



**Year 11 Preliminary Mathematics Advanced  
Assessment Task 2  
Term 3, Week 3, 2021**

Name: \_\_\_\_\_

Teacher: \_\_\_\_\_

**Wednesday 28<sup>th</sup> July 2021**

**Setter: VN**

- All answers are to be written on your own lined paper so they can be scanned and uploaded as a single PDF file to your teacher's google classroom. Make sure that you write your name on each of the lined pages used.
- **Start each question on a new page.**
- Board approved calculators are permitted in this task.
- Marks may be deducted for insufficient or illegible work.
- For any multiple-choice question, **write the letter** that corresponds to your answer on your writing paper.
- Total possible mark: 32
- Time Allowed: **55 minutes inclusive of task downloading then answering on your own lined paper, scanning and uploading a single PDF file for your responses to your teacher's google classroom.**

| <b>Question 1 (5 marks)</b>                                                            | <b>Start a new page</b> | <b>Marks</b> |
|----------------------------------------------------------------------------------------|-------------------------|--------------|
| (a) Which expression is equivalent to $\cot x + \tan x$ ?                              |                         | <b>1</b>     |
| A. 1      B. $\operatorname{cosec} x \sec x$ C. $\sec x + \operatorname{cosec} x$ D. 2 |                         |              |
| (b) Differentiate $y = x^3 - 2x^2 - 5$ with respect to $x$ .                           |                         | <b>1</b>     |
| (c) Find the exact value of $\sin 300^\circ$ .                                         |                         | <b>1</b>     |
| (d) Given $\sqrt[3]{a} = b^5$ , make $x$ the subject of the formula.                   |                         | <b>2</b>     |

**Question 2 (5 marks)****Marks**

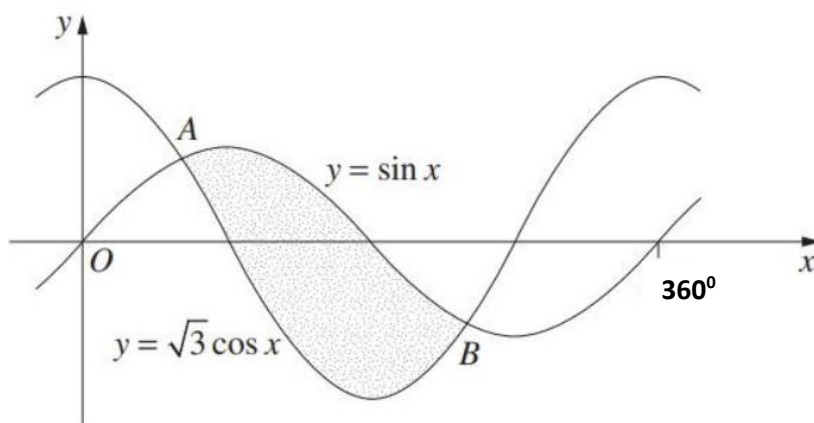
- (a) Which of the following is equal to  $\frac{\log_2 9}{\log_2 3}$ ? 1

A. 2                      B. 3                      C.  $\log_2 3$                       D.  $\log_2 6$

- (b) The graph below shows the curves  $y = \sin x$  and  $y = \sqrt{3} \cos x$ . 2

The first two points of intersection to the right of the  $y$  axis are labelled **A** and **B**.

Solve the equation  $\sin x = \sqrt{3} \cos x$  to find the  $x$ -coordinates of **A** and **B**.



- (c) Solve  $\log_2 x + \log_2(x+7) = 3$ . 2

**Question 3 (5 marks)**      [Start a new page](#)

- (a) What is the derivative of  $\frac{2}{3x^4}$ ? 1

A.  $-\frac{8}{3x^3}$                       B.  $-\frac{8}{3x^5}$                       C.  $\frac{8}{3x^3}$                       D.  $\frac{8}{3x^5}$

- (b) Differentiate with respect to  $x$ , leaving your answer fully factorised and in simplest positive integer or surd form, where appropriate.

(i)  $y = (3x+2)(x-1)^3$  2

(ii)  $f(x) = \frac{x^3-3}{2x+5}$ . 2

**Question 4 (5 marks)****Start a new page****Marks**

(a) Which interval gives the range of the function  $y = 5 + 2 \cos 3x$ ?

