



**Year 11 Mathematics Advanced
Preliminary HSC Assessment Task 2
Term 3, Week 3, 2022**

Name: _____

Wednesday 3rd August 2022

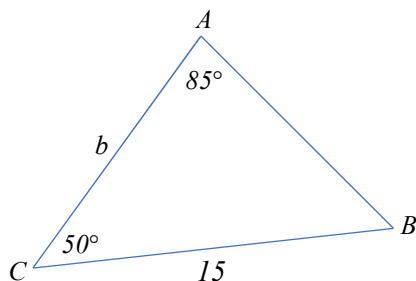
- Answer all questions on the lined paper provided.
- **Start each question on a new page.**
- NESA approved calculators are permitted in this task.
- Marks may be deducted for insufficient or illegible work.
- Total possible mark: **32**
- Time allowed: **45 minutes + 2 minutes reading time**

	Marks
Question 1: (5 marks)	
(a) The value of the expression, $\log_2 8 - -2 $ is,	1
(A) 1	
(B) 5	
(C) 2	
(D) 14	
(b) Solve the equation, $ 3 - 2x = 8$, for x .	2
(c) Find the exact value of $\sec 150^\circ$.	2

Question 2: (5 marks) – *Start a new page*

- (a) Consider the triangle ABC below. The expression which gives the value of b is,

1



- (A) $\frac{15 \sin 50^\circ}{\sin 85^\circ}$ (B) $\frac{15 \sin 85^\circ}{\sin 50^\circ}$
 (C) $\frac{15 \sin 45^\circ}{\sin 85^\circ}$ (D) $\frac{15 \sin 85^\circ}{\sin 45^\circ}$

- (b) A tangent is drawn to the graph of the function $g(x) = \frac{3x}{1-x}$ at the point $P\left(-1, -\frac{3}{2}\right)$.

- (i) Find $g'(x)$. 2
 (ii) Find the equation of the tangent. 2

Question 3: (5 marks) – *Start a new page*

- (a) When evaluated, $\log_3 1234$, lies between which pair of consecutive integers ? 1

- (A) 7 and 8 (B) 6 and 7
 (C) 3 and 4 (D) 411 and 412

- (b) Find the derivative of $y = 5x^3 + \sqrt{x}$ 1

- (c) Given that $\frac{d}{dx}\left(x^2(2x+1)^5\right) = ax(2x+1)^4(bx+1)$, find a and b . 3

Question 4: (5 marks) – *Start a new page*

- (a) Consider the equation $\sin^2 \theta = \frac{2}{9}$. Given that $0^\circ \leq \theta \leq 360^\circ$, the number of solutions is, 1

(A) 0 (B) 1

(C) 2 (D) 4

- (b) (i) Show that $\cot^2 \theta \sin \theta + \sin \theta = \operatorname{cosec} \theta$. 2

- (ii) Hence, or otherwise, solve, 2

$$2 \cot^2 \theta \sin \theta + 2 \sin \theta = -4$$

for all values of θ in the domain $[-180^\circ, 180^\circ]$.

Question 5: (6 marks) – *Start a new page*

- (a) Given that $\cos \theta = -\frac{1}{3}$ and that $\sin \theta > 0$, find the exact value of $\cot \theta$. 2

- (b) A student was asked to differentiate $f(x) = x - x^2$ from first principles. 2

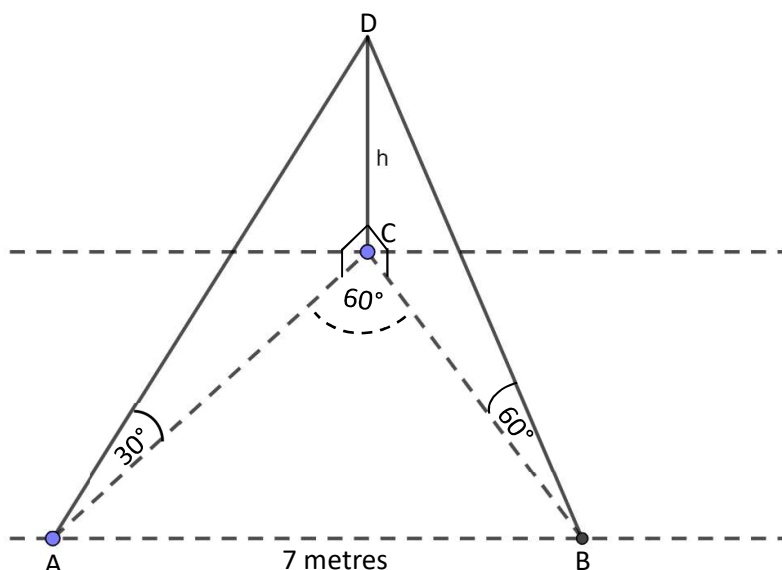
The first line of the student's working was, $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$.

