Year 11 Mathematics 2 Unit
Preliminary HSC Assessment Task 2
Term 3, Week 3, 2020

Name: _____

Teacher: _____

Wednesday 5th August 2020

Setter: CHG

- Answer all questions on the lined paper provided.
- Start each question on a new page.
- Board approved calculators are permitted in this task.
- Marks may be deducted for insufficient or illegible work.
- For any multiple choice question, **write the letter** that corresponds to your answer on your writing paper.
- Total possible mark: **32**
- Time allowed**: 45 minutes + 2 minutes reading time**

Question 1 (8 marks) **Marks**

(a) If $\tan\theta = \sqrt{3}$ and $\sin\theta < 0$, which value for $0 \leq \theta \leq 360°$ satisfies these conditions. **1**

(A) 60°

(B) 120°

(C) 240°

(D) 400°

(b) A formula for the temperature of water after t hours is $T(t) = 100 \times \left(\frac{1}{2}\right)^t$. **1**

Using this formula calculate the time taken for the water to cool to 10°C.
Leave your answer in hours correct to 2 decimal places.

(c) Solve these equations for $[0°, 360°]$:

(i) $\sin 2x = \frac{1}{\sqrt{2}}$ **2**

(ii) $8\sin x = \text{cosec}^2 x$ **2**

(iii) $2\sin^2 x + \cos x = 1$ **2**

End of Question 1

Question 2 starts on a new page

Question 2 (8 marks) *Start a new page* **Marks**

(a) Differentiate: $y = 6x^2 - 4$ **1**

(A) $6x^2$

(B) $12x$

(C) $12x - 1$

(D) $2x^3 - 4x$

Differentiate with respect to x, leaving your answer in simplest positive integer or surd form.

(b) $y = -\dfrac{2}{x^2}$ **1**

(c) $y = \sqrt{9x^2 - 16}$ **2**

(d) $y = \left(2x^2 + 5\right)\left(3x - 5\right)^4$ (Leaving your answer in fully factorised form) **2**

(e) $y = \dfrac{4x^2 - 3}{x + 3}$ **2**

End of Question 2

Question 3 starts on a new page

Question 3 (8 marks) *Start a new page* **Marks**

(a) If $\cos\theta = -0.5$, which values for $0 \leq \theta \leq 360°$ satisfy these conditions? **1**

(A) 120°, 240°

(B) 150°, 330°

(C) 210°, 330°

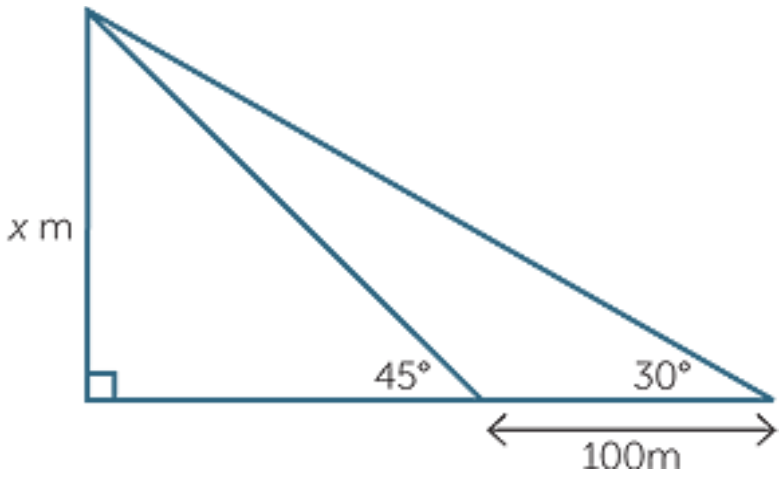
(D) 240°, 300°

(b) Given that $\log_b 3 = 1.2$, find the value of $log_b 3b^2$. **1**

(c) Find the equation of the normal to the curve $y = 2\sqrt{x}$ at the point (4,4). **2**

(d) Prove: $\dfrac{\cos\theta}{1-\sin\theta} - \tan\theta = \sec\theta$ **2**

(e) Calculate the value of x in the diagram below, answer correct to 2 decimal places. **2**