**1**

A. [2,8]

B. [3,7]

C. [4,6]

D. [5,9]

(b) Sketch the curve  $y = \log_2(x+1)$  clearly showing any intercepts and asymptotes.

**2**

(c) Write  $\frac{128}{2^n \times 64^{1-2n}}$  in the form  $2^{a+bn}$ , where  $a$  and  $b$  are integers.

**2****Question 5 (6 marks)****Start a new page**

(a) Given that  $\frac{\sec x}{\operatorname{cosec} x} = \tan x$ , solve,  $3 \sec^2\left(\frac{x}{2}\right) = \operatorname{cosec}^2\left(\frac{x}{2}\right)$  for  $0 \leq x \leq 360^\circ$ .

**2**

(b) Find the equation of the tangent to the parabola  $y = 4x - x^2$  at the point where the gradient is  $-2$ .

**2**

(c) For the function  $f(x) = x^2 + 2$ , show that  $f'(x) = 2x$  by differentiating from first principles.

**2**

**Question 6 (6 marks). Start a new page**

**Marks**

- (a) Given that  $x^2 + y^2 = 23xy$ , express  $\left(\frac{x+y}{5}\right)^2$  in terms of  $xy$ . **3**

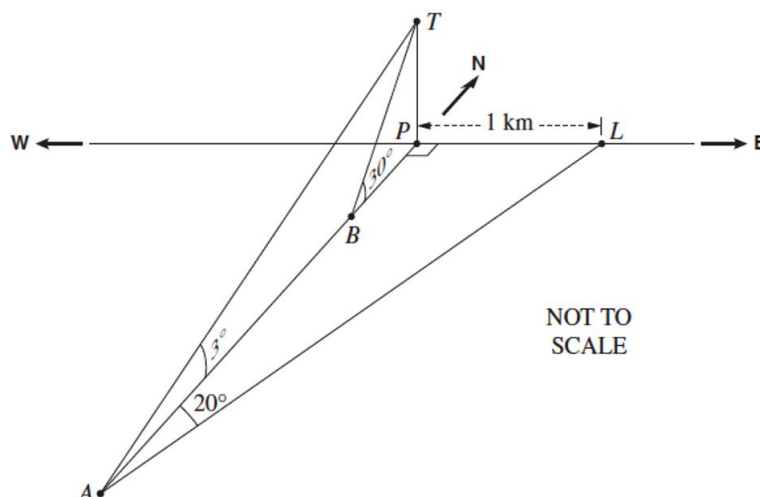
Hence or otherwise show that  $\log\left(\frac{1}{5}(x+y)\right) = \frac{1}{2}(\log x + \log y)$ .

- (b) A boat is sailing due north from a point **A** towards a point **P** on the shore line. The shore line runs from west to east.

In the diagram, **T** represents a tree on a cliff vertically above **P**, and **L** represents a landmark on the shore. The distance **PL** is **1km**.

From **A** the point **L** is on a bearing of  $020^\circ$ , and the angle of elevation to **T** is  $3^\circ$ .

After sailing for some time the boat reaches point **B**, from which the angle of elevation to **T** is  $30^\circ$ .



Show that  $BP = \frac{\sqrt{3} \tan 3^\circ}{\tan 20^\circ}$  kilometres. **3**

**End of Assessment**



# 2021 Year 11 Mathematics Advanced Task 2 SOLUTIONS

| Suggested Solution (s)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Comments |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|
| <p><b>Question 1</b></p> <p>a) <math>\cot x + \tan x</math><br/> <math>= \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}</math><br/> <math>= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}</math><br/> <math>= \frac{1}{\sin x \cos x}</math><br/> <math>= \operatorname{cosec} x \sec x</math><br/> <math>\therefore \mathbf{B}</math> (1 mark)</p> <p>b) <math>y' = 3x^2 - 4x</math> (1 mark)</p> <p>c) <math>y = \sin 300^\circ</math><br/> <math>= -\sin 60^\circ</math><br/> <math>= -\frac{\sqrt{3}}{2}</math> (1 mark)</p> <p>d) <math>\sqrt[x]{a} = b^5</math> OR <math>\frac{1}{x} = \log_a b^5</math> using the definition of a logarithm (1 mark)</p> <p><math>a^{\frac{1}{x}} = b^5</math> <math>x = \frac{1}{\log_a b^5}</math> or <math>x = \frac{1}{5 \log_a b}</math> (1 mark)</p> <p><math>\log a^{\frac{1}{x}} = \log b^5</math> (1 mark)</p> <p><math>\frac{1}{x} \log a = 5 \log b</math></p> <p><math>x = \frac{\log a}{5 \log b}</math> (1 mark)</p> <p><b>Question 2</b></p> <p>a) <math>\frac{\log_2 9}{\log_2 3}</math><br/> <math>= \frac{2 \log_2 3}{\log_2 3}</math><br/> <math>= 2 \therefore \mathbf{A}</math> (1 mark)</p> |          |