Complete the solution.

- (c) If $\log_5 4 = a$ and $\log_5 10 = b$, express $\log_5 20$ in terms of a and b 2

Question 6: (6 marks) – *Start a new page*

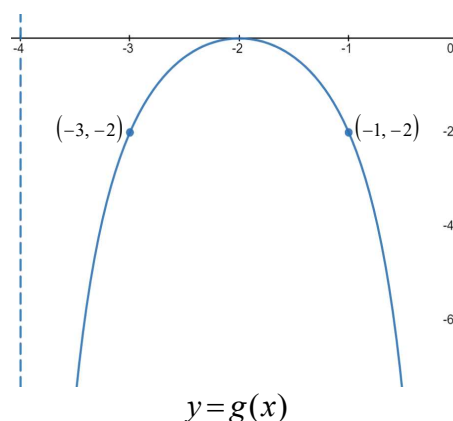
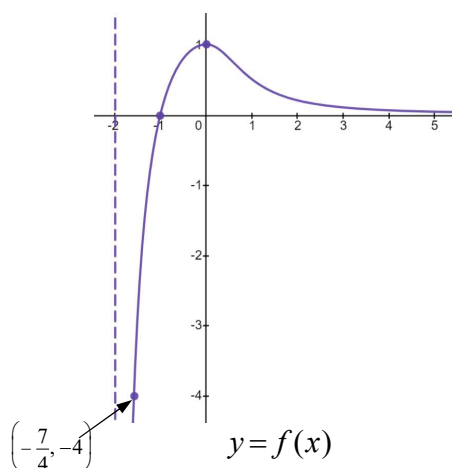
- (a) A road on horizontal ground has two parallel edges.



CD is a vertical flagpole of height h metres which stands with its base C on one edge of the road.

A and B are two points on the other edge of the road such that $AB = 7m$ and $\angle ACB = 60^\circ$. From A and B , the angles of elevation of the top of the flagpole, D , are 30° and 60° respectively.

- (i) Find an expression for AC in terms of h . 1
- (ii) Hence, find the exact height of the flagpole. 3
- (b) Use the graphs of $y = f(x)$ and $y = g(x)$ shown below to determine the domain and range of $g(f(x))$. 2



End of Task



2022 Year 11 Mathematics Advanced Assessment Task 2 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
<u>Question 1:</u> (a) A (b) $3 - 2x = 8$ $\therefore 3 - 2x = 8$ or $3 - 2x = -8$ ✓ $-2x = 5$ $-2x = -11$ $x = -\frac{5}{2}$ $x = \frac{11}{2}$ ✓ (c) $\sec 150^\circ = -\sec 30^\circ$ $= -\frac{1}{\cos 30^\circ}$ $= -\frac{2}{\sqrt{3}}$ Aw1 • an exact answer which is negative • evidence of the equivalence between 150° & 30° • evidence student understands that $\sec \theta = \frac{1}{\cos \theta}$ • $\frac{2}{\sqrt{3}}$ Aw2 • $-\frac{2}{\sqrt{3}}$ AwD • $-1.1547...$	1 for 2 correct equations 2 for both answers	<u>Question 2:</u> (a) C (b) (i) $g(x) = \frac{3x}{1-x}$ $\therefore g'(x) = \frac{(1-x)3 - 3x(-1)}{(1-x)^2}$ ✓ $= \frac{3 - 3x + 3x}{(1-x)^2}$ $= \frac{3}{(1-x)^2}$ ✓ (ii) $g'(-1) = \frac{3}{4}$ ✓ $\therefore y + \frac{3}{2} = \frac{3}{4}(x+1)$ $4y + 6 = 3x + 3$ $3x - 4y - 3 = 0$ } ✓ (or $y = \frac{3}{4}x - \frac{3}{4}$)	

