Year 11 Mathematics Advanced
Assessment Task 2
Term 3 Week 3, 2023

Name: ______________________________

Teacher: ______________________________

Setter: EKA

Wednesday, 2nd August 2023

- Answer Questions in the space provided.
- Answer multiple choice questions on the question paper by circling the correct alternative.
- Show all relevant mathematical reasoning and/or calculations.
- Use blue or black PEN only.
- NESA approved calculators are permitted in this task.
- Marks may be deducted for insufficient or illegible work.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Total possible marks: **32**
- Time allowed**: 45 minutes plus 2 minutes Reading Time**

Q1	Q2	Q3	Q4	Q5	Q6	Q7	TOTAL
/6	/6	/4	/4	/4	/4	/4	/32

Question 1 (6 marks) **Marks**

(a) Circle your correct alternative. **1**

The graph of the function $f(x) = 18$ intersects with the graph of $g(x) = 2^x$ where:

(A) $x = \log_2 18$

(B) $x = \dfrac{1}{18}$

(C) $x = 9$

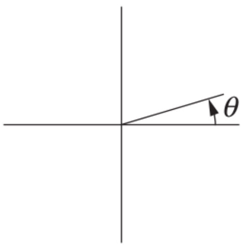
(D) $x = \log_{18} 2$

(b) Circle your correct alternative. **1**

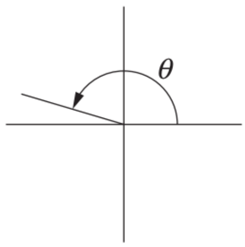
For the angle θ, $\sin\theta = \dfrac{7}{25}$ and $\cos\theta = -\dfrac{24}{25}$.

Which diagram best shows the angle θ?

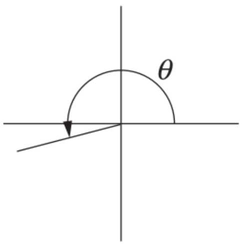
(A)



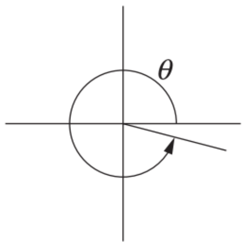
(B)



(C)



(D)



(c) Solve $\cos\theta = 0.63$ for $-180° \leq \theta \leq 0°$, giving your answer to the nearest degree. **1**

Question 1 Continued

(d) Find the derivative of $y = \frac{3x^4 - 1}{x}$. **1**

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(e) Given that $f(x) = x^2 + 1$ and $g(x) = \sqrt{x - 3}$. Find $y = f\big(g(x)\big)$ and state its domain. **2**

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End of Question 1

Question 2 (6 marks) **Marks**

(a) Circle your correct alternative. **1**

What is the exact value of $\cos 135° + \operatorname{cosec} 60°$?

(A) $\dfrac{2\sqrt{2}-\sqrt{3}}{\sqrt{6}}$

(B) $\dfrac{2\sqrt{2}-1}{\sqrt{2}}$

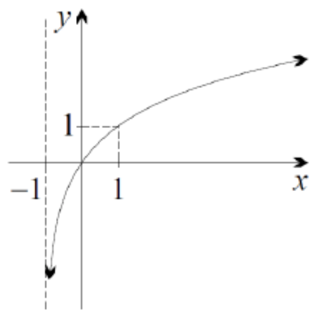
(C) $\dfrac{2\sqrt{2}+\sqrt{3}}{\sqrt{6}}$

(D) $\dfrac{2\sqrt{2}+1}{\sqrt{2}}$

(b) Circle your correct alternative. **1**

Which of the following is the graph of $y = -\log_2(x+1)$?

