



# NORMANHURST BOYS HIGH SCHOOL

## MATHEMATICS ADVANCED

2023 Year 11 Assessment Task 2

Wednesday, 21st June 2023

### General instructions

- Working time – 60 min.  
(plus 5 minutes reading time)
- Write your answers in the space provided.
- Write using a blue or a black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.
- Attempt **all** questions.

**Class** (please ✓)

☐ 11MAA.1 – Miss Chaussivert

**NESA STUDENT #** .....

Marker's use only.

| QUESTION | 1-3            | 4-6             | 7-8            | 9-10            | 11-12          | Total           | % |
|----------|----------------|-----------------|----------------|-----------------|----------------|-----------------|---|
| MARKS    | $\overline{7}$ | $\overline{15}$ | $\overline{8}$ | $\overline{10}$ | $\overline{7}$ | $\overline{47}$ |   |

**Question 1** (2 marks)Find the exact value of  $\tan 600^\circ$ .**2**

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**Question 2** (2 marks)

Find the solutions of:

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

where  $0^\circ \leq \theta \leq 360^\circ$ .**2**

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**Question 3** (3 marks)Solve the following for  $0^\circ \leq x \leq 180^\circ$ :

$$2 \cos^2 x - \cos x = 0$$

**3**

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**Question 4** (6 marks)Solve the following equations for  $x$ :

(a)  $3^{x-1} = 27$

**2**

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(b)  $(\log_3 x)^2 = \log_3(x^5) - 6$

**4**

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**Question 5** (3 marks)

Simplify the following, writing each pronumeral with a positive index.

**3**

$$\frac{\left(\sqrt{mn^{\frac{3}{2}}}\right)^4}{p^{-3}} \times (p^{-2})^{-3}$$

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**Examination continues overleaf...**

**Question 6** (6 marks)

Consider the function  $f(x) = \log_{10}(3x + 1)$ .

- (a) State any vertical and horizontal asymptotes of  $f(x)$ . **1**

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- (b) Find all points where  $f(x)$  intersects the coordinate axes. **2**

