int_B^C f(x) dx$

(D) $\int_A^B g(x) dx - \int_B^C f(x) dx$

End of Section I

Examination continues on next page

Section II

In Questions 5– 13, your response should include relevant mathematical reasoning and/or calculations.

Question 5 (7 marks)

(a) Find

2

$$\int (1 + \sec^2 x) dx$$

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(b) Find

2

$$\int (5x - 1)^9 dx$$

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(c) Find

3

$$\int_0^{\ln 2} \frac{e^{2x}}{4 + e^{2x}} dx$$

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Question 6 (2 marks)

Using Trapezoidal Rule with three function values, find an approximation for

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$$\int_0^2 e^{x^2} dx.$$

(Give your answer to four significant figures)

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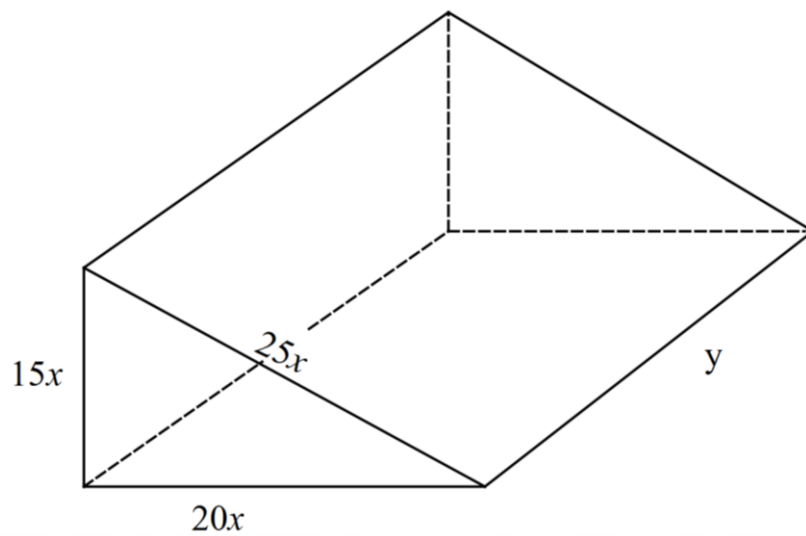
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Question 7 (5 marks)



The figure above shows a triangular prism with a total surface area of 3600 cm^2 . The triangular faces of the prism are right angled with a base of $20x \text{ cm}$ and a height of $15x \text{ cm}$. The length of the prism is $y \text{ cm}$

- (a) Show that the volume is given by $V = 9000x - 750x^3$

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- (b) Find the value of x that gives the largest possible volume

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Question 8 (6 marks)

Consider the curve $y = (x + 1)(x^2 + 1)$

- (a) Show that the curve has no stationary points

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- (b) Find all point/s of inflection.

