Year 11 Mathematics Advanced
Preliminary HSC Assessment Task 1
Term 2, Week 3, 2019 VUL

Name: ____________________

Wednesday 15th May 2019

- Answer all questions on the lined paper provided.
- Start each question on a new page.
- NESA approved calculators are permitted in this task.
- Marks may be deducted for insufficient or illegible work.
- Total possible mark: **50**
- Time allowed**: 80 minutes + 5 minutes reading time**

Question 1 (8 marks) **Marks**

(a) Solve $(x+4)^2 = 9$. **1**

(b) Simplify $\dfrac{2\sqrt{6}\times 3\sqrt{7}}{12\sqrt{21}}$. **1**

(c) Factorise each expression completely.

(i) $9x^2 - 6x - 8$. **1**

(ii) $l^2 - m^2 + 2mn - n^2$. **2**

(d) Find the exact solutions of $\dfrac{5m}{2} = 2 + \dfrac{1}{m}$. **2**

(e) A cyclist rides for 5 hours at a certain speed and then for 4 hours at a speed 6 km/h greater than his original speed. If he rides 294 altogether, what was his original speed? **1**

End of Question 1

Question 2 starts on the next page

Question 2 (8 marks) *Start a new page* **Marks**

(a) State the domain of the following functions:

(i) $y = \sqrt{1-3x}$ **1**

(ii) $y = \dfrac{1}{x^3+1}$. **1**

(b) By the use of a sketch, or otherwise, state the range of $y = -\sqrt{4-x^2}$. **1**

(c) State the definition of a rational number. **1**

(d) Without using a calculator, show that the following expression is rational: **2**

$$\frac{1}{3+\sqrt{6}} + \frac{2}{\sqrt{6}} \ .$$

(e) Solve, by the substitution method, these simultaneous equations: **2**

$$y = 3x^2$$

$$y = x \ .$$

End of Question 2

Question 3 starts on the next page

Question 3 (8 marks) *Start a new page* **Marks**

(a) If $f(x)=\sqrt{1-x^2}$, explain why $f(-2)$ cannot be found. **1**

(b) If $P(x)=x^2-2x-4$, find $P\left(\sqrt{3}-1\right)$ in simplest exact form. **1**

(c) Solve $|1-3x|=2$ for x. **2**

(d) Sketch the parabola $y=-x^2+8x-15$ clearly indicating its vertex and all intercepts with the coordinate axes. **2**

(e) If $g(x)=x^2+5x$, find $\dfrac{g(a)-g(c)}{a-c}$ in simplest form. **2**

End of Question 3

Question 4 starts on the next page

Question 4 (8 marks) *Start a new page* **Marks**

(a) Sketch the graph of $xy = 4$ and classify it as one-to-one, many-to-one, one-to-many or many-to-many. Justify your classification. **2**

(b) Consider the polynomial $y = (1-x)(x+1)^2(x-2)^3$.

(i) Write down the zeroes of the polynomial. **1**

(ii) By using a table of values to test its sign, or otherwise, sketch the polynomial. **2**

(c) The wavelength λ in metres of a musical tone is inversely proportional to its frequency f in vibrations per second.

(i) Express this relationship algebraically. **1**

(ii) The frequency of the musical note "middle C" is 260 vibrations per second and its wavelength is 1.319 metres. Find the wavelength of a musical note with frequency 440 vibrations per second, correct to 3 decimal places. **2**

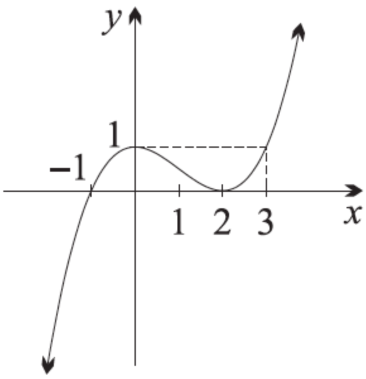
End of Question 4

Question 5 starts on the next page

Question 5 (8 marks) *Start a new page* **Marks**

(a) If $f(x) = x^3 - x$, find $f(-x)$ and hence determine whether the function is odd, even or neither. **1**

(b) The graph shown below is the curve $y = f(x)$.