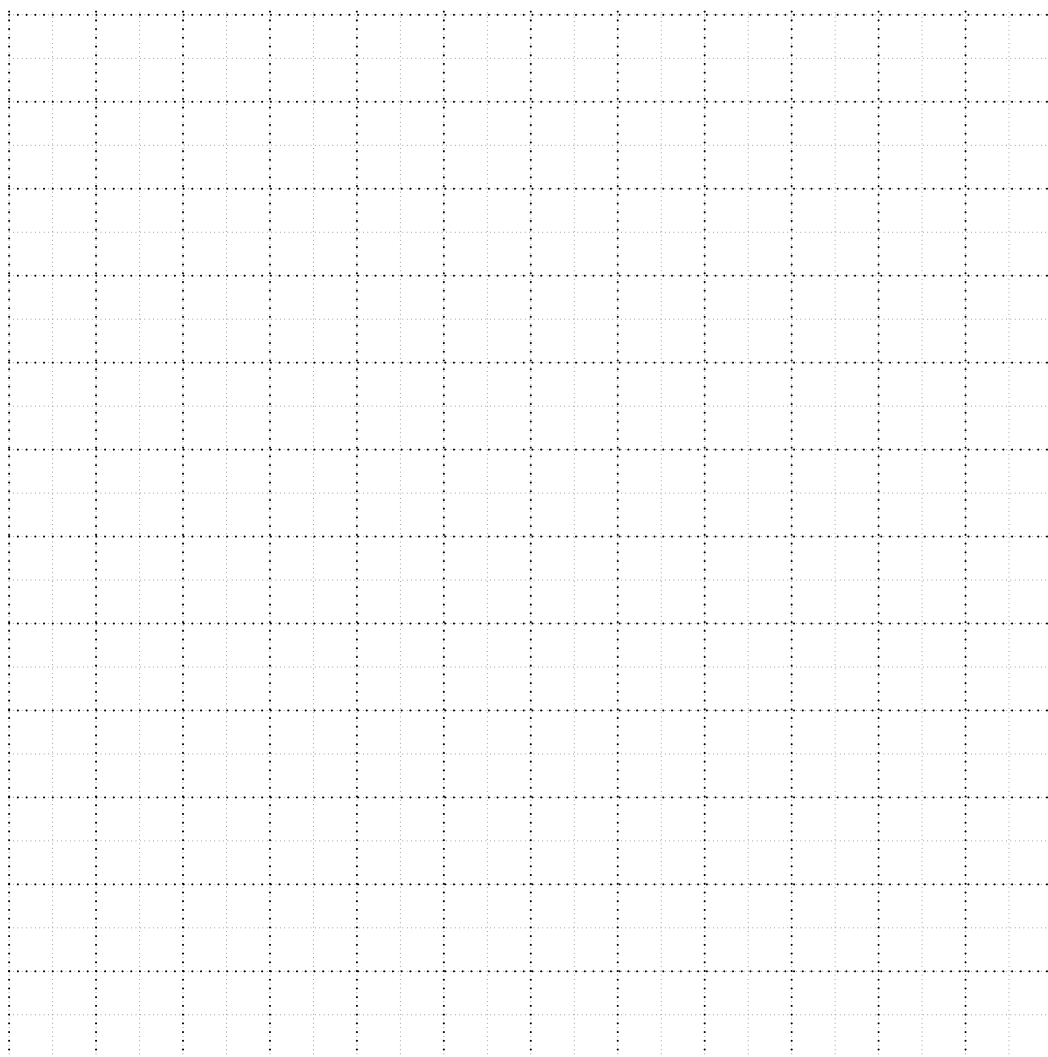
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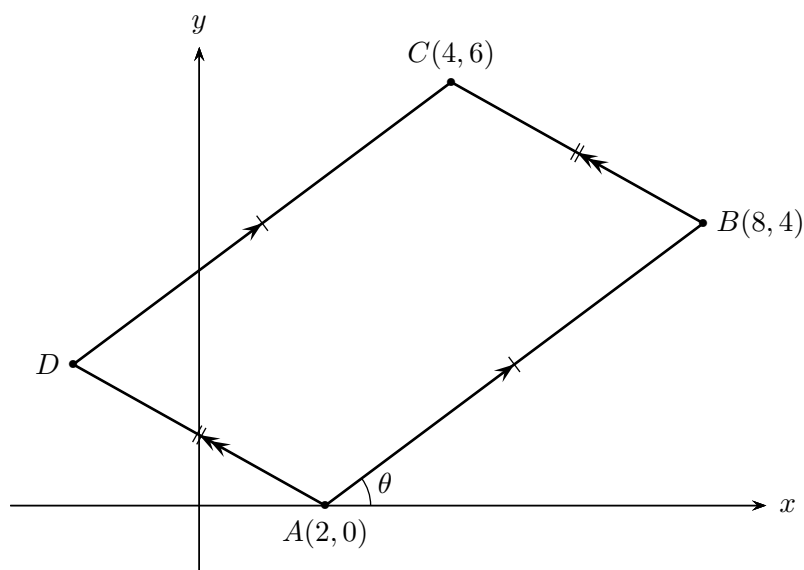
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- (c) Sketch the graph of  $f(x)$ , showing all important features. **3**



**Question 7** (6 marks)

The points  $A(2, 0)$ ,  $B(8, 4)$ ,  $C(4, 6)$  and  $D$  form the four vertices of a parallelogram, as shown in the diagram.



- (a) Find the gradient of the line  $AB$  and hence find the angle  $\theta$  correct to the nearest minute. **2**

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- (b) Show that the equation of line  $CD$ , parallel to  $AB$  and passing through  $C$  is given by  $2x - 3y + 10 = 0$ . **2**

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- (c) State the coordinates of point  $D$ . **2**

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**Examination continues overleaf...**

**Question 8** (2 marks)**2**

Given the points  $A(-1, 2)$ ,  $B(3, 4)$ , and  $C(7, -4)$  in the Cartesian plane, show that  $\triangle ABC$  is a right angled triangle.