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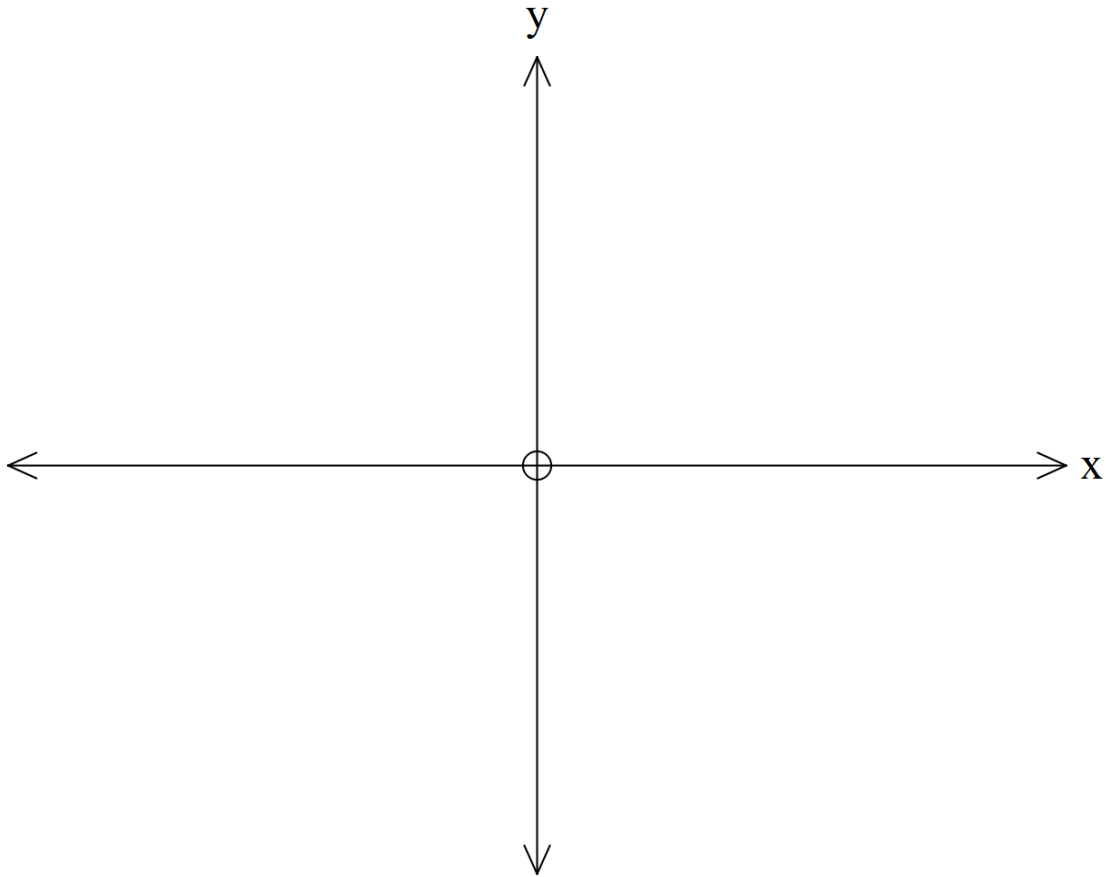
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(c) Hence, sketch the curve

3

$$y = (x + 1)(x^2 + 1)$$



Question 9 (5 marks)

The management of a tyre factory is taking measures to make its operations more environmentally sustainable. A filter was installed at its main chamber exhaust so that the rate of emissions, $\frac{dE}{dt}$, in kilogram per month of pollution can be limited to

$$\frac{dE}{dt} = 100 + \frac{470}{t+1},$$

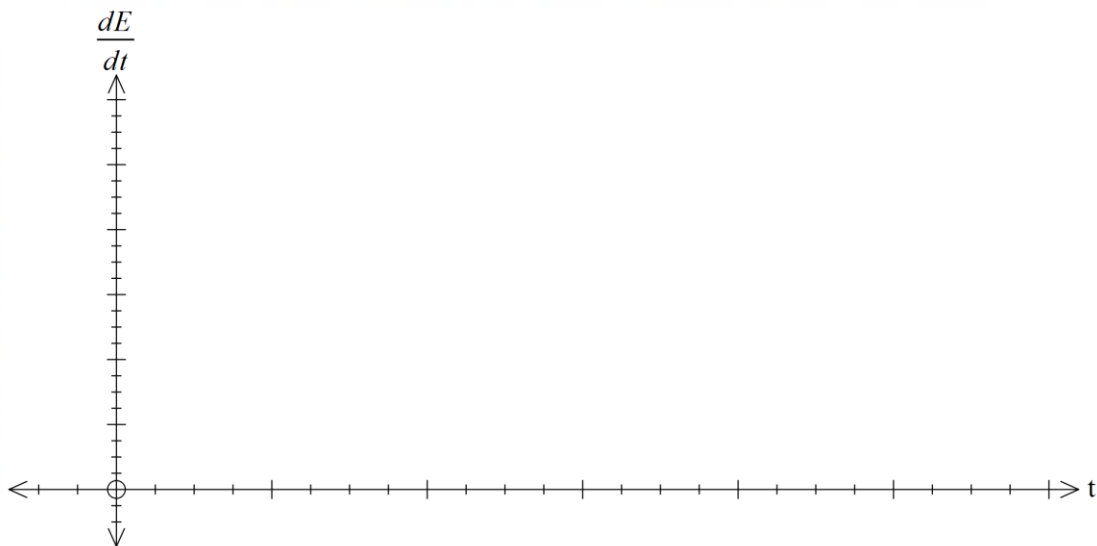
where t is the time in months since the filter was installed.

- (a) Find the initial rate of emission.

1

- (b) Sketch a graph of the rate of emission $\frac{dE}{dt}$ versus time t in months.