(A)



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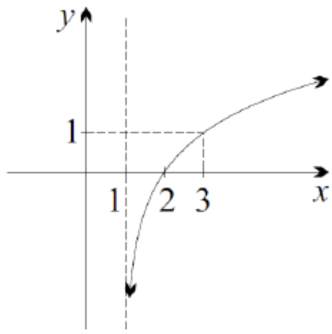

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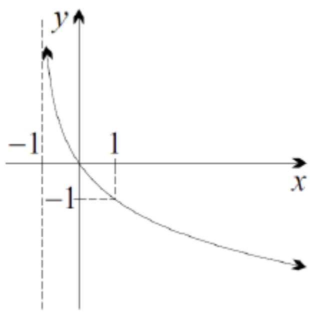
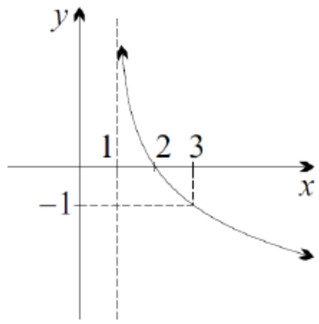


(D)



Question 2 Continued

(c) Solve the equation $|2x - 3| = 6$. **2**

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(d) Differentiate with respect to x. Leave your answer in fully factorised form. **2**

$$y = \frac{2x^2}{2 - x}$$

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End of Question 2

Question 3 (4 marks) **Marks**

(a) Solve $\sin\left(\frac{x}{2}\right) = \frac{1}{2}$ for $0° \leq x \leq 360°$. **2**

(b) Show that $\cot A + \frac{\sin A}{1+\cos A} = \operatorname{cosec} A$. **2**

End of Question 3

Question 4 (4 marks) **Marks**

(a) i) Show that the derivative of $f(x) = x^2 + 3x + 5$ is $f'(x) = 2x + 3$ by **first principles.** **2**

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ii) Hence, find the equation of the normal to the curve $f(x) = x^2 + 3x + 5$ at the point $(1, 9)$. **2**

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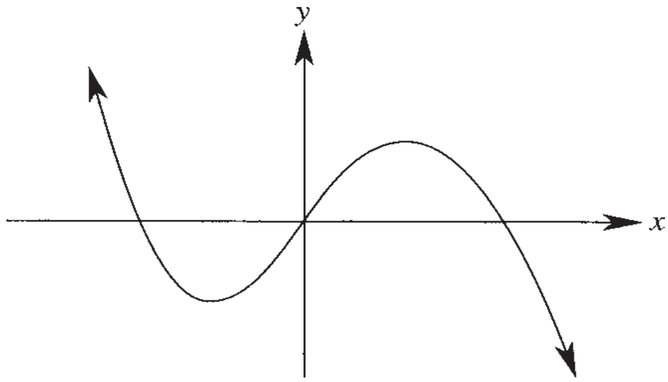
End of Question 4

Question 5 (4 marks) **Marks**

(a) The diagram below shows a sketch of the function $y = f(x)$.
On the same diagram sketch the gradient function $y = f'(x)$. **1**



(b) Show that, $\frac{d}{dx}[\, 3x^2(4x-1)^3] = 6x(10x-1)(4x-1)^2$. **3**

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End of Question 5

Question 6 (4 marks) **Marks**

(a) Differentiate with respect to x. **2**

$$y = \sqrt{x^2 - 2x}$$

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(b) Solve $(\log_2 x)^2 - 8 = \log_2 x^2$. **2**

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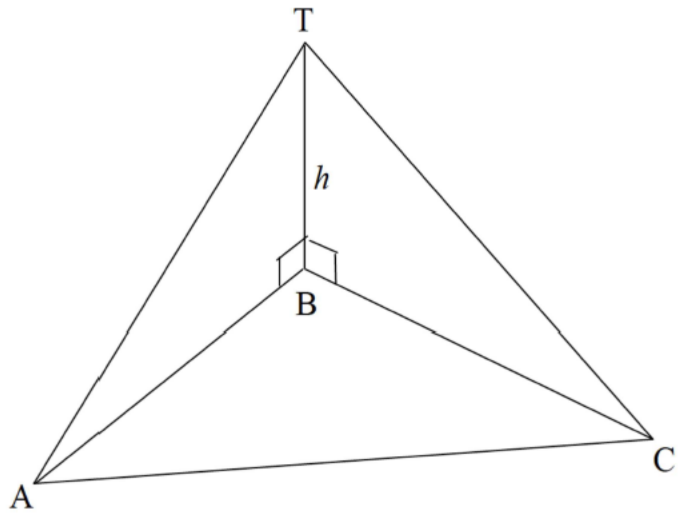
End of Question 6

Question 7 (4 marks) **Marks**

The angle of elevation of a vertical tower BT of height h metres, at a point A due west of it, is $14°$. From the point C, the bearing of the tower is $343°$ and the angle of elevation is $11°$. The points A and C are $1000m$ apart and on the same level as the base B of the tower.



