

NEWINGTON COLLEGE



Assessment 2

11 MATHEMATICS 2011

Time allowed - 2 hours

DIRECTIONS TO CANDIDATES:

- Marks may not be awarded for careless or badly arranged work.
- Approved silent calculators may be used.
- Each bundle must show the candidate's computer number.
- All questions may be attempted.
- The answers to the three questions in this paper are to be returned in separate bundles. Clearly marked Question 1, Question 2 etc.
- The questions are not necessarily arranged in order of difficulty. Candidates are advised to read the whole paper carefully at the start of the examination.

Outcomes to be Assessed:

- P1** Demonstrates confidence in using mathematics to obtain realistic solutions to problems.
- P2** Provides reasoning to support conclusions which are appropriate to the context.
- P3** Performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities.
- P4** Chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques.
- P5** Understands the concept of a function and the relationship between a function and its graph.

Question 1 (20 marks)	Marks
(a) Evaluate $ 2x-1 $ when $x = -5$	1
(b) Evaluate $\left(\frac{1}{16}\right)^{\frac{1}{2}}$	1
(c) Write in index form $\frac{1}{\sqrt[3]{a^4}}$	2
(d) Express this expression without negative or fractional indices $\frac{a^{-3} \times b^{\frac{1}{4}}}{a^{-\frac{1}{2}}}$	3
(e) Evaluate $\frac{2 + \sqrt{16+7}}{14-3^2}$, correct to 2 decimal places	2
(f) Write 0.000030158 in scientific notation, correct to 3 significant figures.	2
(g) The distance to the moon is 3.84×10^5 km. How fast would you have to travel if you wanted to reach the moon in a week? Give your answer in km/h in scientific notation accurate to 2 significant figures.	3
(h) Express 0.281 as a fraction in its simplest terms, showing all working.	2
(i) A shop is having a sale, discounting everything by 30%. If a skirt costs \$59.50 now in the sale, how much did it cost originally?	2
(j) Simplify fully $\sqrt{48} + \sqrt{75} - \sqrt{3}$	2

Question 2 (31 marks)**Start this question in a new booklet.****Marks**

(a) Expand and simplify

8

(i) $(2a+3)^2$

(ii) $(3a+5b)(a-2b)$

(iii) $3\sqrt{2}(4\sqrt{3}-\sqrt{2})$

(iv) $(7-\sqrt{5})(\sqrt{5}+7)$

(b) Factorise fully

(i) x^2-8x+7

1

(ii) $2a^3-8a$

2

(iii) $10x^2-11x-8$

2

(iv) $8x^3+27$

2

(v) $2x^3y-16y^4$

2

(c) Simplify the following algebraic fractions

4

(i) $\frac{a^2-9}{2a-6}$

(ii) $\frac{4k-20}{k^2-3k-10} \times \frac{3k+6}{10}$

(d) Express each of the following as a single fraction

4

(i) $\frac{2}{ab} + \frac{3b}{ac} - \frac{5}{bc}$

(ii) $\frac{3x-1}{2x+8} - \frac{5}{x^2+4x}$

(e) Evaluate x when $\sqrt{x} = 4\sqrt{5}$ **1**

(f) Rationalise the denominator and simplify:

3

(i) $\frac{4\sqrt{3}}{\sqrt{2}}$

(ii) $\frac{5\sqrt{3}}{2+\sqrt{3}}$

(g) A triangle has base of $(3+\sqrt{2})$ cm and a height of $(5-3\sqrt{2})$ cm.

Find the exact area of the triangle

Express it in the form $a+b\sqrt{2}$ where a and b are rational numbers.**2**

Question 3 (28 marks) **Start this question in a new booklet.** **Marks**

(a) Solve the following:

(i) $2x+17=4x+1$ **1**

(ii) $\frac{3x+5}{2} = \frac{x-2}{4}$ **2**

(iii) $|2x+5|=x+7$ **3**

(b) Solve the following inequalities and represent your solutions on a number line.