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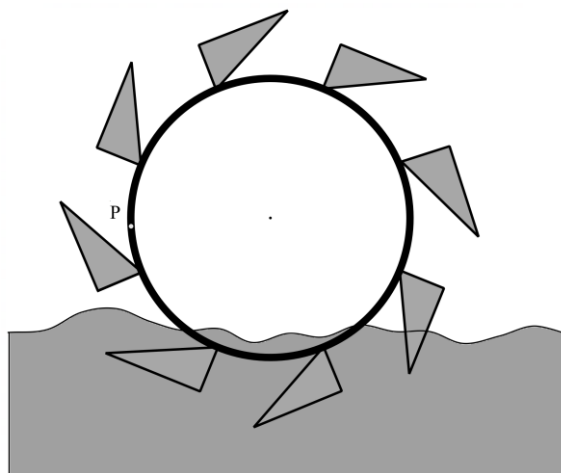


- (c) Calculate the total amount of pollution emitted by the factory in the first year of operation after the filter was installed.

Question 10 (6 marks)

A water wheel 4m in diameter completes a full rotation every 36 seconds. Initially a point, P , is at the lowest point on the water wheel which is 2m above the riverbed. The height of the point, P , on the water wheel can be modelled by the equation

$$P = a + b \cos(nt + \phi)$$



- (a) Show that the position of P can be modelled by

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$$P = 4 - 2 \cos\left(\frac{\pi}{18}t\right)$$

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- (b) Given that the river is 3m deep, what percentage of the time is the point P visible above the water? (You may wish to use the axis below)

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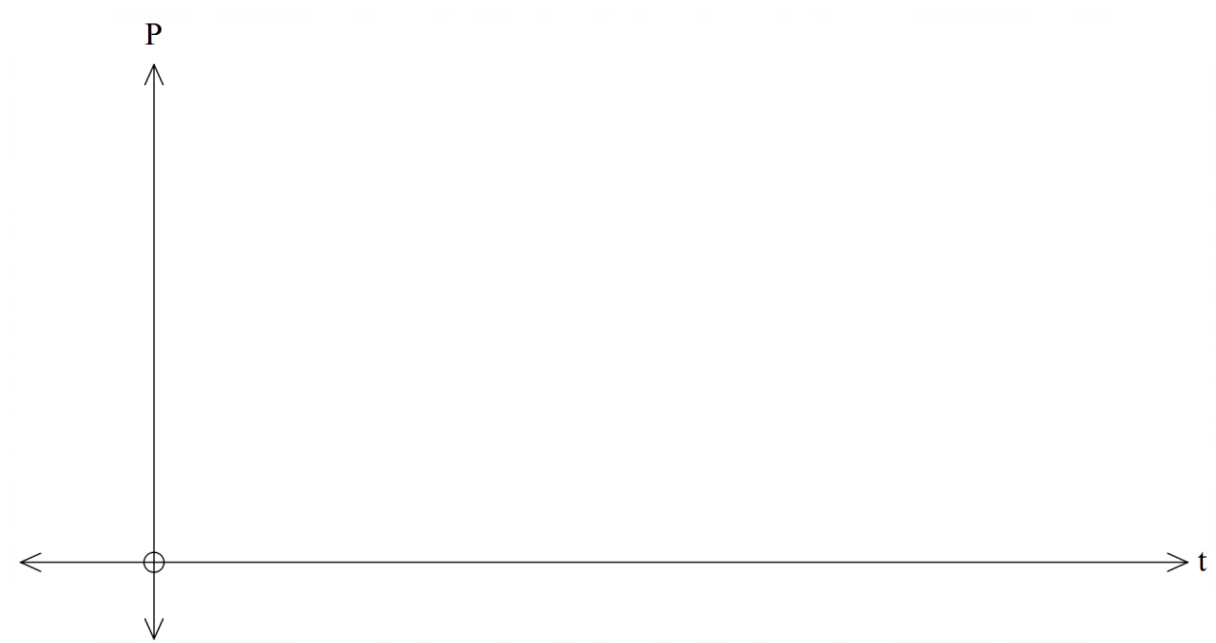
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Question 11 (7 marks)

The velocity V , in m/s, of a particle moving in a straight line is given by $V=18 - 2e^t$, where t is the time in seconds. The particle is initially at $x = 4$.

- (a) Show that the time at which the particle comes to rest is $2 \log_e 3$ seconds.