Use this graph to sketch separate graphs of:

(i) $y = f(x-1)$ **1**

(ii) $y = -f(-x)$. **1**

(c) Sketch $x^2 - 10x + y^2 + 8y + 32 = 0$. **2**

(d) (i) Sketch $y = |x+1| - 1$ carefully showing the intercepts with the coordinate axes. **2**

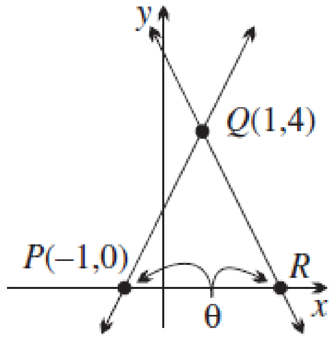
(ii) For what values of k does $|x+1| = k+1$ have no solutions for x? **1**

End of Question 5

Question 6 starts on the next page

Question 6 (10 marks) *Start a new page* **Marks**

(a) The vertices of the triangle in the diagram below are $P(-1,0)$, $Q(1,4)$ and R, where R lies on the x-axis and $\angle QPR = \angle QRP = \theta$.



Not to Scale

(i) Find the midpoint of PQ. **1**

(ii) Find the size of θ correct to the nearest degree. **1**

(iii) Find the equation of the line PQ. **1**

(iv) Explain why the gradient of line QR is -2. **1**

(v) By considering the area of ΔPQR, find the shortest distance from P to QR. **2**

(b) If $L(x) = x+1$ and $Q(x) = x^2 + 2x$;

(i) Find the zeros of $Q\left(L\left(\frac{1}{x+1}\right)\right)$. **1**

(ii) Find the equation of the horizontal asymptote of $y = Q\left(L\left(\frac{1}{x+1}\right)\right)$. **1**

(iii) It is known that as $x \to -1^-$ and as $x \to -1^+$, $y \to \infty$. (**Do not prove this**)
By using a table of values to test the sign of the function, **2**
or otherwise, sketch the graph of $y = Q\left(L\left(\frac{1}{x+1}\right)\right)$, $x \neq -1$.

End of Assessment

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Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 1 (a) $(x+4)^2 = 9$ $x+4 = \pm 3$ $x = -1, -7$ ✓	- correct answers	(d) $\frac{5m}{2} = 2 + \frac{1}{m}$ $\frac{5m^2}{2} = 2m + 1$ $5m^2 - 4m - 2 = 0$ ✓ $m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(-2)}}{2(5)}$ ✓ $= \frac{4 \pm \sqrt{56}}{10}$ $= \frac{4 \pm 2\sqrt{14}}{10}$ $= \frac{2 \pm \sqrt{14}}{5}$	
(b) $\frac{2\sqrt{6} \times 3\sqrt{7}}{12\sqrt{21}}$ $= \frac{6\sqrt{6}}{12\sqrt{3}}$ $= \frac{\sqrt{2}}{2}$ ✓	- must be fully simplified	(e) $s = \frac{d}{t}$ $\therefore d = s \times t$ Let initial speed be s $5s + 4(s+6) = 294$ $9s + 24 = 294$ $9s = 270$ $\therefore s = 30$ km/h ✓	
(c)(i) $9x^2 - 6x - 8$ $= (3x-4)(3x+2)$ ✓ $\quad 3x \times -4,\ 3x \times +2$ (ii) $l^2 - (m^2 - 2mn + n^2)$ $= l^2 - (m-n)^2$ ✓ $= (l - m + n)(l + m - n)$ ✓ [AWO for $(l-m)(l+m) + n(2m-n)$]			

2019 Year 11 Mathematics Advanced Task 1 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 2

(a) (i) $1-3x \geq 0$

$-3x \geq -1$

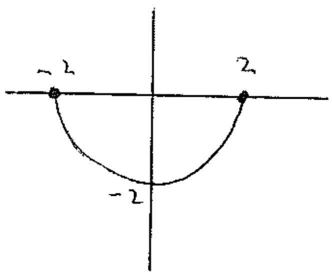
D: $x \leq \frac{1}{3}$ ✓

(ii) $x^3+1 \neq 0$

$x \neq -1$ ✓

$\therefore$ D: All real $x \neq -1$

(b)