# 2021 Year 11 Mathematics Advanced Task 2 SOLUTIONS

| Suggested Solution (s)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Comments                                                                      |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------|
| <p>b) <math>\sin x = \sqrt{3} \cos x</math><br/> <math>\frac{\sin x}{\cos x} = \sqrt{3}</math><br/> <math>\tan x = \sqrt{3}</math> (1 mark)<br/> <math>\therefore x = 60^\circ</math> or <math>x = 240^\circ</math><br/> The x coordinates of the points A and B are <math>x = 60^\circ</math> and <math>x = 240^\circ</math> respectively. (1 mark)</p> <p>c) <math>\log_2 x + \log_2 (x+7) = 3</math><br/> <math>\log_2 x(x+7) = 3 \log_2 2</math><br/> <math>\log_2 x(x+7) = \log_2 2^3</math> (1 mark)<br/> <math>x(x+7) = 2^3</math><br/> <math>x^2 + 7x - 8 = 0</math><br/> <math>(x+8)(x-1) = 0</math><br/> <math>x = -8</math> or <math>x = 1</math><br/> But x cannot be negative (see step 1) hence <math>x = 1</math> (1 mark)</p> <p><b>Question 3</b></p> <p>a) <math>\frac{2}{3x^4}</math><br/> The derivative is: <math>\frac{2}{3} \times -4x^{-5} = -\frac{8}{3}x^{-5} = -\frac{8}{3x^5}</math><br/> The answer is B (1 mark)</p> <p>b)<br/> <math>y' = (x-1)^3 \frac{d}{dx}(3x+2) + (3x+2) \frac{d}{dx}((x-1)^3)</math><br/> <math>y' = 3(x-1)^3 + 3(x-1)^2(3x+2)</math> (1 mark)<br/> <math>y' = 3(x-1)^2((x-1) + (3x+2))</math><br/> <math>y' = 3(x-1)^2(4x+1)</math> (1 mark)</p> | <p><math>x_A = \frac{\pi}{3}, x_B = \frac{4\pi}{3}</math><br/> acceptable</p> |



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c)

$$f'(x) = \frac{(2x+5) \times 3x^2 - 2(x^3-3)}{(2x+5)^2}$$

(1 mark)

$$f'(x) = \frac{6x^3 + 15x^2 - 2x^3 + 6}{(2x+5)^2}$$

$$f'(x) = \frac{4x^3 + 15x^2 + 6}{(2x+5)^2}$$

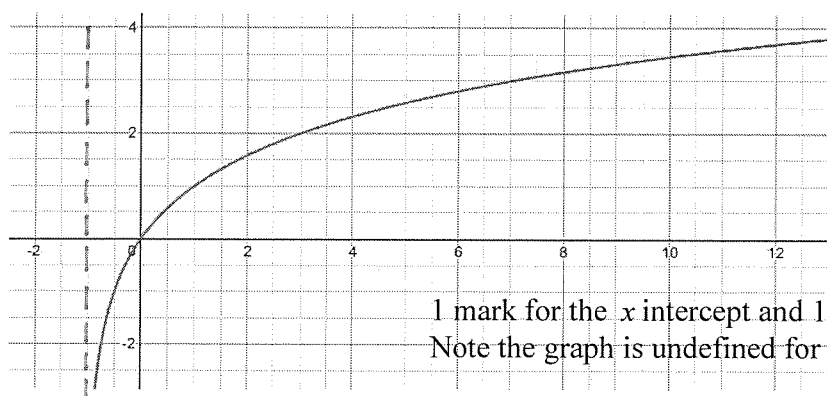
(1 mark)