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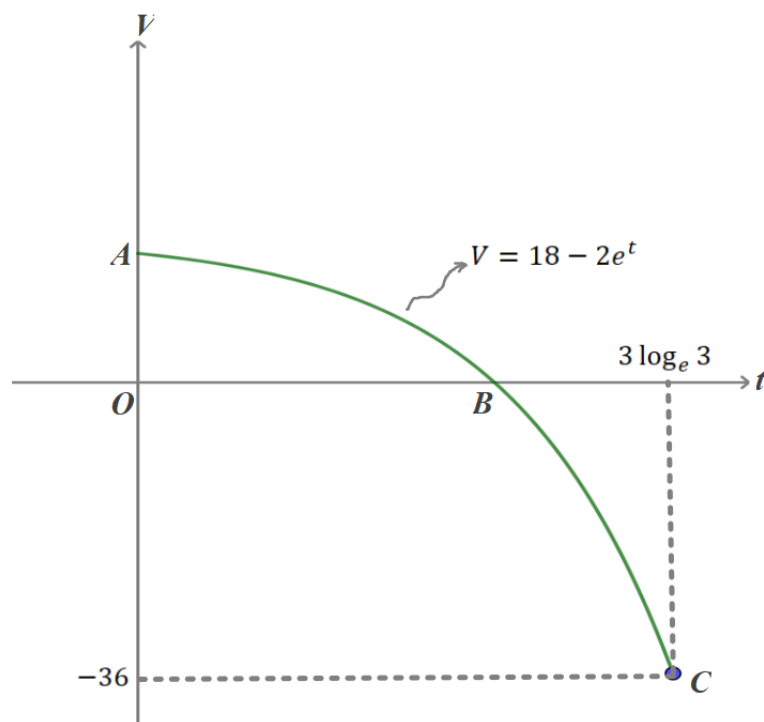
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- (b) The sketch below shows the graph of $V= 18 - 2e^t$.
On the same set of axes, sketch the graph of the displacement – time graph.

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- (c) Hence, or otherwise, find the distance travelled by the particle between $t = 0$ and $t = 3\log_e 3$ seconds.

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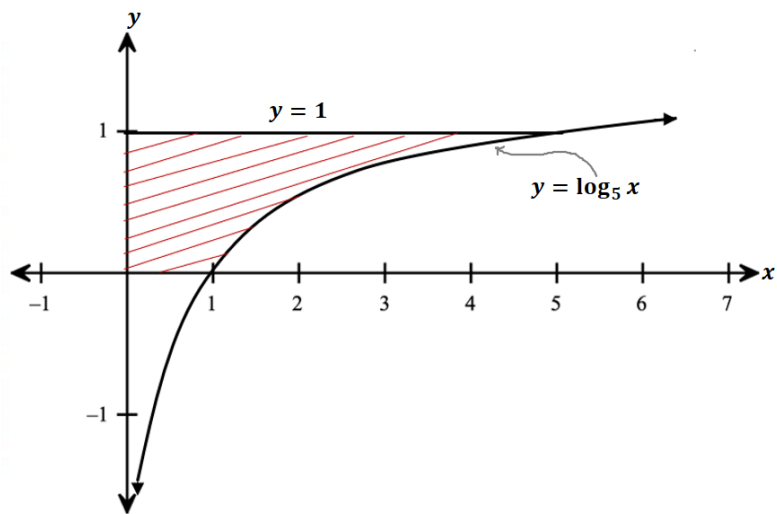
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Question 12 (3 marks)



The diagram shows the graph of $y = \log_5 x$. Find the area of the region bounded by the curve of $y = \log_5 x$, the line $y = 1$, and the x and y axes. 3

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End of Examination

Extra writing space

If you use this space, clearly indicate which question you are answering.