(i) $\frac{3x}{4}+2 \geq 13$ (ii) $\frac{x+2}{3}-1 \leq 10-2x$ **9**

(iii) $4 \leq 6x+5 \leq 10$ (iv) $|2x-9| \leq 7$

(c) Evaluate k when $4^{k+1} = \left(\frac{1}{8}\right)^{2k}$ **3**

(d) Solve $2x^2 + x = 3$ **2**

(e) Solve $3x^2 - 4x - 2 = 0$ leaving your answer in simplest surd form. **2**

(f) Use the completing the square method to solve $x^2 - 2x = 10$ accurate to 2 decimal places. **2**

(g) Solve these simultaneous equations **4**

(i) $3x+5y=19$

(ii) $a+4=2b$

$2x+7y=20$

$a=b^2-5b+8$

END OF PAPER

$$1) a) |2x-1| \quad |2 \times 5 - 1| = |-1| = 1 \quad (1)$$

$$b) \sqrt{\frac{16}{1}} = 4 \quad (1)$$

$$c) \frac{1}{\sqrt[3]{a^4}} = \frac{1}{a^{\frac{4}{3}}} = a^{-\frac{4}{3}} \quad \begin{matrix} (1) \text{ for neg.} \\ (1) \text{ for } \frac{4}{3} \end{matrix}$$

$$d) \frac{\frac{1}{a^3} \times 4\sqrt{b}}{\frac{1}{\sqrt{a}}} = \frac{\sqrt{a} \cdot 4\sqrt{b}}{a^3} \quad (1)$$

$$e) 1.359166305 \quad (1)$$

$$1.36 \quad (1)$$

$$f) 3.02 \times 10^{-5} \quad \begin{matrix} (1) \\ (1) \end{matrix}$$

$$g) S = \frac{D}{T} \quad \text{Speed} = \frac{3.84 \times 10^5 \text{ km}}{7 \times 24 \text{ hours}} = 2285.714286 \text{ km/h} \quad (1)$$

$$2.3 \times 10^3 \quad (1)$$

(1) for formula (1) for rounding to 2 sig. fig. (1) for sci. notation

$$h) x = 0.281$$

$$\begin{matrix} 100x = 28.1 \\ 1000x = 281.1 \\ 900x = 253 \end{matrix} \quad (1) \text{ for working}$$

$$900x = 253 \quad (1) \text{ for answer}$$

$$1) \$59.50 = 70\%$$

$$\div 70 \quad 0.85 = 1\% \quad (1)$$

$$\times 100 \quad \$85 = 100\% \quad (1)$$

$$j) \sqrt{48} + \sqrt{75} - \sqrt{3}$$

$$\sqrt{16}\sqrt{3} + \sqrt{25}\sqrt{3} - \sqrt{3}$$

$$4\sqrt{3} + 5\sqrt{3} - \sqrt{3} = 8\sqrt{3} \quad \begin{matrix} \text{simplifying} (1) \\ \text{collecting} (1) \end{matrix}$$

2 Unit Q2 Mark Scheme

$$\begin{aligned} \text{2) i) } (2a+3)(2a+3) &= 4a^2+6a+6a+9 \quad \textcircled{1} \\ &= 4a^2+12a+9 \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} \text{ii) } (3a+5b)(a-2b) &= 3a^2-6ab+5ab-10b^2 \quad \textcircled{1} \\ &= 3a^2-ab-10b^2 \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} \text{iii) } 3\sqrt{2}(4\sqrt{3}-\sqrt{2}) &= 12\sqrt{6}-3 \times 2 \quad \textcircled{1} \\ &= 12\sqrt{6}-6 \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} \text{v) } (7-\sqrt{5})(\sqrt{5}+7) &= 7\sqrt{5}+49-5-7\sqrt{5} \quad \textcircled{1} \\ &= 44 \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} \text{2) i) } x^2-8x+7 \\ (x-7)(x-1) \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} \text{ii) } 2a^3-8a &= 2a(a^2-4) \quad \textcircled{1} \\ &= 2a(a+2)(a-2) \quad \textcircled{1} \end{aligned}$$

$$\text{iii) } 10x^2-11x-8$$

$$P=-80 \quad 10x^2+5x-16x-8 \quad \textcircled{1} \text{ for working}$$

$$S=-11 \quad 5x(2x+1)-8(2x+1)$$

$$F=5, -16 \quad (5x-8)(2x+1) \quad \textcircled{1}$$

$$\text{iv) } 8x^3+27 \quad a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$a=2x \quad b=3 \quad \textcircled{1}$$

