

Name: _____

Teacher: _____



Knox Grammar
School
2019

Year 11 Advanced Mathematics
Task 2B

Year 11 Mathematics
Advanced
Preliminary
Task 2B In Class Task

Setter:
LEP

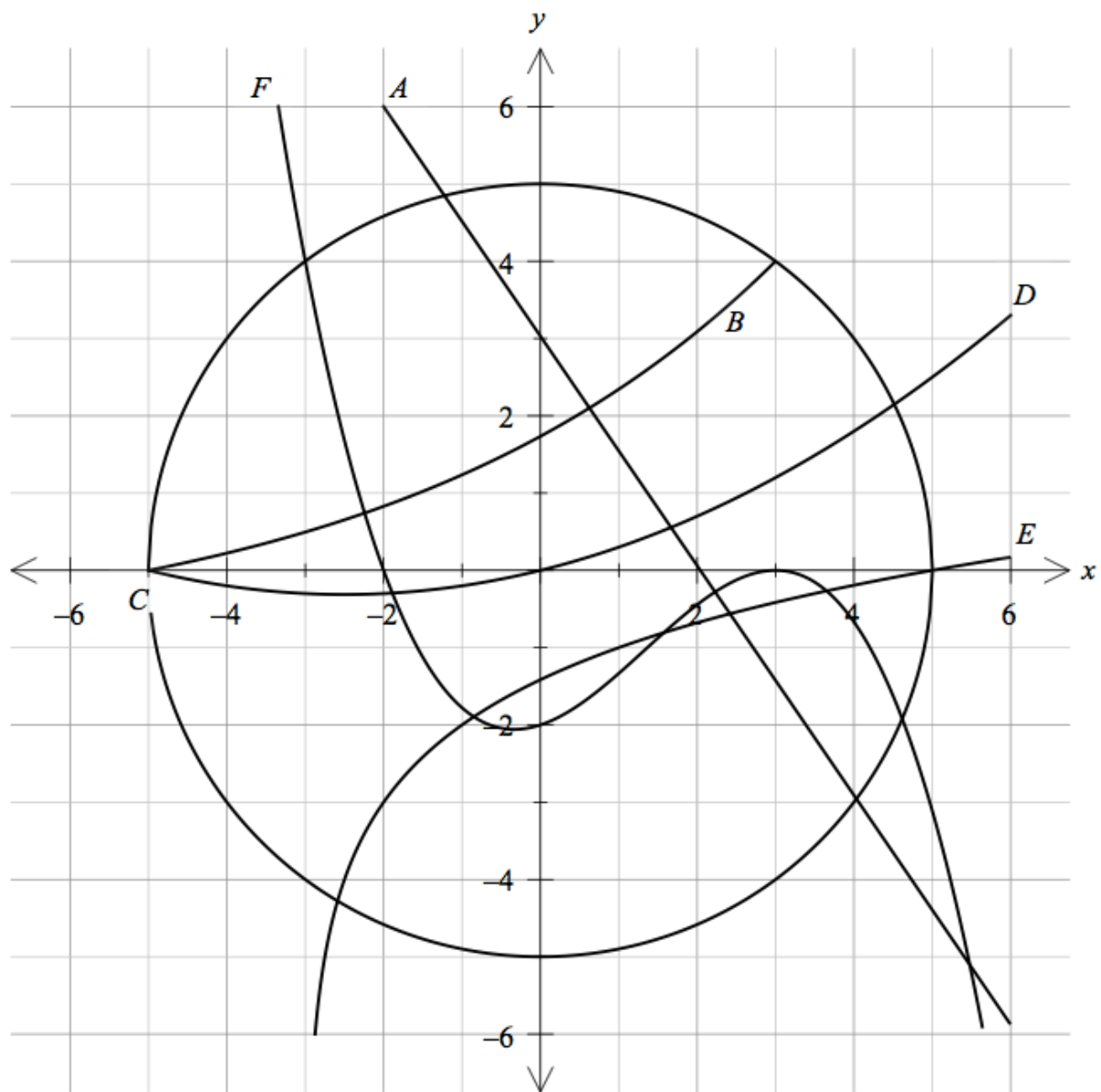
General Instructions:

- Time: 30 mins + 5 mins reading time
- Write using black or blue pen. Black pen is preferred
- NESA-approved calculators may be used
- Marks may be deducted for incorrect work leading to a correct result
- Marks may be deducted for careless or badly arranged work, which compromises the validity of a response
- All required working must be evident for the award of marks

	Marks
Task 2A	/12
Task 2B	
Section I	/4
Section II	/4
Section III	/4
Section IV	/4
Section V	/4
Total	/32

A start-up company have asked you to design a logo for them. The logo lies between the values of $-6 \leq x \leq 6$ and $-6 \leq y \leq 6$.