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Centre Number								
Student Number								

Mathematics Advanced Task 3

Section 1 - Multiple Choice Answer Sheet

Use this multiple-choice sheet for questions 1 – 10. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
 A ☐ B ☒ C ☐ D ☐

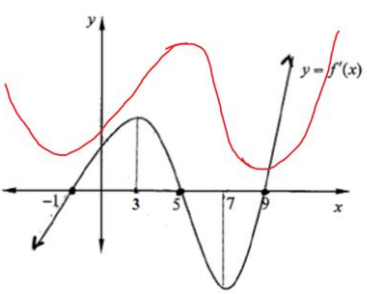
If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A ☒ B ☒ C ☐ D ☐
 correct

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|----|-------------------------|-------------------------|-------------------------|-------------------------|
| 1. | A ☐ | B ☐ | C ☐ | D ☐ |
| 2. | A ☐ | B ☐ | C ☐ | D ☐ |
| 3. | A ☐ | B ☐ | C ☐ | D ☐ |
| 4. | A ☐ | B ☐ | C ☐ | D ☐ |
| 5. | A ☐ | B ☐ | C ☐ | D ☐ |
| 6. | A ☐ | B ☐ | C ☐ | D ☐ |

2022 Advanced Mathematics Task 2 Marking Scheme

1	C		
2	B		
3	B		
4	C		

Question 5

5(a)	$\int (1 + \sec^2 x) dx$ $x + \tan x + C$	1 mark: integrates both terms 1 mark: constant of integration added	
5(b)	$\int (5x - 1)^9 dx$ $\frac{(5x - 1)^{10}}{10 \times 5} + C$ $\frac{(5x - 1)^{10}}{50} + C$	1 mark: gives $\frac{(5x - 1)^{10}}{10} + C$ (ignore C) 1 mark: divides by $f'(x) = 5$ Gives $\frac{(5x - 1)^{10}}{5 \times 10} + C$	
5(c)	$\int_0^{\ln 2} \frac{e^{2x}}{4 + e^{2x}} dx$ $= \frac{1}{2} \int_0^{\ln 2} \frac{2e^{2x}}{4 + e^{2x}} dx$ $= \frac{1}{2} [\ln 4 + e^{2x}]_0^{\ln 2}$ $= \frac{1}{2} \ln \left \frac{4 + e^{2 \ln 2}}{4 + 1} \right $ $= \frac{1}{2} \ln \left(\frac{8}{5} \right)$	2 marks: correctly integrates $\frac{1}{2} \ln 4 + e^{2x} $ Only 1 mark: minor error in integration. (may be $\frac{1}{2}$ or absolute value is missing) 1 mark: correctly evaluates, must give the answer in simplest form	

Question 6

6	<table border="1"><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr></table>	x	0	1	2	2 mark: correct answer from correct working	
x	0	1	2				

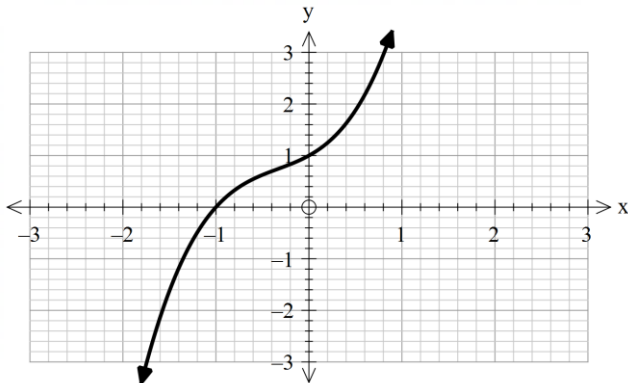
	<table><tr><td>$y = e^{x^2}$</td><td>1</td><td>e</td><td>e^4</td></tr></table> $\int_0^2 e^{x^2} dx \approx \frac{1}{2}[1 + 2 \times e + e^4]$ $= 30.517 \dots$ $= 30.52 \quad (4 \text{ sig. fig.})$	$y = e^{x^2}$	1	e	e^4	1 mark: Evidence of applying Trapezoidal rule correctly	
$y = e^{x^2}$	1	e	e^4				

Question 7

7	$SA = 300x^2 + 60xy = 3600$ $5x^2 + xy = 60$ $xy = 60 - 5x^2$ $V = 150x^2y$ Substitute $xy = 60 - 5x^2$ into V $= 150x(60 - 5x^2)$ $= 9000x - 750x^3$	2 mark: Show through substituting from SA to volume 1 mark: Have formed surface area and volume formulas. Or have correct working with one small errors. Or have rearranged surface area for y , or equivalent.	
7(b)	$V' = 9000 - 2250x^2$ Let $V' = 0$ $9000 - 2250x^2 = 0$ $x^2 = 4$ $x = \pm 2, x \geq 0$ $x = 2$ $V'' = -4500x$ $V''(2) = -4500(2)$ $= -9000$ $\therefore \text{Max volume at } x = 2 \text{ as } V' = 0 \text{ and } V'' < 0$	Find V' Solve for $x = 2$, showing that you discard the negative. Find V'' and substitute $x = 2$ (or equivalent talb Concluding statement. Need all four for 3 marks, only two for 2 marks, one mark for correct derivative	
(c)	$V = 9000(2) - 750(2^3)$ $= 12000\text{cm}^3$	1 mark correct volume	