$$(2x+3)(4x^2-6x+9) \quad \textcircled{1}$$

$$\begin{aligned} \text{v) } 2x^3y-16y^4 &= 2y(x^3-8y^3) \quad \textcircled{1} \\ &= 2y(x-2y)(x^2+2xy+4y^2) \quad \textcircled{1} \quad a^3-b^3=(a-b)(a^2+ab+b^2) \end{aligned}$$

$$2$$

$$c) i) \frac{a^2-9}{2a-6} = \frac{(a+3)(a-3)}{2(a-3)} = \frac{a+3}{2} \quad (1)$$

$$ii) \frac{4k-20}{k^2-3k-10} \times \frac{3k+6}{10}$$

$$\frac{4(k-5)}{(k+2)(k-5)} \times \frac{3(k+2)}{10} \quad (1) \text{ factorising all}$$

$$= \frac{12}{10} = \frac{6}{5} \quad (1) \text{ simplifying}$$

$$d) \frac{2c}{abc} + \frac{3ab^2}{abc} - \frac{5a}{abc}$$

$$= \frac{2c+3b^2-5a}{abc} \quad (1)$$

$$e) \frac{3x-1}{2(x+4)} - \frac{5}{x(x+4)}$$

$$\frac{x(3x-1)}{2x(x+4)} - \frac{10}{2x(x+4)} \quad (1) \text{ for factorising}$$

full marks for this stage

$$= \frac{3x^2-x-10}{2x(x+4)} = \frac{(3x+5)(x-2)}{2x(x+4)}$$

Numerator:

$$\begin{aligned} 2 &= -30 & 3x^2 - 6x + 5x - 10 \\ 3 &= -1 & \\ 5 &= -6 & = 3x(x-2) + 5(x-2) \\ & & (3x+5)(x-2) \end{aligned}$$

$$e) 4\sqrt{5} = \sqrt{16}\sqrt{5} = \sqrt{16 \times 5}$$

$$= \sqrt{80}$$

$$\text{so } x=80 \quad (1)$$

$$2) i) \frac{4\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{4\sqrt{6}}{2} = 2\sqrt{6} \quad (1)$$

$$ii) \frac{5\sqrt{3}}{2+\sqrt{3}} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})} \stackrel{(1)}{=} \frac{10\sqrt{3}-15}{4-3} = 10\sqrt{3}-15 \quad (1)$$

$$g) \frac{1}{2} b \times h$$

$$\frac{1}{2} (3+\sqrt{2})(5-3\sqrt{2})$$

$$\frac{1}{2} [15 - 9\sqrt{2} + 5\sqrt{2} - 3 \times 2]$$

$$\frac{1}{2} [9 - 4\sqrt{2}]$$

(1) for some start of the multiplication!