(a) By drawing a diagram showing the bearings, explain why $\angle ABC = 107°$. **1**

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(b) Show that $AB = h\ \tan 76°$ and find a similar expression for CB. **1**

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(c) Hence, find the height of the tower, correct to the nearest metre. **2**

End of Question 7

End of Assessment Task

Extra writing space

If you are are using this space indicate clearly which question(s) you are answering & write 'see extra writing space' on the original question space provided for that question.

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Name: Solutions

Teacher: ______________________

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Question 1 (6 marks) **Marks**

(a) Circle your correct alternative. **1**

The graph of the function $f(x) = 18$ intersects with the graph of $g(x) = 2^x$ where:

(A) $x = \log_2 18$ (B) $x = \dfrac{1}{18}$

(C) $x = 9$ (D) $x = \log_{18} 2$

$2^x = 18$

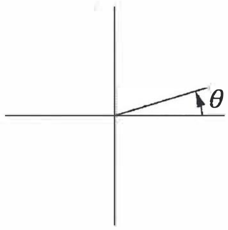
$x = \log_2 18$

(b) Circle your correct alternative.

For the angle θ, $\sin\theta = \dfrac{7}{25}$ and $\cos\theta = -\dfrac{24}{25}$.

Which diagram best shows the angle θ?

(A)



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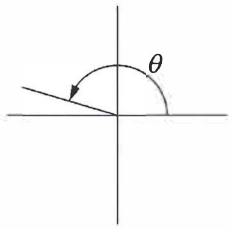
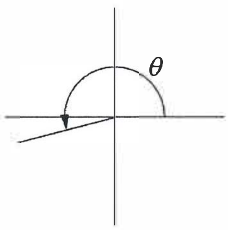
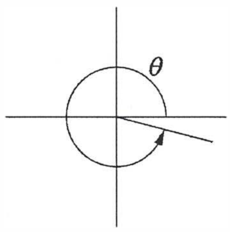


(C)

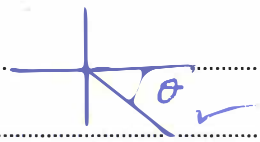


(D)



(c) Solve $\cos\theta = 0.63$ for $-180° \leq \theta \leq 0°$, giving your answer to the nearest degree. **1**

$\cos\theta = 0.63$



ref $\angle = \cos^{-1}(0.63)$

$= 51°$

$\theta = -51°$ ✓

Question 1 Continued

(d) Find the derivative of $y = \frac{3x^4 - 1}{x}$. **1**

$y = 3x^3 - x^{-1}$

$\frac{dy}{dx} = 9x^2 - (-1 \times x^{-2})$

$= 9x^2 + \frac{1}{x^3}$

(e) Given that $f(x) = x^2 + 1$ and $g(x) = \sqrt{x-3}$. Find $y = f(g(x))$ and state its domain. **2**

$f(g(x)) = (\sqrt{x-3})^2 + 1$

$= x - 3 + 1$

$= x - 2$ 1

domain of $g(x) = x \geq 3$

$\therefore$ domain of $f(g(x)) = x \geq 3$

End of Question 1

Question 2 (6 marks) **Marks**

(a) Circle your correct alternative. **1**

What is the exact value of $\cos 135° + cosec\ 60°$?

(A) $\dfrac{2\sqrt{2}-\sqrt{3}}{\sqrt{6}}$

(B) $\dfrac{2\sqrt{2}-1}{\sqrt{2}}$

(C) $\dfrac{2\sqrt{2}+\sqrt{3}}{\sqrt{6}}$

(D) $\dfrac{2\sqrt{2}+1}{\sqrt{2}}$

$\cos 135° + cosec\ 60°$

$= -\cos 45° + \dfrac{1}{\sin 60°}$

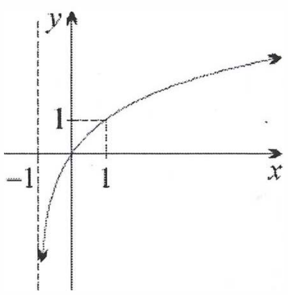
$= -\dfrac{1}{\sqrt{2}} + \dfrac{2}{\sqrt{3}}$

$= \dfrac{-\sqrt{3} + 2\sqrt{2}}{\sqrt{6}}$

(b) Circle your correct alternative. **1**

Which of the following is the graph of $y = -\log_2(x+1)$?