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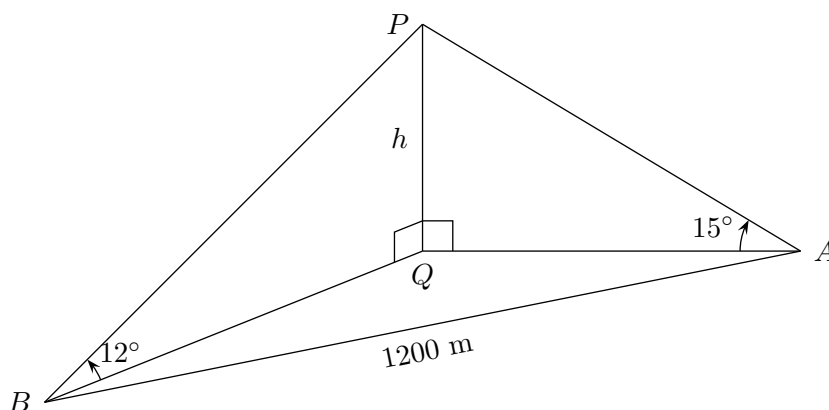
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**Question 9** (7 marks)

A tower  $PQ$  of height  $h$  metres has an angle of elevation of  $15^\circ$  from a point  $A$  due east of it. From another point  $B$ , the bearing of the tower is  $055^\circ\text{T}$  and the angle of elevation is  $12^\circ$ . The points  $A$  and  $B$  are 1200 metres apart on the same level as the tower's base  $Q$ .



- (a) Show that  $\angle AQB = 145^\circ$

**1**

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- (b) Consider the triangle  $APQ$  and show that  $AQ = h \cot 15^\circ$ .

**1**

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- (c) Find a similar expression for  $BQ$ .

**1**

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- (d) Show that  $h = \frac{1200^2}{\cot 15^\circ \cot 12^\circ - 2 \cot 15^\circ \cot 12^\circ \cos 145^\circ}$  **3**

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- (e) Hence, calculate the value of  $h$ , correct to the nearest metre. **1**

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**Question 10** (3 marks)

Given that  $\cos \alpha = -\frac{2}{\sqrt{5}}$  and  $180^\circ \leq \alpha \leq 360^\circ$ , find the exact value of  $\cot \alpha$ .

**3**

Show all relevant working.

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**Examination continues overleaf...**

**Question 11** (4 marks)

A newly bought car is worth \$10 000. After  $t$  years, the value of the car (in dollars) is given by the formula:

$$V = Ae^{-0.2t}$$

- (a) Find the value of  $A$ . **1**

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 .....

- (b) Find the value of the car after 6 years, correct to the nearest cent . **1**

.....

- (c) How many years (correct to 2 decimal places) will it take for the car's value to reach \$1 000? **2**

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**Question 12** (3 marks)

Express:

$$\log(x^2\sqrt{y^3z})$$

in the form of  $a \log x + b \log y + c \log z$ .

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 .....

**End of paper.**



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NESA STUDENT #

*Solutions*

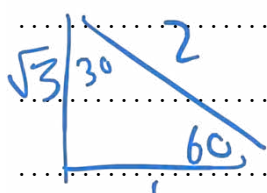
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| QUESTION | 1-3            | 4-6             | 7-8            | 9-10            | 11-12          | Total           | % |
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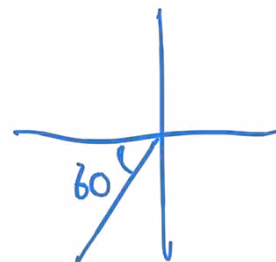
**Question 1** (2 marks)Find the exact value of  $\tan 600^\circ$ .