Your first draft of the logo is shown below.



Section I

Marks

- a) Consider the function F .

2

Find the gradient of the line passing through the y -intercept and the positive x -intercept of function F .

- b) Function D is a parabola. The point $(4, 1.8)$ also lies on this curve.

2

Show that the equation of function D is:

$$y = \frac{1}{20} x(x + 5)$$

Section II

Marks

- a) Function A has the equation $49x + 33y = 100$.

2

Find the point of intersection between functions A and D that lies within the domain of the logo.

Section II continues on the next page

- b) Find the equation of the line that is perpendicular to function A through the point found in part a) above. **2**

Section III

Marks

4

The equation used to graph function E is $y = \log_2(x + 3) - 3$. However, the domain needs to be restricted in order to keep it within the logo, and as a result, the range will also be reduced.

Find the domain and range in exact form of the function E for the given logo.

Section IV

Marks

2

- a) The equation of the cubic function (function F) in the logo is:

$$f(x) = \frac{-1}{9}(x+2)(x-3)^2$$

The company wants you to move the cubic function to a different place. They want it to be reflected in the y -axis and then translated down 1 unit.

Let $g(x)$ be the equation of this new cubic function.

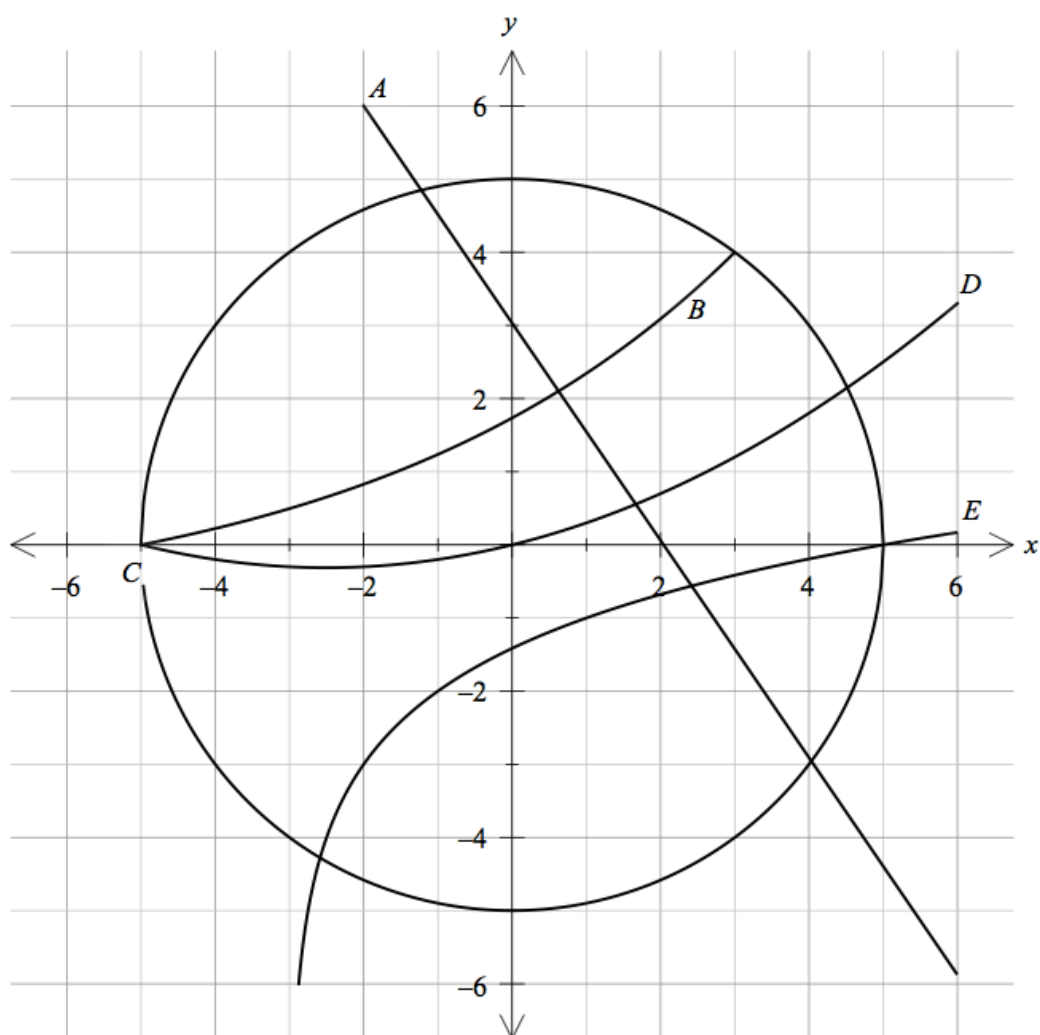
What will the equation of $g(x)$ be in simplest form?