(A)



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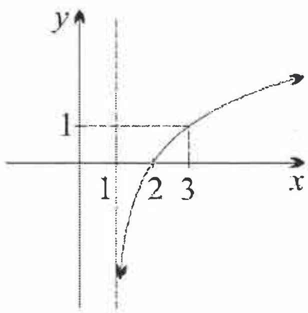

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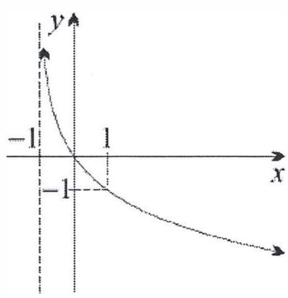
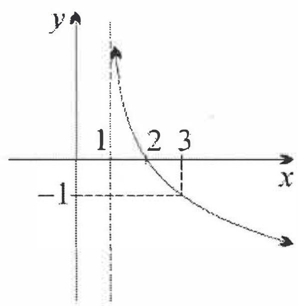


(D)



Question 2 Continued

(c) Solve the equation $|2x - 3| = 6$. **2**

$2x-3 = 6$ or $2x-3 = -6$

$2x = 9$ $2x = -3$

$x = 9/2$ or $x = -3/2$

✓ ✓

(d) Differentiate with respect to x. Leave your answer in fully factorised form. **2**

$$y = \frac{2x^2}{2-x}$$

$u = 2x^2$ $v = 2-x$

$u' = 4x$ $v' = -1$

$y' = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{(2-x) \times 4x - 2x^2(-1)}{(2-x)^2}$ 1

$= \dfrac{8x - 4x^2 + 2x^2}{(2-x)^2}$

$= \dfrac{8x - 2x^2}{(2-x)^2}$

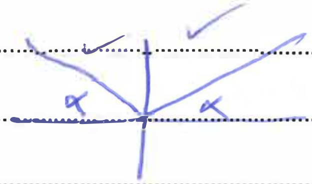
$= \dfrac{2x(4-x)}{(2-x)^2}$ ✓ ✓

End of Question 2

Question 3 (4 marks) **Marks**

(a) Solve $\sin\left(\frac{x}{2}\right) = \frac{1}{2}$ for $0° \leq x \leq 360°$ **2**

$\sin\left(\frac{x}{2}\right) = \frac{1}{2}$



ref ∠ = 30°

$0 \leq x \leq 360°$

$0 \leq \frac{x}{2} \leq 180°$

$\frac{x}{2} = 30°, \ 150°$ 1

$x = 60°, \ 300°$

(b) Show that $\cot A + \frac{\sin A}{1+\cos A} = \operatorname{cosec} A$. **2**

$LHS = \cot A + \frac{\sin A}{1+\cos A}$

$= \frac{\cos A}{\sin A} + \frac{\sin A}{1+\cos A}$

$= \frac{\cos A(1+\cos A) + \sin^2 A}{\sin A(1+\cos A)}$ 1

$= \frac{\cos A + \cos^2 A + \sin^2 A}{\sin A(1+\cos A)}$

$= \frac{\cancel{1+\cos A}}{\sin A\cancel{(1+\cos A)}}$

$= \operatorname{cosec} A$

$= RHS$

or

$= \frac{\cos A}{\sin A} + \frac{\sin A}{1+\cos A} \times \frac{(1-\cos A)}{(1-\cos A)}$

