Year 11 Mathematics Advanced
Assessment Task 1
Term 2, Week 3, 2024

Name: ______________________________

Teacher: ______________________________

Setter: BRA

Wednesday , 15th May 2024

- Answer Questions in the space provided.
- Answer multiple choice questions on the question paper by circling the correct alternative.
- Show all relevant mathematical reasoning and/or calculations.
- Use blue or black PEN only.
- NESA approved calculators are permitted in this task.
- Your own Formula Sheet can be used with this task.
- Marks may be deducted for insufficient or illegible work.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Total possible marks: **32**
- Time allowed**: 45 minutes plus 2 minutes Reading Time**

Q1	Q2	Q3	Q4	Q5	Q6	Q7	TOTAL
/6	/6	/4	/4	/4	/4	/4	/32

Question 1 (6 marks) **Marks**

(a) Circle your correct alternative. **1**

If $ab = 12, bc = 20$ and $ac = 15$, then a possible value for abc is:

(A) – 600

(B) 60

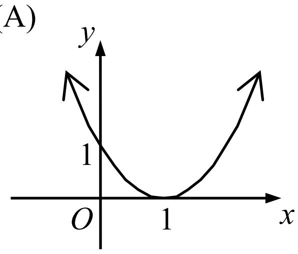
(C) 360

(D) 600

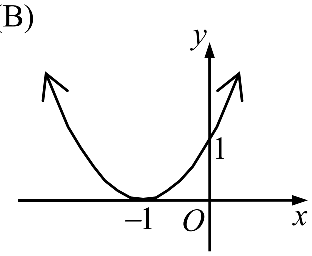
(b) Circle your correct alternative. **1**

Which graph drawn below best represents $y = (x + 1)^2$?

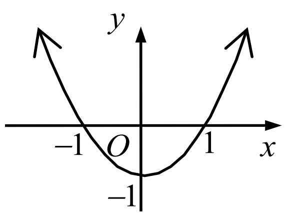
(A)



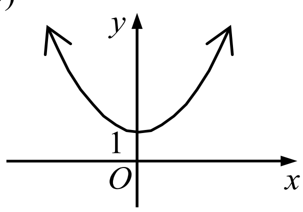
(B)



(C)



(D)



Question 1 Continued

(c) Given $M = \dfrac{m}{\sqrt{1 - \dfrac{v^2}{c^2}}}$, find M if $m = 550$, $v = 2{\cdot}75 \times 10^8$ and $c = 3 \times 10^8$. **2**

Give your answer correct to five significant figures.

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(d) Simplify completely $\dfrac{4a^2b - 8ab^2}{4ab^2 - 8a^2b}$. **2**

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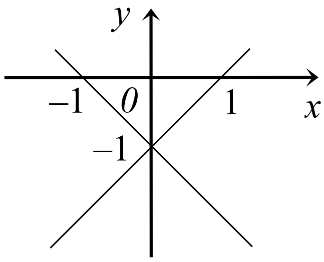
End of Question 1

Question 2 (6 marks) **Marks**

(a) Circle your correct alternative. **1**

The equation $x^2 - (y+1)^2 = 0$ represents a relation between x and y in the xy-plane. Its sketch is shown below.



The relation can be classified as:

(A) One-to-one

(B) One-to-many

(C) Many-to-one

(D) Many-to-many

(b) Circle your correct alternative. **1**

Consider the following two statements:

I. Two irrational zeroes of a quadratic function cannot be equal.

II. If α and β are distinct zeroes of the quadratic function $f(x) = ax^2 + bx + c$, where a, b and c are real constants, then $a\alpha + b + a\beta = 0$.

Which of the above statements are true?

(A) Neither I nor II

(B) I only

(C) II only

(D) Both I and II

Question 2 Continued

(c) Solve $\frac{x-5}{4} = \frac{6}{x}$. **2**

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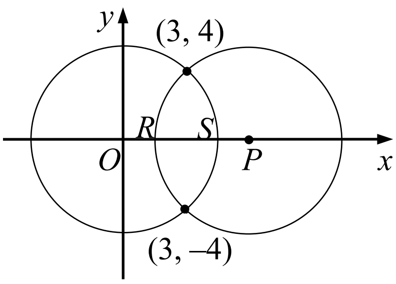
(d) **2**



Not to Scale

In the diagram above, O and P are centres of circles with equal radii, which intersect at (3, 4) and (3, –4). The circles cut the x-axis at R and S as shown.

Find the value of the length RS. Explain your reasoning carefully.