Section IV continues on the next page

b) The logo below has had the original cubic function removed.

2

Neatly sketch where $g(x)$ will be on the updated logo, showing all key features.



Section V

Marks

- a) In this design, function B is an exponential function that has the form $y = a^{x+b} - 1$ in the domain $-5 \leq x \leq 3$. It intersects the circle at its endpoints.

3

Find the exact values of a and b .

- b) Rewrite the equation of the exponential function above so that it has an integer base.

1

End of Paper

Name: Solns

Teacher: _____



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**Year 11 Advanced Mathematics
Task 2B**

**Year 11 Mathematics
Advanced
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Task 2B In Class Task**

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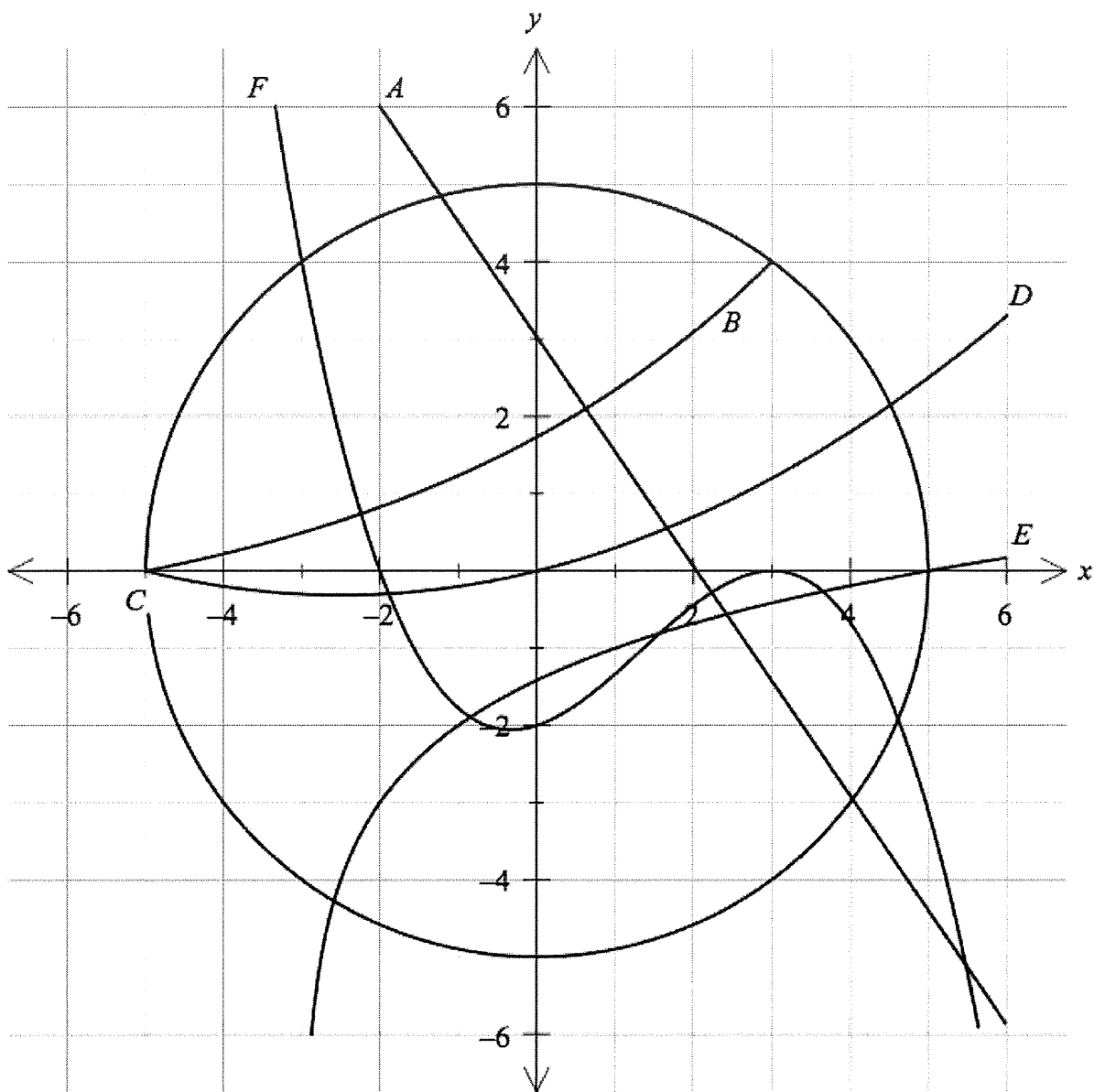
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Task 2B	
Section I	/4
Section II	/4
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Total	/32

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Your first draft of the logo is shown below.