$\therefore R: -2 \leq y \leq 0$ ✓

(c) A rational number can be expressed in the form $\frac{p}{q}$ where $p, q \neq 0$ are integers with no common factor other than 1. ✓

(d) $\frac{1}{3+\sqrt{6}} + \frac{2}{\sqrt{6}}$

$= \frac{3-\sqrt{6}}{9-6} + \frac{2\sqrt{6}}{6}$ ✓

$= \frac{6-2\sqrt{6}+2\sqrt{6}}{6}$

$= 1$ ✓ which is rational.

(e) $y = 3x^2$

$y = x$

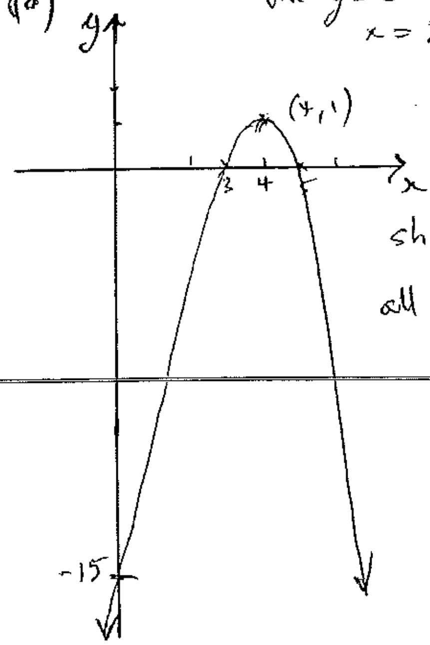
$3x^2 - x = 0.$

$x(3x-1) = 0$

$\therefore x = 0$ or $\frac{1}{3}$ ✓

$y = 0$ or $\frac{1}{3}$ ✓

2019 Year 11 Mathematics Advanced Task 1 SOLUTIONS

Suggested Solution (s)	Comments
Question 3	
(a) $f(-2) = \sqrt{1-(-2)^2}$ $= \sqrt{-3}$ which is not real. (You can't take the square root of a negative number).	
(b) $P(\sqrt{3}-1) = (\sqrt{3}-1)^2 - 2(\sqrt{3}-1) - 4$ $= 4 - 2\sqrt{3} - 2\sqrt{3} + 2 - 4$ $= 2 - 4\sqrt{3}$ ✓	
(c) $\lvert 1-3x \rvert = 2$ $1-3x = \pm 2$ ✓ $3x = -1 \quad 3x = 3$ $x = -\frac{1}{3} \quad x = 1$ ✓	or equivalent both answers
(d) when $y=0$ $x = 3$ or 5 	shape and vertex ✓ all intercepts ✓

Suggested Solution (s)	Comments
(e) $g(x) = x^2 + 5x$ $\frac{g(a)-g(c)}{a-c} = \frac{a^2+5a-(c^2+5c)}{a-c}$ ✓ $= \frac{a^2-c^2+5a-5c}{a-c}$ $= \frac{(a-c)(a+c)+5(a-c)}{a-c}$ $= \frac{\cancel{(a-c)}(a+c+5)}{\cancel{a-c}}$ $= a+c+5$ ✓	