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End of Question 2

Question 3 (4 marks) **Marks**

(a) Sketch a function $f(x)$ that possesses all of the following properties: **3**

1. $f(x)$ is an odd function.
2. $f(x)$ has horizontal asymptotes $y = \pm 2$.
3. $f(x)$ has vertical asymptotes $x = \pm 2$.
4. $f(x)$ has domain $x < -2$ together with $x > 2$ as well as $x = 0$.

(b) Write $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ in the form of $a + b\sqrt{x}$, where a, b and x are integers. **1**

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End of Question 3

Question 4 (4 marks) **Marks**

A function is defined by the formula $f(x)=\dfrac{\sqrt{-x}}{\sqrt{x+1}}$.

(a) Determine the domain of the function. **2**

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(b) Determine the range of the function. **1**

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(c) Given f is a one-to-one function, what is the relationship between x_1 and x_2, if $f(x_1)=f(x_2)$? Explain your reasoning carefully. **1**
You may assume x_1 and x_2 are in the domain of f.

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End of Question 4

Question 5 (4 marks)

(a) If $f(x) = \sqrt{x}$ sketch the graph of $y = f(x-1)$ showing intercepts with the coordinate axes. **1**

(b) $P(1, 2)$ is a point on the line $3x + y - 5 = 0$. Find the coordinates of the points on this line, so that their distance from P is $\sqrt{10}$ units. **3**

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End of Question 5

Question 6 (4 marks)

(a) Simplify completely $\sqrt{54x} \div \sqrt{6}$ **1**

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(b) Factorise fully $a^2 - 2ab + b^2 + 3a - 3b + 2$. **3**

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End of Question 6

Question 7 (4 marks)

(a) How many integers are between $\sqrt{50}$ and $\sqrt{500}$? **1**

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(b) By definition, a positive integer that can be written as the square of a positive integer is called a square number or a perfect square. For example, 9 is a perfect square since $9 = 3^2$. **3**

For x a positive integer, how many positive integers are there such that both x and $x + 99$ are perfect squares? Explain carefully your reasoning.
Hint: Let $x = m^2$ and $x + 99 = n^2$, where m and n are positive integers.

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End of Assessment Task

Extra writing space

If you are are using this space indicate clearly which question(s) you are answering & write ‘see extra writing space’ on the original question space provided for that question.

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Term 2 Week 3, 2024

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Name: SOLUTIONS

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Question 1 (6 marks) **Marks**

(a) Circle your correct alternative. **1**

If $ab = 12, bc = 20$ and $ac = 15$, then a possible value for abc is:

(A) -600

(B) 60

(C) 360

(D) 600

$(ab)(ac) = 12 \times 15$

$\Rightarrow a^2bc = 180$ & $bc = 20$

$\Rightarrow 20a^2 = 180$

$\Rightarrow a^2 = 9$

$\Rightarrow a = \pm 3$

So $abc = a \times (bc)$

$= \pm 3 \times 20$

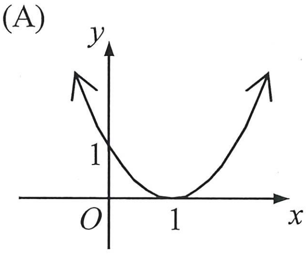
$= \pm 60$

$\therefore$ (B)

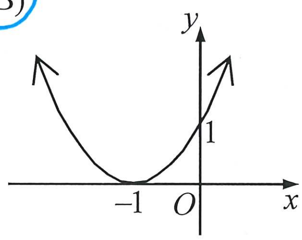
(b) Circle your correct alternative. **1**

Which graph drawn below best represents $y = (x + 1)^2$?

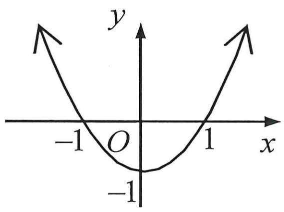
(A)



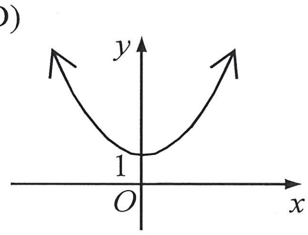
(B)



(C)



(D)



$y = (x+1)^2$ is a translation of $y = x^2$ one unit to the LEFT. $\therefore$ (B).