$= \frac{\cos A}{\sin A} + \frac{\sin A(1-\cos A)}{1-\cos^2 A}$ 1

$= \frac{\cos A}{\sin A} + \frac{\cancel{\sin A}(1-\cos A)}{\sin^2 A}$

$= \frac{\cos A + 1 - \cos A}{\sin A}$

$= \frac{1}{\sin A}$

$= \operatorname{cosec} A$

$= RHS$

End of Question 3

Question 4 (4 marks) **Marks**

(a) i) Show that the derivative of $f(x) = x^2 + 3x + 5$ is $f'(x) = 2x + 3$ by **first principles.** **2**

$f(x) = x^2 + 3x + 5$

$f(x+h) = (x+h)^2 + 3(x+h) + 5$

$= x^2 + 2xh + h^2 + 3x + 3h + 5$

$f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

must have on every line

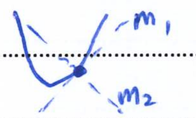
$= \lim_{h \to 0} \dfrac{(x^2 + 2xh + h^2 + 3x + 3h + 5) - (x^2 + 3x + 5)}{h}$ 1

$= \lim_{h \to 0} \dfrac{\cancel{h}(2x + h + 3)}{\cancel{h}}$ ✓✓

$= 2x + 3$ as required

must show 'h' cancels for 2 marks

ii) Hence, find the equation of the normal to the curve $f(x) = x^2 + 3x + 5$ at the point $(1, 9)$. **2**



$f(x) = (2x + 3)$

$f(1) = 5$

$m_1 m_2 = -1$

$m_2 = -\frac{1}{5}$ 1

$y - y_1 = m(x - x_1)$

$y - 9 = -\frac{1}{5}(x - 1)$

$y - 9 = -\frac{1}{5}x + \frac{1}{5}$

$y = -\frac{1}{5}x + 9 + \frac{1}{5}$

$y = -\frac{1}{5}x + \frac{46}{5}$

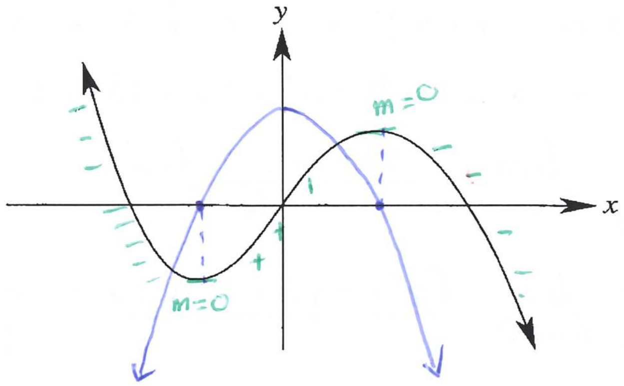
$5y = -x + 46$ ✓✓

$5y + x - 46 = 0$

End of Question 4

Question 5 (4 marks) **Marks**

(a) The diagram below shows a sketch of the function $y = f(x)$. On the same diagram sketch the gradient function $y = f'(x)$. **1**



✓

(b) Show that, $\frac{d}{dx}[3x^2(4x-1)^3] = 6x(10x-1)(4x-1)^2$. **3**

$y = 3x^2(4x-1)^3$

$u = 3x^2 \qquad v = (4x-1)^3$

$u' = 6x \qquad v' = 3(4x-1)^2 \times 4$

$\qquad\qquad = 12(4x-1)^2$

1

$\frac{d}{dx}[3x^2(4x-1)^3] = uv' + vu'$

$= 3x^2 \times 12(4x-1)^2 + (4x-1)^3 \times 6x$ → 1

$= 6x(4x-1)^2(6x + 4x - 1)$

$= 6x(4x-1)^2(10x-1)$ ✓✓

factorising should be there for 3rd mark.

End of Question 5

Question 6 (4 marks) **Marks**

(a) Differentiate with respect to x. **2**

$$y = \sqrt{x^2 - 2x}$$

$y = \sqrt{x^2 - 2x}$

$y = (x^2 - 2x)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(x^2 - 2x)^{-1/2}(2x - 2)$ 1