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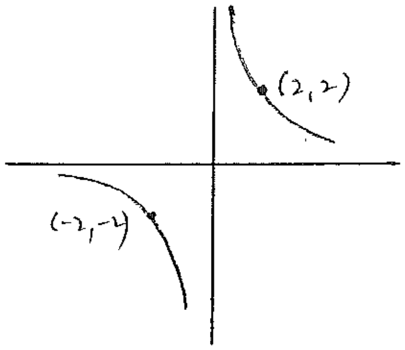
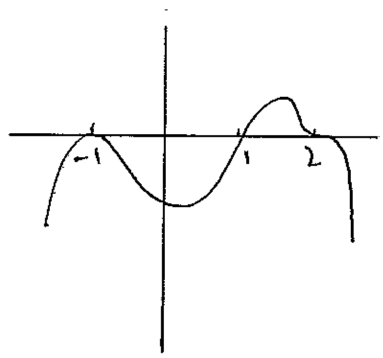
Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 4 (a)  This is a "one-to-one" function as there is 1 x-value for each y-value in its range	✓ graph ✓ correct classification with reason.	c) i) $\lambda \propto \frac{1}{f}$ ✓ or $\lambda = \frac{k}{f}$ ii) $1.319 = \frac{k}{260}$ $\therefore k = 260 \times 1.319$ ✓ $\therefore$ if $f = 440$ $\lambda = \frac{260 \times 1.319}{440}$ $= 0.779$ m. ✓	either
i) $-1, 1, 2$ ✓ (b)  ✓ - intercepts at zero's and $y \to -\infty$ for extreme values of x ✓ - correct shape at zeros.			

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Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 5

(a) $f(x) = x^3 - x$

$f(-x) = (-x)^3 - (-x)$

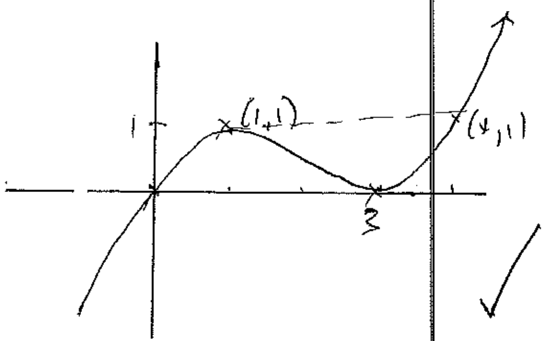
$= -x^3 + x$

$= -(x^3 - x)$ ✓ with correct conclusion.

$= -f(x)$

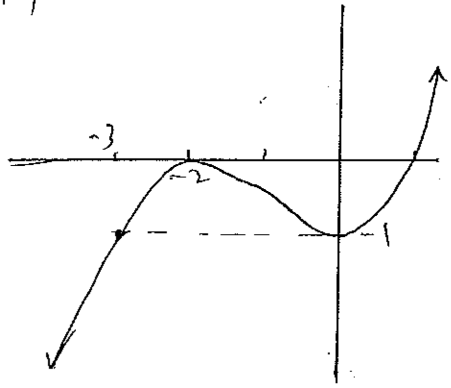
$\therefore f(x)$ is ODD

(b) (i)



✓

(ii)

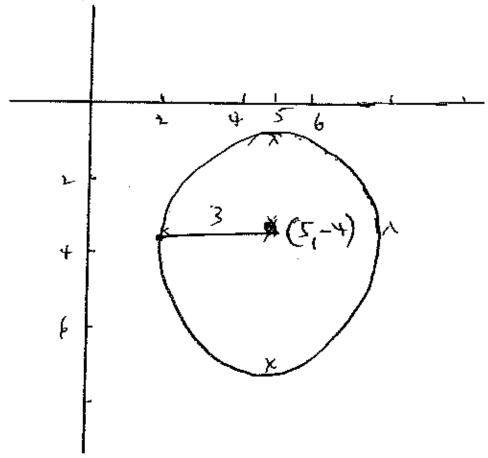


✓

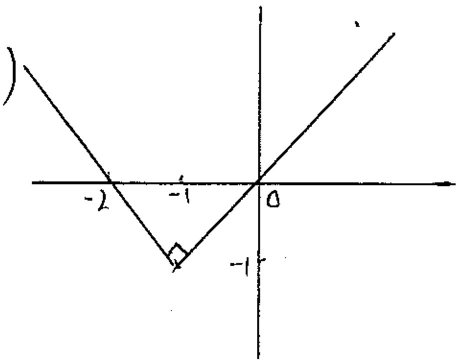
c) $x^2 - 10x + 25 + y^2 + 8y + 16 = -32 + 25 + 16$

$\therefore (x-5)^2 + (y+4)^2 = 9$

$\therefore C(5,-4)\quad r = 3$



d) (i)



✓ the "V" shape with either one of the translations

✓ correct solution.