Question 1 Continued

(c) Given $M = \dfrac{m}{\sqrt{1-\dfrac{v^2}{c^2}}}$, find M if $m = 550$, $v = 2{\cdot}75 \times 10^8$ and $c = 3 \times 10^8$. **2**

Give your answer correct to five significant figures.

$$M = \frac{550}{\sqrt{1-\left(\dfrac{2.75\times10^8}{3\times10^8}\right)^2}}$$

$$= \frac{550}{\sqrt{1-\left(\dfrac{2.75}{3}\right)^2}}$$

N.B. $\dfrac{2.75}{3} = \dfrac{11}{12}$ & so equivalent calculation is $\dfrac{550}{\sqrt{1-\left(\dfrac{11}{12}\right)^2}}$

$= 1376{\cdot}195\ 133\ldots$ (Calculator)

$= 1376{\cdot}2$ (5 s.f.)

✓ for correct calculation

✓ for correct # of sig. figures.

Correct bald answer : 2 MARKS

(d) Simplify completely $\dfrac{4a^2b-8ab^2}{4ab^2-8a^2b}$. **2**

$$= \frac{4ab(a-2b)}{4ab(b-2a)}$$ ✓ 1mark

$$= \frac{a-2b}{b-2a}$$ ✓ 1mark

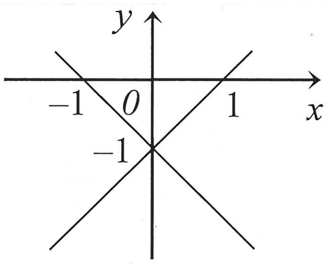
BALD ANSWER : 1 mark only.

End of Question 1

Question 2 (6 marks) **Marks**

(a) Circle your correct alternative. **1**

The equation $x^2-(y+1)^2=0$ represents a relation between x and y in the xy-plane. Its sketch is shown below.



The relation is NOT a function and so (A) & (C) are immediately eliminated. As the relation fails the horizontal line test as well, then it is classified as many-to-many. So (D)

The relation can be classified as:

(A) One-to-one

(B) One-to-many

(C) Many-to-one

(D) Many-to-many

(b) I. is false. Counterexample:
$f(x) = x^2 - 2\sqrt{2}x + 2$, i.e.,
$f(x) = (x-\sqrt{2})^2$ whose zeroes are $\sqrt{2}, \sqrt{2}$ or equivalently $\sqrt{2}(2)$.

II. is true. As α and β are zeroes of $f(x)$, then:–

$a\alpha^2 + b\alpha + c = 0$ &
$a\beta^2 + b\beta + c = 0$

(b) Circle your correct alternative. **1**

Consider the following two statements:

I. Two irrational zeroes of a quadratic function cannot be equal.

II. If α and β are distinct zeroes of the quadratic function $f(x)=ax^2+bx+c$, where a, b and c are real constants, then $a\alpha+b+a\beta=0$.

Subtracting the above equations yields $a(\alpha^2-\beta^2)+b(\alpha-\beta)=0$
$\Rightarrow a(\alpha-\beta)(\alpha+\beta)+b(\alpha-\beta)=0$
$\Rightarrow a(\alpha+\beta)+b=0$ as $\alpha\neq\beta$
$\Rightarrow a\alpha+b+a\beta=0$, as required.
$\therefore$ (C)

Which of the above statements are true?

(A) Neither I nor II

(B) I only

(C) II only

(D) Both I and II

Question 2 Continued

(c) Solve $\frac{x-5}{4} = \frac{6}{x}$. **2**

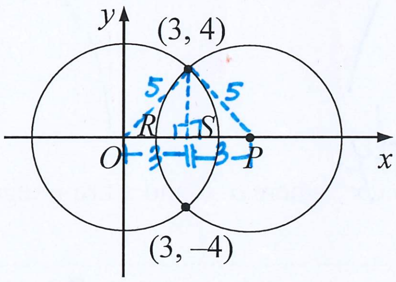
$\Rightarrow x(x-5) = 24$

$\Leftrightarrow x^2 - 5x - 24 = 0$ ✓ correct quadratic equation

$\Leftrightarrow (x-8)(x+3) = 0$

$\Leftrightarrow x = -3$ or 8 ✓ correct answers.

(d) **2**



Not to Scale

In the diagram above, O and P are centres of circles with equal radii, which intersect at (3, 4) and (3, –4). The circles cut the x axis at R and S as shown.

Find the value of the length RS. Explain your reasoning carefully.

Radius of both circles found by a simple application of Pythagoras' theorem or the distance formula applied to O (0,0) & (3,4). Radius = 5 units.
From the same right-angled triangle constructions, it is evident that OP = 3 + 3 or 6 units. ∴ OR = OP − RP = 6 − 5 or 1 unit. Finally, RS = OS − OR

✓ Correct answer = Radius − OR

✓ Valid reasoning. = 5 − 1 or 4 units.