Section I

Marks

2

- a) Consider the function F .

Find the gradient of the line passing through the y -intercept and the positive x -intercept of function F .



$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{0 - (-2)}{3 - 0} \\
 &= \frac{2}{3} \quad (1)
 \end{aligned}$$

- b) Function D is a parabola. The point $(4, 1.8)$ also lies on this curve.

2

Show that the equation of function D is:

$$y = \frac{1}{20} x(x + 5)$$

$$y = ax(x + 5) \quad (1)$$

$$\text{when } x = 4, y = 1.8$$

$$1.8 = 4a(4 + 5)$$

$$1.8 = 36a$$

$$a = \frac{1.8}{36}$$

$$= \frac{1}{20}$$

$$\therefore y = \frac{1}{20} x(x + 5) \quad (1)$$

Section II

Marks

2

- a) Function A has the equation $49x + 33y = 100$.

Find the point of intersection between functions A and D that lies within the domain of the logo.

$$49x + 33y = 100$$

$$y = \frac{1}{20}x(x+5)$$

$$\therefore 49x + \frac{33}{20}x(x+5) = 100 \quad (1)$$

$$980x + 33x^2 + 165x = 2000$$

$$33x^2 + 1145x - 2000 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1145 \pm \sqrt{1145^2 + 4 \times 33 \times 2000}}{66}$$

$$= \frac{-1145 \pm 1255}{66}$$

$$= \frac{5}{3} \text{ or } \frac{-400}{11}$$

$$\text{but } -6 \leq x \leq 6$$

$$\therefore x = \frac{5}{3}$$

$$y = \frac{1}{20} \times \frac{5}{3} \left(\frac{5}{3} + 5 \right)$$

$$= \frac{5}{9}$$

$\therefore \left(\frac{5}{3}, \frac{5}{9} \right)$ is the point of intersection. (1)

Section II continues on the next page

- b) Find the equation of the line that is perpendicular to function A through the point found in part a) above.

2

$$y = \frac{-49}{33}x + \frac{100}{33}$$

$$\therefore m_A = \frac{-49}{33}$$

$$m_{\perp} = \frac{33}{49} \quad (1)$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{5}{9} = \frac{33}{49} \left(x - \frac{5}{3} \right)$$

$$y - \frac{5}{9} = \frac{33}{49}x - \frac{55}{49}$$

$$y = \frac{33}{49}x - \frac{250}{441} \quad (1)$$

Section III

Marks

4

The equation used to graph function E is $y = \log_2(x + 3) - 3$. However, the domain needs to be restricted in order to keep it within the logo, and as a result, the range will also be reduced.

Find the domain and range in exact form of the function E for the given logo.

$$y = \log_2(x + 3) - 3$$

when $x = 6$

$$y = \log_2 9 - 3$$

①

when $y = -6$

$$-6 = \log_2(x + 3) - 3$$

$$\log_2(x + 3) = -3$$

$$x + 3 = 2^{-3}$$

$$x = -3 + 2^{-3}$$

$$= -\frac{23}{8} \quad \text{①}$$

$$\therefore D: -\frac{23}{8} \leq x \leq 6 \quad \text{①}$$

$$R: -6 \leq y \leq \log_2 9 - 3 \quad \text{①}$$

Section IV

Marks

2

- a) The equation of the cubic function (function F) in the logo is:

$$f(x) = \frac{-1}{9}(x+2)(x-3)^2$$

The company wants you to move the cubic function to a different place. They want it to be reflected in the y-axis and then translated down 1 unit.

Let $g(x)$ be the equation of this new cubic function.