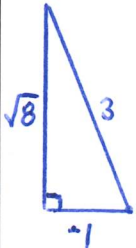


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<u>Question 3:</u> (a) B (b) $y = 5x^3 + \sqrt{x}$ $= 5x^3 + x^{\frac{1}{2}}$ $\therefore y' = 15x^2 + \frac{1}{2}x^{-\frac{1}{2}}$ ✓ (c) $\frac{d}{dx} [x^2(2x+1)^5]$ $= x^2 \cdot 5(2x+1)^4 \cdot 2 + (2x+1)^5 \cdot 2x$ ✓ $= 10x^2(2x+1)^4 + 2x(2x+1)^5$ $= 2x(2x+1)^4 [5x + 2x + 1]$ $= 2x(2x+1)^4 (7x+1)$ ✓ $\therefore a = 2, b = 7$ ✓		<u>Question 4:</u> (a) D (b) LHS = $\cot^2 \theta \sin \theta + \sin \theta$ $= \sin \theta (1 + \cot^2 \theta)$ $= \sin \theta \cdot \operatorname{cosec}^2 \theta$ ✓ $= \frac{1}{\operatorname{cosec} \theta} \cdot \operatorname{cosec}^2 \theta$ $= \operatorname{cosec} \theta$ ✓ $= \text{RHS}$ ✓ (ii) $2\cot^2 \sin \theta + 2\sin \theta = -4$ $2(\cot^2 \sin \theta + \sin \theta) = -4$ from (i) $2\operatorname{cosec} \theta = -4$ ✓ $\operatorname{cosec} \theta = -2$ $\sin \theta = -\frac{1}{2}$ $\therefore \theta = -30^\circ, -150^\circ$ ✓	Evidence of $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$ Ans 2: Correct solution with detail



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<u>Question 5</u> (a) $\cos \theta = -\frac{1}{3}$ $\therefore \tan \theta = -\sqrt{8}$ $\cot \theta = -\frac{1}{\sqrt{8}}$  (b) $f(x+h)$ $= x+h - (x+h)^2$ $= x+h - x^2 - 2xh - h^2$ $\therefore f'(x)$ $= \lim_{h \rightarrow 0} \frac{x+h - x^2 - 2xh - h^2 - x + x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h - 2xh - h^2}{h}$ $= \lim_{h \rightarrow 0} 1 - 2x - h$ $= 1 - 2x$ (c) $\log_5 20$ $= \log_5 2 + \log_5 10$ $= \log_5 4^{\frac{1}{2}} + \log_5 10 \checkmark$ $= \frac{1}{2} \log_5 4 + \log_5 10$ $= \frac{1}{2} a + b \checkmark$	Aw 1 for calc of $\sqrt{8}$ 1 for expr'n involving $\log_5 4$ & $\log_5 10$ 2 for answer (Aw 2 for bald answer)	<u>Question 6 :</u> (a) (i) $\tan 30^\circ = \frac{h}{AC}$ $AC = \frac{h}{\tan 30}$ $= \sqrt{3}h \checkmark$ (ii) Similarly, $BC = \frac{h}{\tan 60}$ $= \frac{h}{\sqrt{3}} \checkmark$ $\therefore 7^2 = (\sqrt{3}h)^2 + \left(\frac{h}{\sqrt{3}}\right)^2 - 2 \cdot \sqrt{3}h \cdot \frac{h}{\sqrt{3}} \cos 60^\circ \checkmark$ $49 = 3h^2 + \frac{h^2}{3} - h^2$ $147 = 9h^2 + h^2 - 3h^2$ $7h^2 = 147$ $h^2 = 21$ $h = \sqrt{21} \text{ m} \checkmark$ (b) Domain: $-\frac{7}{4} < x < -1 \checkmark$ Range: $y \leq 0 \checkmark$	R/W