End of Question 2

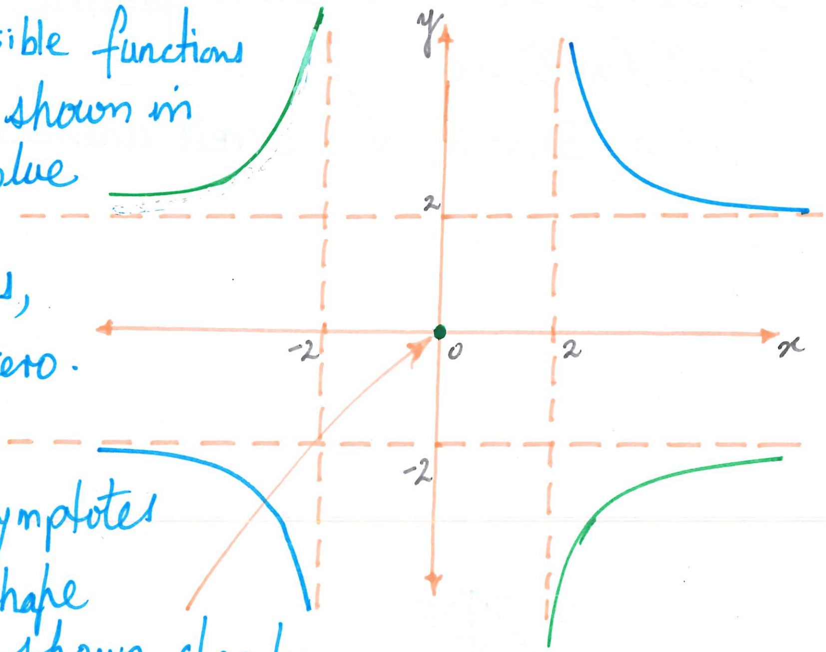
Question 3 (4 marks) **Marks**

(a) Sketch a function $y = f(x)$ that possesses all of the following properties: **3**

1. $f(x)$ is an odd function.
2. $f(x)$ has horizontal asymptotes $y = \pm 2$.
3. $f(x)$ has vertical asymptotes $x = \pm 2$.
4. $f(x)$ has domain $x < -2$ together with $x > 2$ as well as $x = 0$.

Two possible functions available, shown in green & blue

In both cases, $f(0)$ must be zero.



✓ correct asymptotes

✓ correct shape

✓ $f(0) = 0$ shown clearly.

(b) Write $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ in the form of $a + b\sqrt{x}$, where a, b and x are integers. **1**

$$\frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{(\sqrt{2}+1)^2}{1}$$

$$= (\sqrt{2})^2 + 2 \times 1 \times \sqrt{2} + 1^2$$

$$= \underline{\underline{3 + 2\sqrt{2}}} \quad ✓$$

Correct answer.

End of Question 3

Question 4 (4 marks) **Marks**

A function is defined by the formula $f(x) = \dfrac{\sqrt{-x}}{\sqrt{x+1}}$.

(a) Determine the domain of the function. **2**

Require $-x \geq 0$ & $x+1 > 0$ ✓ Correct inequalities

$\Rightarrow x \leq 0$ & $x > -1$ ✓ Correct simplification.

i.e. $-1 < x \leq 0$

(b) Determine the range of the function. **1**

Clearly $f(x)$ is non-negative. Indeed, when $x=0$, $f(0)=0$ and $f \to +\infty$ as $x \to -1^{+}$ (Calculator check is available.)

So range is $y \geq 0$ ✓ Correct answer

(c) Given f is a one-to-one function, what is the relationship between x_1 and x_2, if $f(x_1) = f(x_2)$? Explain your reasoning carefully. **1**

You may assume x_1 and x_2 are in the domain of f.

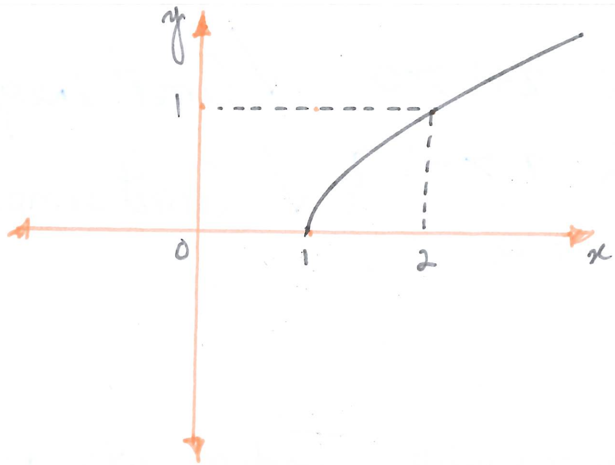
As f is a 1-1 function, then any horizontal line cuts the curve at most once. So if $f(x_1) = f(x_2)$, then $x_1 = x_2$.

