Year 11 Mathematics Advanced
Preliminary HSC Assessment Task 1
Term 2, Week 3, 2020

Name: _____

Wednesday 13th May 2020

Period 4 or Period 5

- Answer all questions on your own lined paper.
- This is an "Open book" Task.
- Start each question on a new page.
- Board approved calculators are permitted in this task.
- Marks may be deducted for insufficient or illegible work.
- Total possible mark: **32**
- **Time allowed: 55 Minutes inclusive of downloading the task, then answering on your own lined-paper, scanning and uploading a single PDF file of your responses to your teacher's google classroom. It is your responsibility to ensure that your marker can read your responses in the PDF file that you create.**

Question 1 (8 marks) **Marks**

(a) (i) Simplify $\dfrac{\left(-5x^2y\right)^3}{15x}$. **1**

(ii) Expand and simplify $(x+2)^2-(x+2)(x-2)$. **1**

(b) Solve $(3x+4)(x-3)=16$ for x. **2**

(c) Simplify $\dfrac{2}{x^2-4}-\dfrac{3}{x+2}$. **2**

(d) Find a and b if $\dfrac{\sqrt{2}+3}{\sqrt{2}-1}=a+\sqrt{b}$. **2**

End of Question 1

Question 2 starts on the next page

Question 2 (8 marks) *Start a new page* **Marks**

(a) If $f(x) = x^2 - 3x$ find the value of $f\left(\sqrt{3}-1\right)$ in simplest surd form. **1**

(b) Write down the domain of the function $f(x) = \dfrac{1}{\sqrt{x+4}}$. **1**

(c) (i) Sketch the function $f(x) = -\sqrt{9-x^2}$. **1**

(ii) Hence state the range of $f(x)$. **1**

(d) Sketch a "one to many" relation. **1**

(e) The diameter of a balloon varies inversely with the thickness of the rubber.
The diameter of the balloon is 80mm when the rubber is 2mm thick.

(i) Find an equation for the diameter *D* as a function of its thickness *x* . **1**

(ii) What is the thickness of the rubber when the diameter is 115.3mm? Give your answer to 1 decimal place. **1**

(f) Determine the maximum value of $y = 6x - x^2$ by completing the square **or** a graphical method of your choice. You must show your working. **1**

End of Question 2

Question 3 starts on the next page

Question 3 (8 marks) *Start a new page* **Marks**

(a) Describe the transformation to the hyperbola $y=\frac{1}{x}$ which

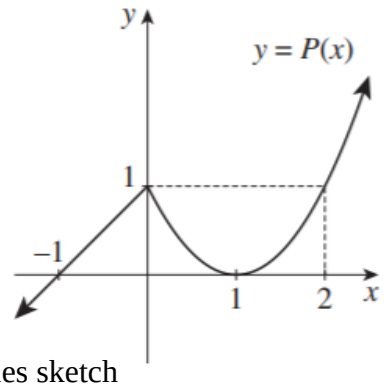
can be used to sketch $y=\frac{1}{x+2}$. **1**

(b) Show that the function $f(x)=x^3+5x$ is odd. **1**

(c) (i) Sketch $y=|x+3|$ and $y=2$ on the same axes. **1**

(ii) Hence, or otherwise, solve $|x+3|=2$. **1**

(d) Consider the graph of $y=P(x)$.



On separate number planes sketch

(i) $y=P(-x)$. **1**

(ii) $y=1-P(-x)$. **1**

(e) If $f(x)=(x+1)(x-2)$ and $g(x)=x^2$.

(i) Write down an expression for $g(f(x))$. **1**

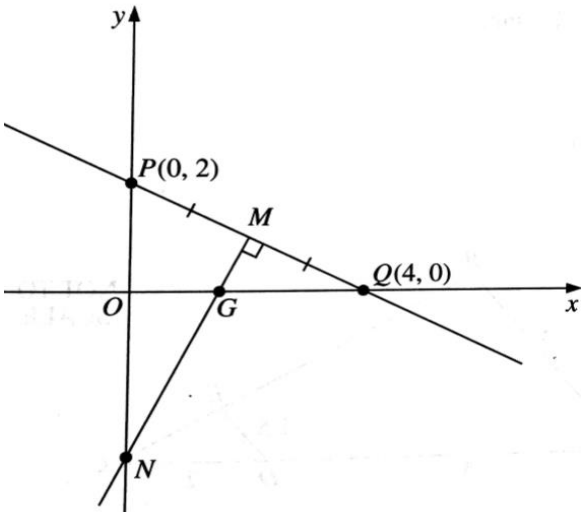
(ii) Sketch the polynomial function $y=g(f(x))$. **1**

End of Question 3

Question 4 starts on the next page

Question 4 (8 marks) *Start a new page* **Marks**

(a) The diagram below show the points $P(0,2)$ and $Q(4,0)$. The point M is is the midpoint of PQ. The line MN is perpendicular to PQ and meets the x-axis at G and the y-axis at N.