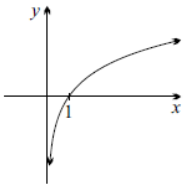
End of Question 3

Question 4 starts on a new page

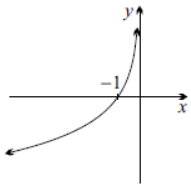
Question 4 (8 marks) *Start a new page* **Marks**

(a) Which graph represents $y = -\log(x)$? **1**

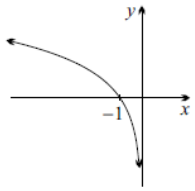
(A)



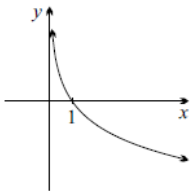
(B)



(C)



(D)



(b) Tim ran 2km on a bearing of 040°, turned and ran 3km on a bearing of 150°. Calculate the distance he is now from his starting position to the nearest km. **1**

(c) Find the x-coordinates on the curve $y = 2x^3 - 3x^2 - 12x + 10$ where the tangent is parallel to the x-axis. **2**

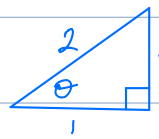
(d) Differentiate $f(x) = x^2 - 4x$ using first principles. **2**

(e) Prove $\log_2 5$ is irrational. **2**

End of Assessment

2020 11 Advanced AT2 Solutions

(1) a)



$\sin\theta = \frac{\sqrt{3}}{2}$ $\quad \frac{S|A}{T|C}$

$\angle_{related} = 60°$

$\theta = 180+60$

$= 240°$ (C) ✓

b) $10 = 100\left(\frac{1}{2}\right)^t$

$0.1 = \left(\frac{1}{2}\right)^t$

$t = \frac{\log(0.1)}{\log(0.5)}$

≈ 3.32 hours ✓

c)i) $\sin 2x = \frac{1}{\sqrt{2}}$ $\quad 0 \le x \le 360$

$2x = 45, 135, 405, 495$ $\quad 0 \le 2x \le 720$

$x = 22.5°, 67.5°, 202.5°, 247.5°$ ✓✓

(1) 45 and 135°

ii) $8\sin x = \operatorname{cosec}^2 x$

$8\sin x = \frac{1}{\sin^2 x}$

$8\sin^3 x = 1$ (1)

$\sin^3 x = \frac{1}{8}$

$\sin x = \sqrt[3]{\frac{1}{8}}$

$\sin x = \frac{1}{2}$

$x = 30, 150°$ ✓✓

iii) $2\sin^2 x + \cos x = 1$

$2(1-\cos^2 x) + \cos x - 1 = 0$

$2 - 2\cos^2 x + \cos x - 1 = 0$

$-2\cos^2 x + \cos x + 1 = 0$

$2\cos^2 x - \cos x - 1 = 0$ (1)

$(\cos x - 1)(2\cos x + 1) = 0$

$\therefore \cos x - 1 = 0$ or $2\cos x + 1 = 0$

$\cos x = 1$ $\quad \cos x = -\frac{1}{2}$

$x = 0, 360°$ $\quad x = 120°, 240°$

$\therefore x = 0, 120°, 240°, 360°$ ✓✓

(2) a)(B) $y = 6x^2 - 4$

$y' = 12x$ ✓

b) $y = 2x^{-2}$

$y' = \frac{4}{x^3}$ ✓

c) $y = (9x^2 - 16)^{1/2}$

$y' = \frac{1}{2}(18x)(9x^2-16)^{-1/2}$ (1)

$= \frac{18x}{2\sqrt{9x^2-16}}$

$= \frac{9x}{\sqrt{9x^2-16}}$ ✓✓

(2) d) $y=(2x^2+5)(3x-5)^4$

(1) $= 4x(3x-5)^4 + 12(2x^2+5)(3x-5)^3$

$= (3x-5)^3(4x(3x-5) + 24x^2+60)$

$= (3x-5)^3(12x^2-20x+24x^2+60)$

$= 4(3x-5)^3(9x^2-5x+15)$ ✓✓

e) $y = \dfrac{4x^2-3}{x+3}$

$= \dfrac{8x(x+3) - (4x^2-3)}{(x+3)^2}$ (1)