✓ for $x_1 = x_2$ with valid reasoning.

End of Question 4

Question 5 (4 marks)

(a) If $f(x)=\sqrt{x}$ sketch the graph of $y=f(x-1)$ showing intercepts with the coordinate axes. **1**



✓ correct shape with x-intercept at $x=1$.

(b) $P(1,\ 2)$ is a point on the line $3x+y-5=0$. Find the coordinates of the points on this line, so that their distance from P is $\sqrt{10}$ units. **3**

Let $Q(x,y)$ be any other point on the line $3x+y-5=0$.
Applying the distance formula:–

$$(x-1)^2+(y-2)^2=(\sqrt{10})^2 \quad [PQ^2=10].$$

$$\Rightarrow (x-1)^2+(y-2)^2=10. \quad \ldots (1)$$

✓ 1 mark

However, as $Q(x,y)$ lies on the line, then $y=5-3x$ (from $3x+y-5=0$). So (1) becomes:–

$$(x-1)^2+(5-3x-2)^2=10$$

$$\Leftrightarrow (x-1)^2+(3-3x)^2=10$$

$$\Leftrightarrow x^2-2x+1+9-18x+9x^2=10$$

$$\Leftrightarrow 10x^2-20x+10=10$$

$$\Leftrightarrow 10x(x-2)=0$$

1 mark

$\Rightarrow$ $x=0$ or $x=2$

$\Rightarrow$ $y=5$ or $y=-1$ respectively.

$\therefore$ required co-ordinates are $(0,5)$ & $(2,-1)$.

✓ correct answers.

End of Question 5

Question 6 (4 marks)

(a) Simplify completely $\sqrt{54x} \div \sqrt{6}$ **1**

$= \dfrac{\sqrt{54x}}{\sqrt{6}}$ or $\sqrt{\dfrac{54x}{6}}$

$= \sqrt{9x}$ or $\underline{3\sqrt{x}}$ ✓ for correct answer

(b) Factorise fully $a^2 - 2ab + b^2 + 3a - 3b + 2$. **3**

1 mark.

$= (a-b)^2 + 3a - 3b + 2$

$= (a-b)^2 + 3(a-b) + 2$

Let $X = a - b$. Algebra is equivalent to

$X^2 + 3X + 2$

$\Rightarrow (X+2)(X+1)$

$\Rightarrow (a-b+2)(a-b+1)$

2 marks or equivalent logic.

End of Question 6

Question 7 (4 marks)

(a) How many integers are between $\sqrt{50}$ and $\sqrt{500}$? **1**

Noting $\sqrt{8^2} = \sqrt{64}$ or 8 is the least & $\sqrt{22^2} = \sqrt{484}$ or 22 is the largest, we require an inclusive count from 8 through to 22. This is $22 - 8 + 1$ or 15.
Hence, there are 15 integers between $\sqrt{50}$ & $\sqrt{500}$.

1 mark.

(b) By definition, a positive integer that can be written as the square of a positive integer is called a square number or a perfect square. For example, 9 is a perfect square since $9 = 3^2$. **3**

For x a positive integer, how many positive integers are there such that both x and $x + 99$ are perfect squares? Explain carefully your reasoning.
Hint: Let $x = m^2$ and $x + 99 = n^2$, where m and n are positive integers.

Using the hint $n^2 - m^2 = (x+99) - x$
i.e., $(n+m)(n-m) = 99$; $n > m$

1 mark or equivalent

Consider the factors of 99 which are 1, 3, 9, 11, 33 & 99.
Thus, possible combinations for n & m are:-

$\begin{cases} n+m = 99 \\ n-m = 1 \end{cases}$; $\begin{cases} n+m = 33 \\ n-m = 3 \end{cases}$; $\begin{cases} n+m = 11 \\ n-m = 9 \end{cases}$

Solving these systems of simultaneous equations gives:-
$n = 50, m = 49$; $n = 18, m = 15$; $n = 10, m = 1$ respectively.

1 mark for systems of linear equations

Thus, x could be $\begin{cases} 49^2 \text{ or } 2401 \\ 15^2 \text{ or } 225 \\ 1^2 \text{ or } 1 \end{cases}$

Consequently, there are exactly three positive integers satisfying the crideria.

1 mark for correct answer/conclusion following from argument.

End of Assessment Task

N.B. No credit for "correct" bald answer.