What will the equation of $g(x)$ be in simplest form?

$$\begin{aligned} g(x) &= -\frac{1}{9}(-x+2)(-x-3)^2 - 1 \\ &= -\frac{1}{9}(-x+2)(x+3)^2 - 1 \\ &= \frac{1}{9}(x-2)(x+3)^2 - 1 \end{aligned}$$

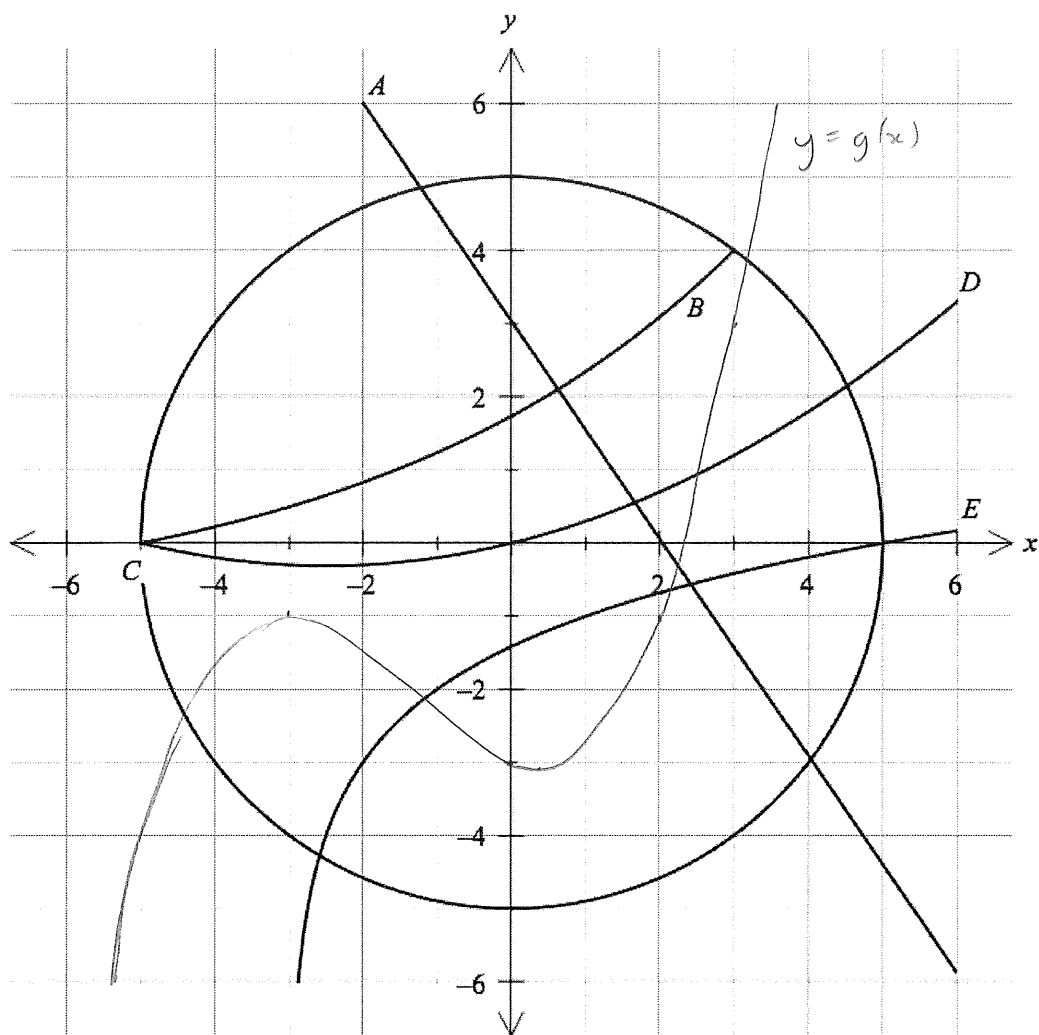
- ① for -1 at the end of the function
- ① for applying algebraic reflection in y-axis correctly

Section IV continues on the next page

b) The logo below has had the original cubic function removed.

2

Neatly sketch where $g(x)$ will be on the updated logo, showing all key features.



① intercepts of $f(x)$ transformed correctly

② shape of cubic and neatness.

Section V

Marks

3

- a) In this design, function B is an exponential function that has the form $y = a^{x+b} - 1$ in the domain $-5 \leq x \leq 3$. It intersects the circle at its endpoints.

Find the exact values of a and b .

The endpoints are $(-5, 0)$ and $(3, 4)$ (1)

at $(-5, 0)$

$$0 = a^{b-5} - 1$$

$$a^{b-5} = 1$$

$$\therefore b-5 = 0$$

$$b = 5 \quad (1)$$

at $(3, 4)$

$$4 = a^{b+3} - 1$$

$$\text{but } b = 5$$

$$\therefore 5 = a^8$$

$$a = \sqrt[8]{5} \quad (1)$$

- b) Rewrite the equation of the exponential function above so that it has an integer base. 1

$$y = (\sqrt[8]{5})^{x+5} - 1$$

$$= (5^{\frac{1}{8}})^{x+5} - 1$$

$$y = 5^{\frac{1}{8}(x+5)} - 1 \quad (1)$$

End of Paper