2

$$\tan(600) = \tan(240) \checkmark$$



$$= \sqrt{3} \checkmark$$

**Question 2** (2 marks)

Find the solutions of:

2

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

where  $0^\circ \leq \theta \leq 360^\circ$ .

3rd and 4th quadrant:  
related angle:  $60^\circ \checkmark$

$$\theta = 180 + 60, 360 - 60 = 240^\circ, 300^\circ \checkmark$$

**Question 3** (3 marks)Solve the following for  $0^\circ \leq x \leq 180^\circ$ :

3

$$2 \cos^2 x - \cos x = 0$$

$$\cos x (2 \cos x - 1) = 0$$

$$\cos x = 0$$

$$\text{or } 2 \cos x - 1 = 0$$

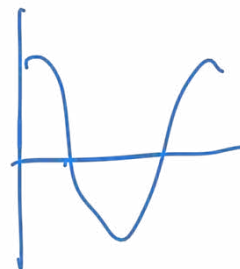
$$x = 90$$

$$2 \cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$x = 60^\circ, 90^\circ \checkmark$$

$$x = 60^\circ$$



**Question 4** (6 marks)Solve the following equations for  $x$ :

(a)  $3^{x-1} = 27$

2

$$3^{x-1} = 3^3 \quad \checkmark$$

$$x-1 = 3$$

$$x = 4 \quad \checkmark$$

(b)  $(\log_3 x)^2 = \log_3(x^5) - 6$

4

$$(\log_3 x)^2 = 5 \log_3 x - 6 \quad \checkmark$$

$$\text{let } u = \log_3 x \quad u^2 = 5u - 6 \quad \checkmark$$

$$u^2 - 5u + 6 = 0$$

$$(u-3)(u-2) = 0$$

$$u = 3, 2 \quad \checkmark \Rightarrow \log_3 x = 3 \quad \log_3 x = 2$$

$$x = 27, 9 \quad \checkmark$$

**Question 5** (3 marks)

3

Simplify the following, writing each pronumeral with a positive index.

$$\frac{(\sqrt{mn^{\frac{3}{2}}})^{\frac{1}{2} \times 4}}{p^{-3}} \times (p^{-2})^{-3}$$

$$= \frac{m^2 n^3 p^6}{p^{-3}} \quad \checkmark$$

$$= m^2 n^3 p^9 \quad \checkmark$$

Examination continues overleaf...

**Question 6** (6 marks)

Consider the function  $f(x) = \log_{10}(3x + 1)$ .

- (a) State any vertical and horizontal asymptotes of  $f(x)$ .

vertical asymptote:  $x = -\frac{1}{3}$  ✓

1

- (b) Find all points where  $f(x)$  intersects the coordinate axes.