### Question 4

a) B

(1 mark)

b)



1 mark for the  $x$  intercept and 1 mark for the shape  
Note the graph is undefined for  $x \leq -1$

$$\text{c) } \frac{128}{2^n \times 64^{1-2n}}$$

$$= \frac{2^7}{2^n \times (2^6)^{1-2n}}$$

$$= \frac{2^7}{2^n \times 2^{6-12n}}$$

$$= \frac{2^7}{2^{6-11n}}$$

$$= 2^{7-(6-11n)}$$

$$= 2^{1+11n}$$

(1 mark)

(1 mark)



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### Question 5

a)  $3 \sec^2\left(\frac{x}{2}\right) = \operatorname{cosec}^2\left(\frac{x}{2}\right)$  for  $0 \leq \frac{x}{2} \leq 180^\circ$

$$\frac{\sec^2\left(\frac{x}{2}\right)}{\operatorname{cosec}^2\left(\frac{x}{2}\right)} = \frac{1}{3}$$

$$\left(\frac{\sec\left(\frac{x}{2}\right)}{\operatorname{cosec}\left(\frac{x}{2}\right)}\right)^2 = \frac{1}{3}$$

$$\frac{\sec\left(\frac{x}{2}\right)}{\operatorname{cosec}\left(\frac{x}{2}\right)} = \pm \frac{1}{\sqrt{3}}$$

(1 mark to get to this point)

$$\tan\left(\frac{x}{2}\right) = \pm \frac{1}{\sqrt{3}}$$

$$\therefore \frac{x}{2} = 30^\circ, 150^\circ$$

This means  $\therefore x = 60^\circ$  or  $300^\circ$

(1 mark for the correct answer)

*Radian Measure acceptable* and so

$$\therefore \frac{x}{2} = \frac{\pi}{6}, \frac{5\pi}{6}$$

This means  $x = \frac{\pi}{3}$  or  $\frac{5\pi}{3}$ .

b)  $y' = 4 - 2x$

when  $y' = -2$

$$4 - 2x = -2$$

$$x = 3$$

When  $x = 3$ ,  $y = 3$

(1 mark for the correct  $x$  and  $y$  value)

$$y - 3 = -2(x - 3)$$

$y = -2x + 9$ , which is the equation of the tangent.

**tangent)**

(1 mark for the equation of the



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c)

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 2 - (x^2 + 2)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 2 - x^2 - 2}{h} \quad (1 \text{ mark for this step and 1 mark to get to the penultimate step})$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} 2x + h$$

$$f'(x) = 2x$$

### Question 6

a)

$$\left(\frac{x+y}{5}\right)^2 = \frac{x^2 + 2xy + y^2}{25}$$

$$\left(\frac{x+y}{5}\right)^2 = \frac{23xy + 2xy}{25}$$

$$\left(\frac{x+y}{5}\right)^2 = xy \quad (1 \text{ mark})$$

$$\therefore \log\left(\frac{1}{5}(x+y)\right)^2 = \log(xy) \quad (1 \text{ mark})$$

$$2\log\left(\frac{1}{5}(x+y)\right) = \log x + \log y$$

$$\log\left(\frac{1}{5}(x+y)\right) = \frac{1}{2}(\log x + \log y) \quad (1 \text{ mark})$$

b) In right triangle *APL*

$$\tan 20^\circ = \frac{1}{AP}$$

$$AP = \frac{1}{\tan 20^\circ} \quad (1 \text{ mark})$$



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In right triangle *APT*

$$\tan 3^\circ = \frac{TP}{AP}$$

$$\therefore \tan 3^\circ = TP \div \frac{1}{\tan 20^\circ}$$

$$\frac{\tan 3^\circ}{\tan 20^\circ} = TP$$

(1 mark)

Finally, in right triangle *BPT*

$$\tan 30^\circ = \frac{TP}{BP}$$

$$\therefore BP = \frac{TP}{\tan 30^\circ}$$

$$BP = \frac{\tan 3^\circ}{\tan 20^\circ} \div \frac{1}{\sqrt{3}}$$

$$BP = \frac{\sqrt{3} \tan 3^\circ}{\tan 20^\circ}$$

(1 mark)