$$= 4.5 - 2\sqrt{2} \quad (1) \quad a=4.5 \quad b=-2$$

Q3 Mark Scheme

a) i) $2x+17=4x+1$

$$16=2x$$

$$x=8 \quad \textcircled{1}$$

ii) $\frac{3x+5}{2} = \frac{x-2}{4}$

$$\frac{6x+10}{4} = \frac{x-2}{4}$$

or $12x+20=2x-4 \quad \textcircled{1}$

$$6x+10=x-2$$

$$5x=-12$$

$$x=-\frac{12}{5} \quad x=-2\frac{2}{5} \quad \textcircled{1}$$

iii) $|2x+5|=x+7$

$$2x+5=x+7$$

$$x=2 \quad \textcircled{1}$$

OR

$$-2x-5=x+7 \quad \textcircled{1}$$

~~-2x~~

$$-12=3x$$

$$x=-4 \quad \textcircled{1}$$

Test $|2 \times 2+5| = 2+7$

$$|9|=9 \checkmark$$

$$|2 \times -4+5| = -4+7$$

$$|-3|=3 \checkmark$$

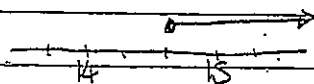
b) i) $\frac{3x}{4} + 2 \geq 13$

$$\frac{3x}{4} \geq 11$$

$\textcircled{1}$ working

$$3x \geq 44$$

$$x \geq 14\frac{2}{3} \quad \textcircled{1} \text{ ans}$$



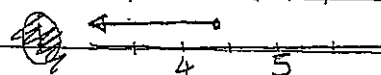
ii) $\frac{x+2}{3} + 1 \leq 10-2x$

$$\frac{x+2}{3} \leq 11-2x$$

$$\textcircled{1} \quad x+2 \leq 33-6x$$

$$7x \leq 31$$

$$\textcircled{1} \quad x \leq 4\frac{3}{7}$$

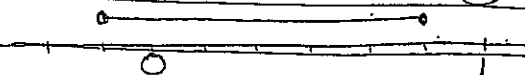


iii) $4 \leq 6x+5 \leq 10$

$$-1 \leq 6x \leq 5$$

$$-\frac{1}{6} \leq x \leq \frac{5}{6} \quad \textcircled{1}$$

$\textcircled{1}$ number line



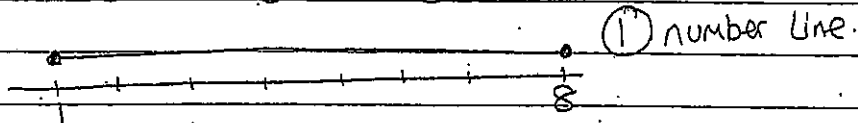
3
cont.

b) iv) $|2x-9| \leq 7$

$$-7 \leq 2x-9 \leq 7 \quad \textcircled{1}$$

$$2 \leq 2x \leq 16$$

$$1 \leq x \leq 8 \quad \textcircled{1}$$



c) $4^{k+1} = \left(\frac{1}{8}\right)^{2k}$

$$(2^2)^{k+1} = \left(\frac{1}{2^3}\right)^{2k} \quad \textcircled{1} \text{ same bases}$$

$$2^{2k+2} = (2^{-3})^{2k}$$

so $2k+2 = -6k \quad \textcircled{1}$

$$8k = -2$$

$$k = -\frac{2}{8} \quad k = -\frac{1}{4} \quad \textcircled{1}$$

d) $2x^2 + x = 3$

$$2x^2 + x - 3 = 0$$

(1) any correct method

(1) both answers

$p=6$ $2x^2 + 3x - 2x - 3 = 0$

$q=1$ $x(2x+3) - 1(2x+3) = 0$

$r=3-2$ $(x-1)(2x+3) = 0$ so $x=1$ or $2x+3=0$

$$x = -\frac{3}{2}$$

e) $3x^2 - 4x - 2 = 0$

$a=3$ $b=-4$ $c=-2$

substituting into formula (1)

$$x = \frac{4 \pm \sqrt{16 - 4 \times 3 \times -2}}{2 \times 3}$$

$$x = \frac{4 \pm \sqrt{40}}{6} = \frac{4 \pm 2\sqrt{10}}{6}$$

$$x = \frac{2 \pm \sqrt{10}}{3} \quad \textcircled{1}$$

$$x^2 - 2x = 11$$

$$x^2 - 2x + 1 = 11 + 1 \quad (1)$$

$$(x-1)^2 = 11$$

$$x-1 = \pm\sqrt{11}$$

$$x = 1 \pm \sqrt{11} \quad (1)$$

g) i) (1) $3x + 5y = 19$

(2) $2x + 7y = 20$

(1) $\times 2$ $6x + 10y = 38$ (3)

(2) $\times 3$ $6x + 21y = 60$ (4) } creating new equations (1)

(4) - (3) $11y = 22$

$y = 2$

sub $y = 2$ into (1) $3x + 10 = 19$

$3x = 9$

$x = 3$

answer $x = 3$ $y = 2$ (1)

ii) $a + 4 = 2b$

$a = b^2 - 5b + 8$

$2b - 4 = b^2 - 5b + 8$

$0 = b^2 - 7b + 12$ (1) for this stage

$0 = (b-4)(b-3)$

$b = 4$ or $b = 3$

If $b = 4$ $a + 4 = 8$

$a = 4$

OR If $b = 3$ $a + 4 = 6$

$a = 2$



(1)

for answers