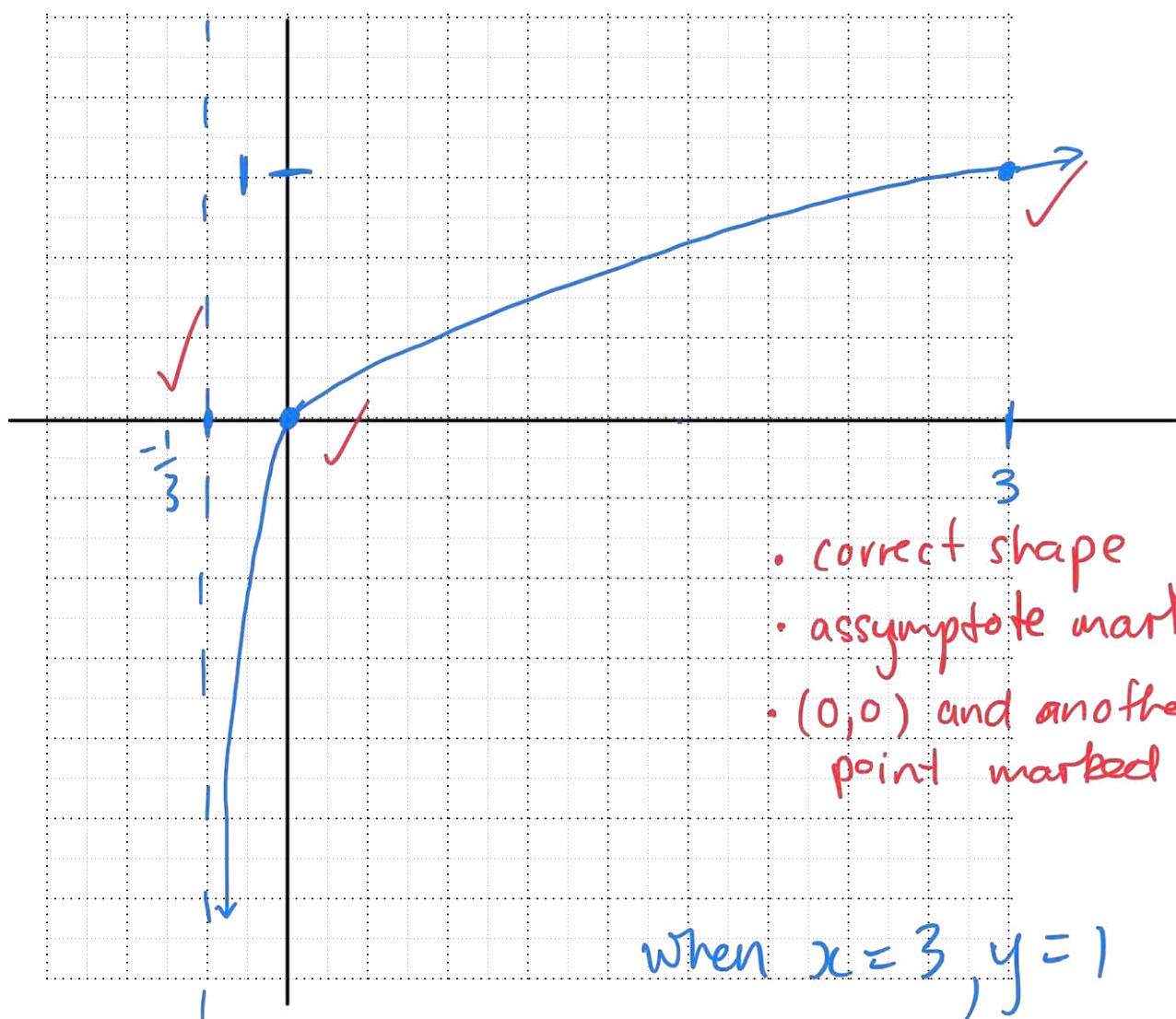
let  $x = 0$ :  $y = \log_{10}(1) = 0$  ✓

2

so  $f(x)$  intersects the axes at  $(0,0)$  ✓

- (c) Sketch the graph of  $f(x)$ , showing all important features.