(i) Find the coordinates of M. **1**

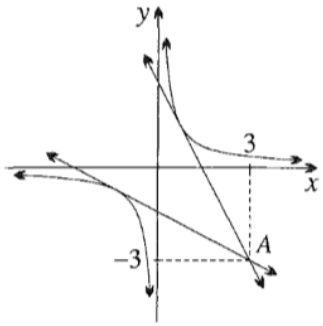
(ii) Find the equation of the line MN. **2**

(iii) Show that N has coordinates $(0,-3)$. **1**

(iv) Find the distance NQ. **1**

(v) Give the equation of the circle with centre N and radius NQ. **1**

(b) Find the gradients of the two tangents to the hyperbola $y=\frac{3}{x}$ which pass through the point $A(3,-3)$ as shown in the diagram below. **2**



End of Assessment

2020 Year 11 Mathematics Advanced Assessment Task 1 Solutions

Suggested Solution (s)

Question 1

(a) (i) $\dfrac{(-5x^2y)^3}{15x}$

$= \dfrac{-125x^6y^3}{15x}$

$= -\dfrac{25x^5y^3}{3}$ ✓

ii) $(x+2)^2 - (x+2)(x-2)$

$= x^2+4x+4 - (x^2-4)$

$= 4x+8$ ✓

(b) $(3x+4)(x-2) = 16$

$3x^2 - 5x - 28 = 0$ ✓

$(3x+7)(x-4) = 0$

$\therefore x = -\frac{7}{3}, 4$ ✓ for both ans.

c) $\dfrac{2}{x^2-4} - \dfrac{3}{x+2}$

$= \dfrac{2}{(x-2)(x+2)} - \dfrac{3(x-2)}{(x-2)(x+2)}$

✓ establishing L.C.D.

$= \dfrac{8-3x}{(x-2)(x+2)}$ ✓ correct ans.

d) $\dfrac{\sqrt{2}+3}{\sqrt{2}-1} \times \dfrac{\sqrt{2}+1}{\sqrt{2}+1} = \dfrac{5+4\sqrt{2}}{1}$

$a = 5$ ✓ $b = 32$ ✓

Question 2

(a) $f(\sqrt{3}-1)$

$= (\sqrt{3}-1)^2 - 3(\sqrt{3}-1)$

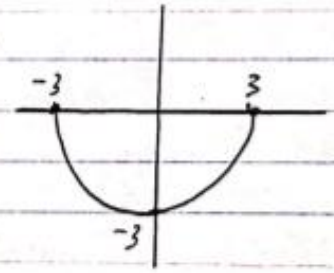
$= 4 - 2\sqrt{3} - 3\sqrt{3} + 3$

$= 7 - 5\sqrt{3}$ ✓

(b) D: $x+4 > 0$

$\therefore x > -4$ ✓

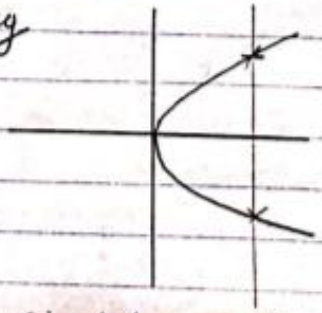
(c) (i)



✓ for graph clearly showing length of radius

(ii) R: $-3 \leq y \leq 0$ ✓

(d) eg



Note: Sketch must "fail" vertical line "pass" horizontal line test.