Question 8

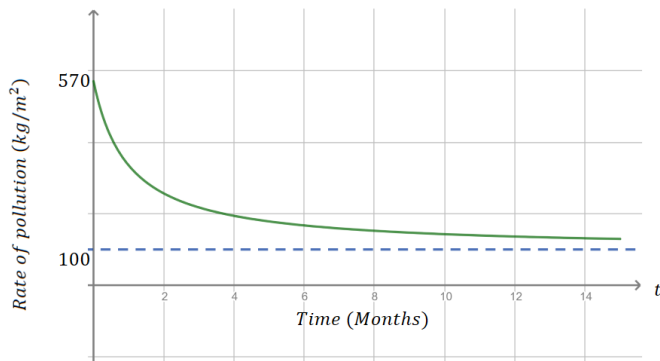
8(a)	$y = x^3 + x + x^2 + 1$ $y' = 3x^2 + 1 + 2x$ $\Delta = 4 - 4 \times 3 \times 1$ $= -8$ As $\Delta < 0$ y' has no solutions	1 mark: correct expansion and correct differentiation 2 mark: Use discriminant to show that the derivative has no solutions	
8(b)	$y'' = 6x + 2$ let $y'' = 0$ $6x + 2 = 0$ $x = -\frac{2}{6} = -\frac{1}{3}$ Test points	1 mark: Find the point of inflection, showing the test either side	

	$\therefore \left(-\frac{1}{3}, \frac{20}{27}\right)$ is a point of inflection		
		Correct shape Correct x and y intercepts Correct, and labelled point of inflection	

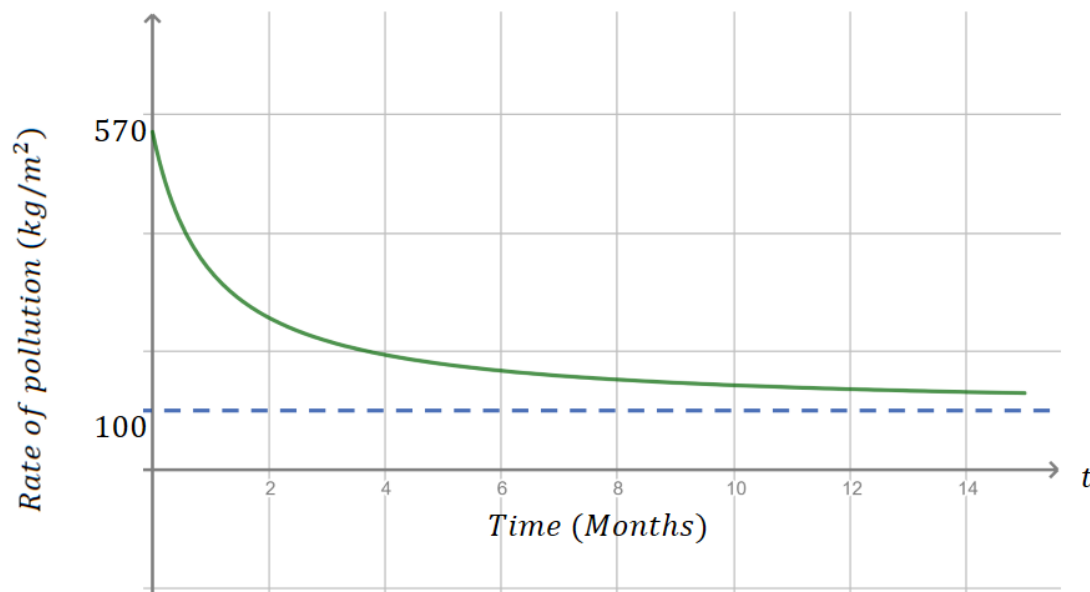
Question 9

9(a)	$\frac{d}{dx}(\log_e \cos x) = -\frac{\sin x}{\cos x} = -\tan x$	1 mark: correct answer from correct working	
9(b)	$\int \tan x - 1 \, dx$ $= -\log_e \cos x - x + C$ Using (a)	1 mark: Uses the anti-derivative from (a) 1 mark: correct integration, including '+C'	