$= \frac{1}{\cancel{2}\sqrt{x^2 - 2x}} \times \cancel{2}(x - 1)$

$= \frac{x - 1}{\sqrt{x^2 - 2x}}$ ✓✓

or

$(x - 1).(x^2 - 2x)^{-\frac{1}{2}}$

(b) Solve $(\log_2 x)^2 - 8 = \log_2 x^2$. **2**

$(\log_2 x)^2 - 8 = \log_2 x^2$

$(\log_2 x)^2 - 8 - 2\log_2 x = 0$ $\quad u = \log_2 x$

$u^2 - 2u - 8 = 0$

$(u - 4)(u + 2) = 0$ 1

$\log_2 x = 4$ or $\log_2 x = -2$

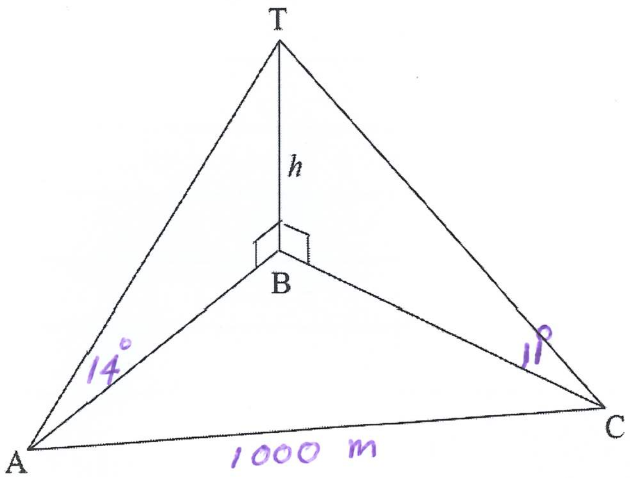
$x = 2^4$ as $x = 2^{-2}$

$x = 2^4$ or $x = \frac{1}{4}$ ✓✓

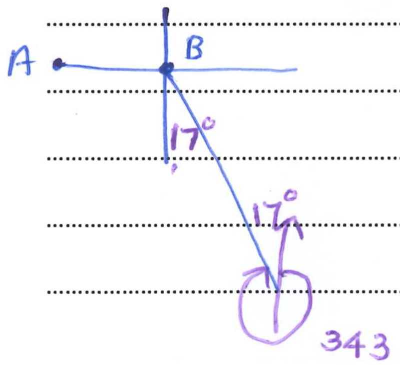
End of Question 6

Question 7 (4 marks) **Marks**

The angle of elevation of a vertical tower BT of height h metres, at a point A due west of it, is 14°. From the point C, the bearing of the tower is 343° and the angle of elevation is 11°. The points A and C are $1000m$ apart and on the same level as the base B of the tower.



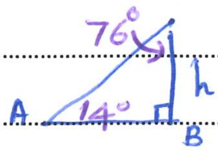
(a) By drawing a diagram showing the bearings, explain why $\angle ABC = 107°$. **1**

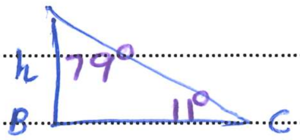


$\angle ABC = 90° + 17°$

$= 107°$

(b) Show that $AB = h\ \tan 76°$ and find a similar expression for CB. **1**

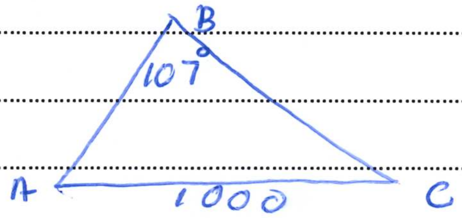




$\tan 76° = \frac{AB}{h}$ $\qquad$ $\tan 79° = \frac{BC}{h}$

$AB = h \tan 76°$ $\qquad$ $CB = h \tan 79°$

(c) Hence, find the height of the tower, correct to the nearest metre. **2**



$1000^2 = AB^2 + BC^2 - 2\,AB \times BC \times \cos 107°$

$= h^2\tan^2 76° + h^2\tan^2 79 - 2h^2\tan 76°\tan 79°\cos 107°$

$= h^2(\tan^2 76° + \tan^2 79 - 2\tan 76° \times \tan 79° \times \cos 107°)$ 1

$h^2 = \dfrac{1000^2}{\tan^2 76° + \tan^2 79° - 2 \times \tan 76° \times \tan 79° \times \cos 107°}$

$h = 135.310$

$h \approx 135\text{ m}$

End of Question 7

End of Assessment Task