3

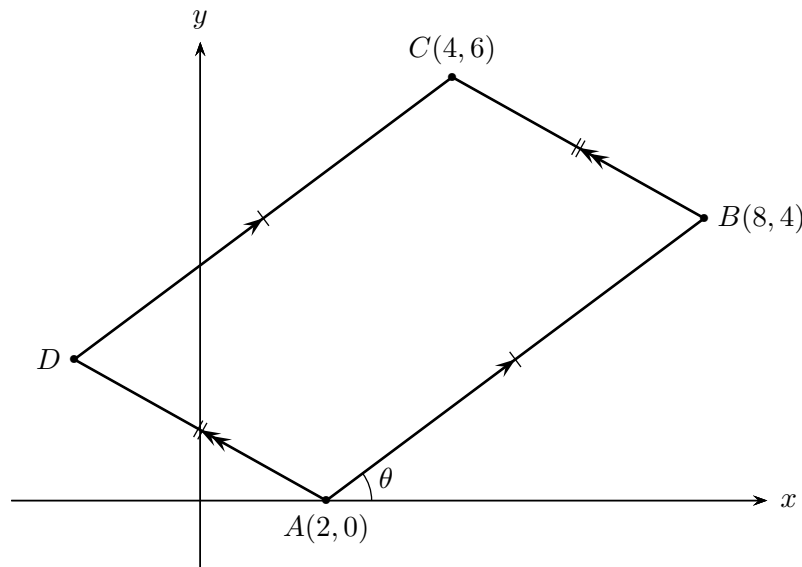


$$3x + 1 = 10$$

$$3x = 9 \quad x = 3$$

**Question 7** (6 marks)

The points  $A(2, 0)$ ,  $B(8, 4)$ ,  $C(4, 6)$  and  $D$  form the four vertices of a parallelogram, as shown in the diagram.



- (a) Find the gradient of the line  $AB$  and hence find the angle  $\theta$  correct to the nearest minute. 2

$$m = \frac{4-0}{8-2} = \frac{4}{6} = \frac{2}{3} \quad \tan \theta = \frac{2}{3} \quad \theta = 33^\circ 41'$$

- (b) Show that the equation of line  $CD$ , parallel to  $AB$  and passing through  $C$  is given by  $2x - 3y + 10 = 0$ . 2

$$\begin{aligned} y - 6 &= \frac{2}{3}(x - 4) \\ 3y - 18 &= 2x - 8 \\ 0 &= 2x + 10 - 3y \end{aligned} \quad \rightarrow \quad 2x - 3y + 10 = 0$$

- (c) State the coordinates of point  $D$ . 2

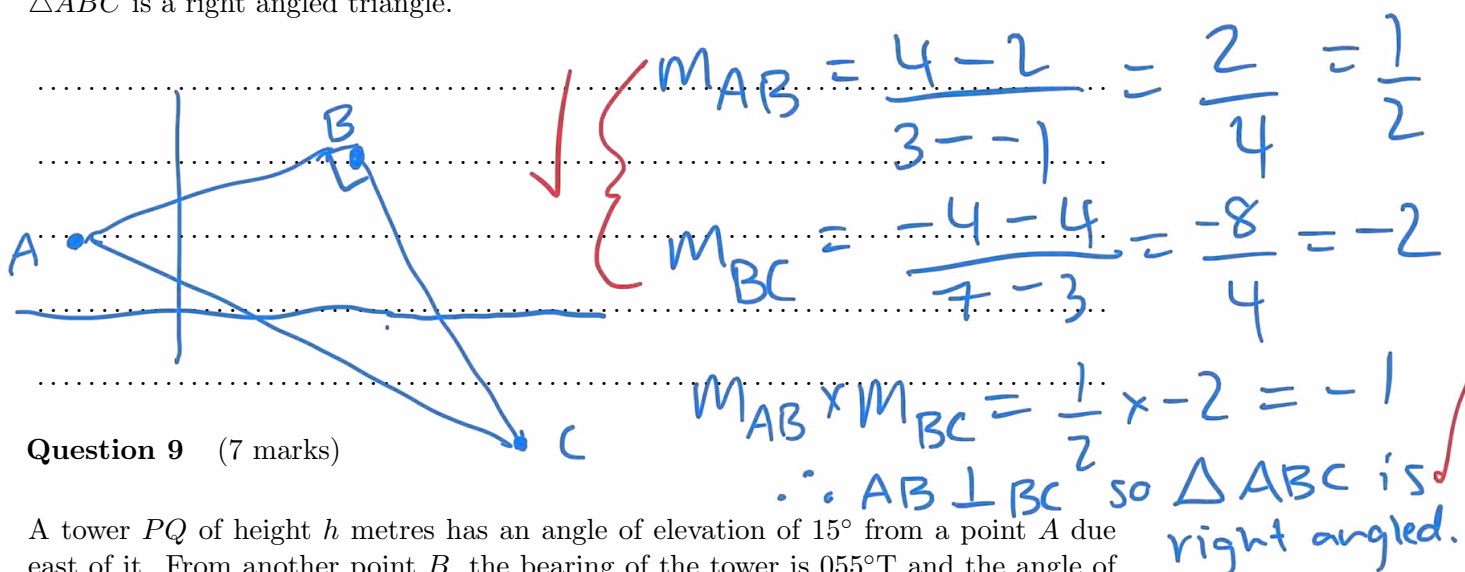
$$\begin{aligned} &\text{across 4, up 2} \\ &\Rightarrow (-2, 2) \end{aligned}$$

Examination continues overleaf...