✓

2020 Year 11 Mathematics Advanced Assessment Task 1 Solutions

Suggested Solution (s)

Question 2 Cont

(e) (i) $D = \frac{160}{x}$ ✓

(ii) $115.3 = \frac{160}{x}$

$\therefore x = \frac{160}{115.3}$

$= 1.4$ mm ✓

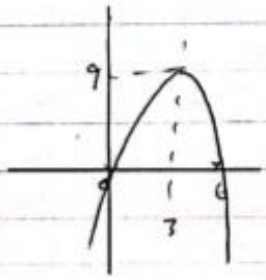
(f) $y = 6x - x^2$

$= -(x^2 - 6x + 9 - 9)$

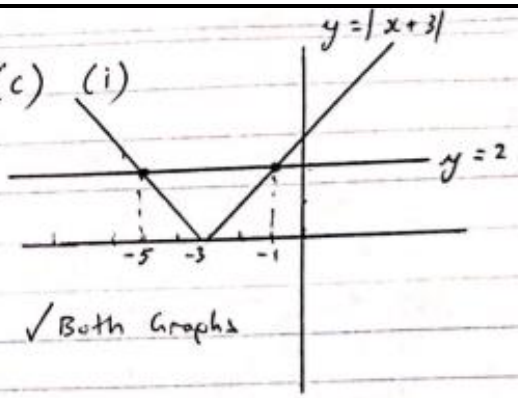
$= 9 - (x-3)^2$

$\therefore$ Max value is 9 ✓

or



(c) (i)



✓ Both Graphs

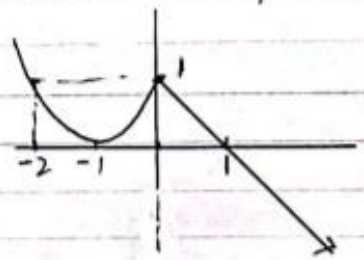
(ii) By inspection ✓

$x = -1$ or -5

or by solving.

(d)

(i) Reflection in y-axis



✓ must pass thru $(-2,1)$, $(-1,0)$, $(0,1)$ and $(1,0)$

Question 3

(a) Translation 2 units to the left ✓

(slide, shifted, moved)

(b) $f(-x) = (-x)^3 + 5(-x)$

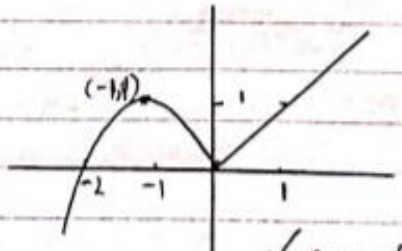
$= -x^3 - 5x$

$= -(x^3 + 5x)$ ✓ process

$= -f(x)$

$\therefore$ Odd

(ii) Graph in part (i) reflected in x-axis and raised 1 unit.



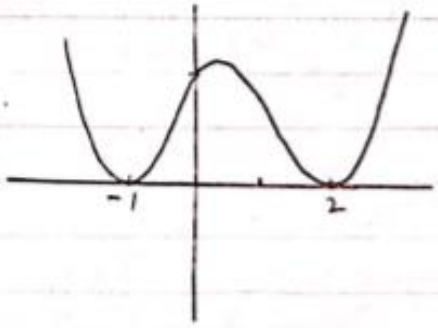
✓ pass thru $(-2,0)$, $(-1,1)$

2020 Year 11 Mathematics Advanced Assessment Task 1 Solutions

Suggested Solution (s)

(e) (i) $y = g(f(x))$
$= ((x+1)(x-2))^2$ ✓
$= (x+1)^2(x-2)^2$

(ii)



✓ double roots at $x = -1, 2$ and correct shape. (y-int not required).

(b) Lines are in form

$y + 3 = m(x - 3)$
$y = mx - (3m + 3)$

Solving simultaneously with $y = \frac{3}{x}$ yields a quadratic:

$mx^2 - 3(m+1)x - 3 = 0$
✓ or equivalent

which will have only 1 solution (double zero) if the lines are tangential.

Double zero's when $\Delta = 0$

$9(m+1)^2 + 12m = 0$

expanding, simplifying and factorising we have

$(m+3)(3m+1) = 0$
$m = -3$ or $-\frac{1}{3}$
✓ both answers

Question 4.

(a) (i) $M(2,1)$ ✓

(ii) $m_{PQ} = -\frac{1}{2}$
$\therefore m_{NN} = 2$ ✓
$y - 1 = 2(x-2)$ ✓
$y = 2x - 3$

(iii) when $x = 0$, $y = 2(0) - 3 = -3$ ✓
$\therefore N(0,-3)$

(iv) $|NQ| = 5$ ✓

(v) $x^2 + (y+3)^2 = 25$ ✓