Question 9

9(a)	$t=0, \frac{dE}{dt} = 100 + \frac{470}{0+1} = 570 \text{ kg/month}$	1 mark: correct answer from correct substitution	
9(b)	 When $t \rightarrow \infty, \frac{470}{t+1} \rightarrow 0$ Then, $\frac{dE}{dt} = 100 + \frac{470}{t+1} \rightarrow 100$	1 mark: correct y-intercept and asymptote 1 mark: correct concavity and shape t must be greater than or equal to zero)	
9(c)	Total pollution = $\int_0^{12} \left(100 + \frac{470}{t+1}\right) dt$	1 mark: correct integral	

	$= [100t + 470 \ln t + 1]_0^{12}$ $= 100(12 - 0) + 470 \ln\left(\frac{13}{1}\right)$ $= 1200 + 470 \ln 13 \text{ kg}$ $\approx 2405.5 \text{ kg}$	1 mark: correct answer	
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Question 10

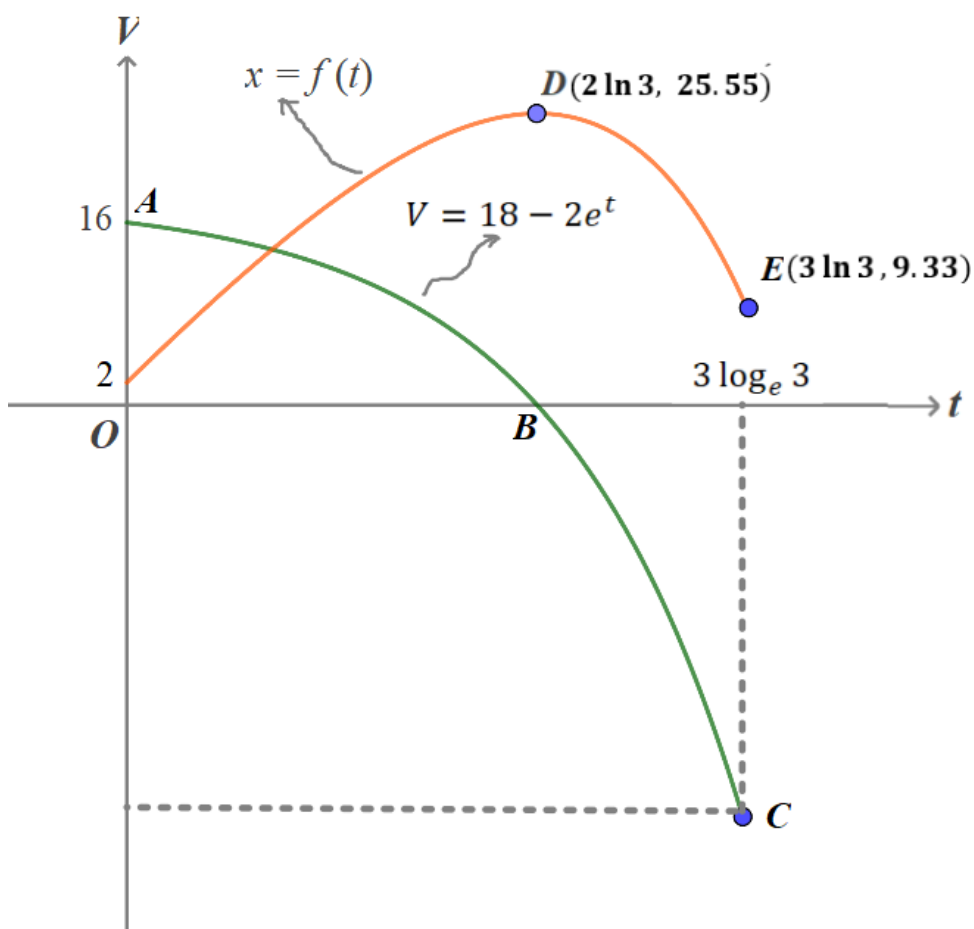
11(a)	Diameter is 4m, means amplitude 2m The lowest point being 2m above the river bed means that the centre of motion is 2m above that, at 4m Starting at the lowest point means you either start with $-\cos nt$ or you start π into a rotation at $\cos(\pi + nt)$ which simplifies to $-\cos(nt)$ The period is 36 so the coefficient of t is $\frac{2\pi}{36} = \frac{\pi}{18}$ Therefore $P = 4 - 2 \cos\left(\frac{\pi}{18}t\right)$	3 mark: Explains and calculates amplitude, period, centre of motion and explains why it is a negative cosine curve 2 mark: correct amplitude, centre of motion and explains why it is a negative cosine curve (Period calculation may be incorrect) 1 mark: Correct explanation for any two values	
11(b)	Set $P = 3$ $4 - 2 \cos\left(\frac{\pi}{18}t\right) = 3$	1 mark: sets $P=3$, or equivalent merit 2 marks: Correctly calculates the points at which P reaches 3 3 marks: correctly calculates the percentage of time that the point	