$= \dfrac{8x^2+24x-4x^2+3}{(x+3)^2}$

$= \dfrac{4x^2+24x+3}{(x+3)^2}$ ✓✓

3a) (4) $\angle$related $= 60^\circ$

$\theta = 180-60, 180+60$

$= 120^\circ, 240^\circ$ ✓

b) $= \log_b 3b^2$

$= \log_b 3 + 2\log_b b$

$= 1.2 + 2$

$= 3.2$ ✓

(3) c) $y = 2x^{\frac{1}{2}}$

$y' = 2\left(\frac{1}{2}\right)x^{-\frac{1}{2}}$

$= \dfrac{1}{\sqrt{x}}$

using $m_T = \dfrac{1}{\sqrt{4}} = \dfrac{1}{2}$ (1) and $(4,4)$

$y-4 = \frac{1}{2}(x-4)$

$y = \frac{x}{2} + 2$ ✓✓

d) Prove $\dfrac{\cos\theta}{1-\sin\theta} - \tan\theta = \sec\theta$

$LHS = \dfrac{1}{\sec\theta - \tan\theta} - \tan\theta$

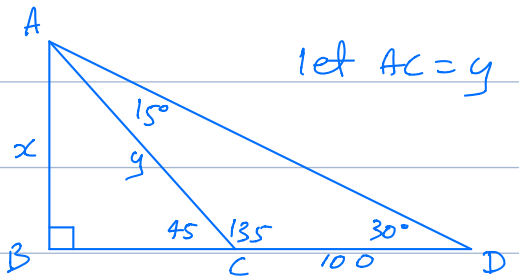
$= \dfrac{\sec\theta+\tan\theta}{\sec^2\theta - \tan^2\theta} - \tan\theta$ (1)

$= \sec\theta + \tan\theta - \tan\theta$

$= \sec\theta$ ✓✓

$= RHS$

e)



$\dfrac{y}{\sin 30} = \dfrac{100}{\sin 15}$

$y = \dfrac{100\sin 30}{\sin 15}$ ≈ 193 (1)

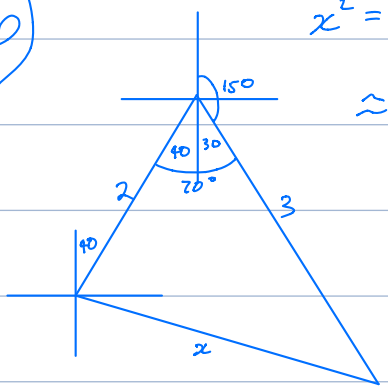
$\sin 45 = \dfrac{x}{\frac{100\sin 30}{\sin 15}}$

$x = (\sin 45)\dfrac{100\sin 30}{\sin 15}$

$\approx 136.6\,m$ ✓✓

(4) a) (D) ✓

b)



$x^2 = 2^2 + 3^2 - 2(2)(3)\cos 70$

$\approx 3.0\,m$ ✓

c) $y = 2x^3 - 3x^2 - 12x + 10$

$y' = 6x^2 - 6x - 12$

let $y' = 0$

$0 = 6x^2 - 6x - 12$

$= 6(x^2 - x - 2)$

$= 6(x+1)(x-2)$ (1)

$\therefore x+1 = 0$ or $x-2 = 0$

$x = -1$ $x = 2$ ✓✓

d) (1) or $-(x^2 - 4x)$

$f(x) = x^2 - 4x$

$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - 4(x+h) - x^2 + 4x}{h}$

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 4x - 4h - x^2 + 4x}{h}$

$= \lim_{h\to 0} \frac{2xh + h^2 - 4h}{h}$

$= \lim_{h\to 0} \frac{h(2x + h - 4)}{h}$

$= \lim_{h\to 0} 2x + h - 4$ ✓✓

$= 2x - 4$

e) Assume $\log_2 5$ is rational.

$\log_2 5 = \frac{m}{n}$

where m & n are integers, $n \neq 0$ and have no common factors.

$2^{\frac{m}{n}} = 5$

$2^m = 5^n$ (1)

As 2 is even so is 2^m

As 5 is odd so is 5^n

$\therefore 2^m \neq 5^n$ ✓✓

This is a contradiction so the original assumption is wrong.

$\therefore \log_2 5$ is irrational

2 marks for saying one is even and one is odd $\therefore$ can't be true