(ii) $|x+1| - 1 = k$

no solutions if $k < -1$ ✓

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Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 6 (a) (i) $M\left(\frac{-1+1}{2}, \frac{0+4}{2}\right)$ $= M(0, 2)$ (ii) $\tan\theta = m_{PQ}$ $= \frac{4-0}{1-(-1)}$ $\tan\theta = 2$ $\therefore \theta \doteqdot 63^\circ$ nearest degree (ii) $y - 4 = 2(x-1)$ ✓ $y = 2x + 2$ or equivalent (iii) Since $\angle QRP = \theta = 63.43\ldots^\circ$ $m_{QR} = \tan(180^\circ - 63.43\ldots^\circ)$ $= -2$ or equivalent algebraic argument (iv) Since ΔPQR is isosceles Area $= \left(\frac{1}{2} \times 2 \times 4\right) \times 2$ $= 8\ u^2$ ✓		Equation of QR $y - 4 = -2(x-1)$ $y = -2x + 6$ thus R is $(3, 0)$ $d_{QR} = \sqrt{(3-1)^2 + (0-4)^2}$ $= \sqrt{4+16}$ $= \sqrt{20} = 2\sqrt{5}$ Shortest distance (d) is perpendicular distance thus $\frac{1}{2} \times \lvert QR \rvert \times d = 8$ $\frac{1}{2} \times 2\sqrt{5} \times d = 8$ $\therefore d = \frac{8}{\sqrt{5}}$ ✓ units.	

2019 Year 11 Mathematics Advanced Task 1 SOLUTIONS

Suggested Solution (s)	Comments

Question 6

(b) $L(x) = x + 1$

$Q(x) = x^2 + 2x$

(i) $L\left(\frac{1}{x+1}\right) = \frac{1}{x+1} + 1$

$= \frac{x+2}{x+1}$

$\therefore Q\left(L\left(\frac{1}{x+1}\right)\right) = Q\left(\frac{x+2}{x+1}\right)$

$= \left(\frac{x+2}{x+1}\right)^2 + 2\left(\frac{x+2}{x+1}\right)$

$= \frac{(x+2)^2}{(x+1)^2} + \frac{2(x+2)(x+1)}{(x+1)^2}$

$= \frac{(x+2)[(x+2) + 2(x+1)]}{(x+1)^2}$

$= \frac{(x+2)(3x+4)}{(x+1)^2}$

which has zeros

$x = -2, -\frac{4}{3}$ ✓

(ii) as $x \to \infty$, $y \to 3^+$

$x \to -\infty$, $y \to 3^-$

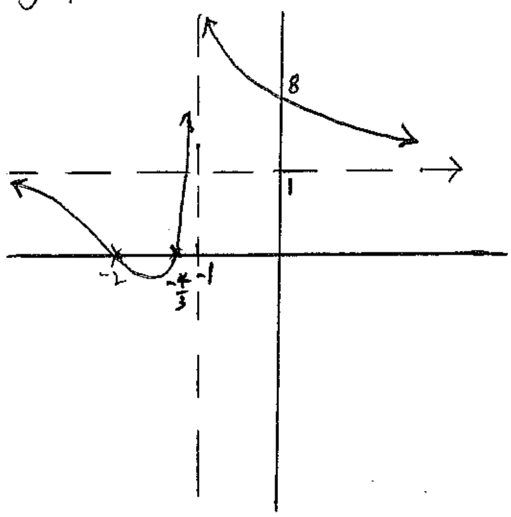
$\therefore$ Horizontal asymptote

is $x = 3$ ✓

Suggested Solution (s)	Comments

(iii)

x	$-\infty$	-3	-2	-1.5	$-1\frac{1}{3}$	-1^-	-1	-1^+	∞
y	3^-	$+$	0	$-$	0	∞	NA	∞	3^+



✓ correct behaviour near both $x = -1$ and $y = 3$

✓ correct sign and shape between zeros and asymptotes.