	$-2 \cos\left(\frac{\pi}{18}t\right) = -1$ $\cos\left(\frac{\pi}{18}t\right) = \frac{1}{2}$ $\frac{\pi}{18}t = \frac{\pi}{3}, \frac{5\pi}{3}$ $t = 6, 30$ Percentage of time is $\frac{30-6}{36} \times 100$ $66\frac{2}{3}\%$ of the time above the water line	would be above water/visible.	
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Question 11

11(a)	At rest when $V = 0$ $\therefore 0 = 18 - 2e^t$ $e^t = 9$ $t = \log_e 9$ $= 2 \ln 3$	2 mark: correct answer from correct working 1 mark: Sets $V=0$ and attempts to solve	
11(b)	From the graph, at B, the particle has zero velocity. Hence, from (a). $t = 2 \log_e 3$ at B. $B(2 \log_e 3, 0)$ From $t = 0$ to B, velocity is positive, hence the particle continues to move right. Distance travelled 0 to B = $\int_0^{2 \ln 3} 18 - 2e^t dt = [18t - 2e^t]_0^{2 \ln 3}$ $= 18 \ln 9 - 18 - 0 + 2$ $= 36 \ln 3 - 16$ Thus B has coordinates $(2 \ln 3, 36 \ln 3 - 16 + 2)$ That is $B(2 \ln 3, 25.55)$ From $t = B$ to $t = 3 \ln 3$, the particle is moving left. Distance travelled B to $t = 3 \ln 3 =$ $\int_{2 \ln 3}^{3 \ln 3} 18 - 2e^t dt = [18t - 2e^t]_{2 \ln 3}^{3 \ln 3}$ $= 18 \ln \frac{27}{9} - 2(27 - 9)$ $= 18 \ln 3 - 36$ Thus, C has the y-coordinate of $36 \ln 3 - 16 + 2 - (18 \ln 3 - 36)$ Thus $C(3 \ln 3, 9.33)$	*Correctly interprets the time at B *$x - t$ graph has positive gradient $t=0$ to $t=B$ * Correctly calculates the area under the curve $t=0$ to $t=B$ *Calculates the displacement at $t=B$ *$x - t$ graph has maximum turning point at $t=B$ * Calculates and labels maximum turning point *$x - t$ graph has negative gradient $t=B$ to $t = 3 \ln 3$ * Correctly calculates the area under the curve $t=B$ to $t = 3 \ln 3$ * Calculates and labels the coordinate of E. 4 marks: All of the 9 aspects are demonstrated 3 marks: At least 7 aspects are demonstrated 2 marks: At least 5 aspects are demonstrated	

		1 marks: At least 3 aspects are demonstrated	
11(c)	Distance travelled = $36 \ln 3 - 16 + 18 \ln 3 - 36 $ $= 36 \ln 3 - 16 + 36 - 18 \ln 3$ $= 18 \ln 3 + 20 \approx 39.78 \text{ m}$	1 mark: Uses the values from 7(b) to calculate the distance travelled.	



Question 12

12	$y = \log_5 x$ Finding limits of integration: $\log_5 x = 1$ $\therefore x = 5$ Limits of y are 0 and 5 $x = 5^y$ Area with y -axis=	1 mark: correctly finds the limits of integration 2 marks: correct expression for area and correctly integrates, and gives the answer to exact unit ² Award only 1 mark: If any minor error	
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	$\int_0^5 5^y dy = \frac{1}{\ln 5} [5^y]_0^5$ $= \frac{1}{\ln 5} (5^5 - 1)$ $= \frac{3124}{\ln 5} \text{ unit}^2$		
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