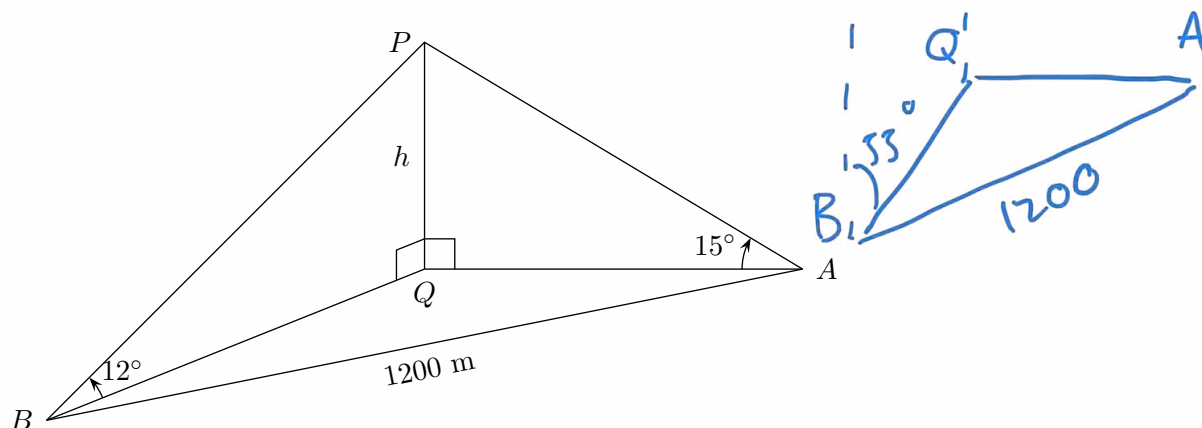
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2

**Question 9** (7 marks)

A tower  $PQ$  of height  $h$  metres has an angle of elevation of  $15^\circ$  from a point  $A$  due east of it. From another point  $B$ , the bearing of the tower is  $055^\circ T$  and the angle of elevation is  $12^\circ$ . The points  $A$  and  $B$  are 1200 metres apart on the same level as the tower's base  $Q$ .



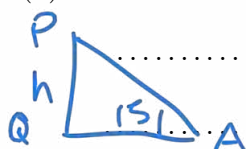
- (a) Show that  $\angle AQB = 145^\circ$

$\text{reflex } \angle AQB = 90 + (180 - 55) = 215^\circ$   
 $\therefore \angle AQB = 360 - 215 = 145^\circ$

1

- (b) Consider the triangle  $APQ$  and show that  $AQ = h \cot 15^\circ$ .

1



$\cot(15) = \frac{AQ}{h} \Rightarrow AQ = h \cot(15)$

- (c) Find a similar expression for  $BQ$ .

1

$BQ = h \cot(12)$

- (d) Show that  $h = \frac{1200^2}{\cot 15^\circ \cot 12^\circ - 2 \cot 15^\circ \cot 12^\circ \cos 145^\circ}$  3

cosine rule :  $1200^2 = AQ^2 + BQ^2 - 2AQBQ \cos AQB$   
 $\triangle ABQ$   
 $1200^2 = h^2 \cot^2 15 + h^2 \cot^2 12 - 2h^2 \cot 15 \cot 12$   
 $1200^2 = h^2 (\cot^2 15 + \cot^2 12 - 2 \cot 15 \cot 12)$  ✓

$$h^2 = \frac{1200^2}{\cot^2 15 + \cot^2 12 - 2 \cot 15 \cot 12}$$
 ✓

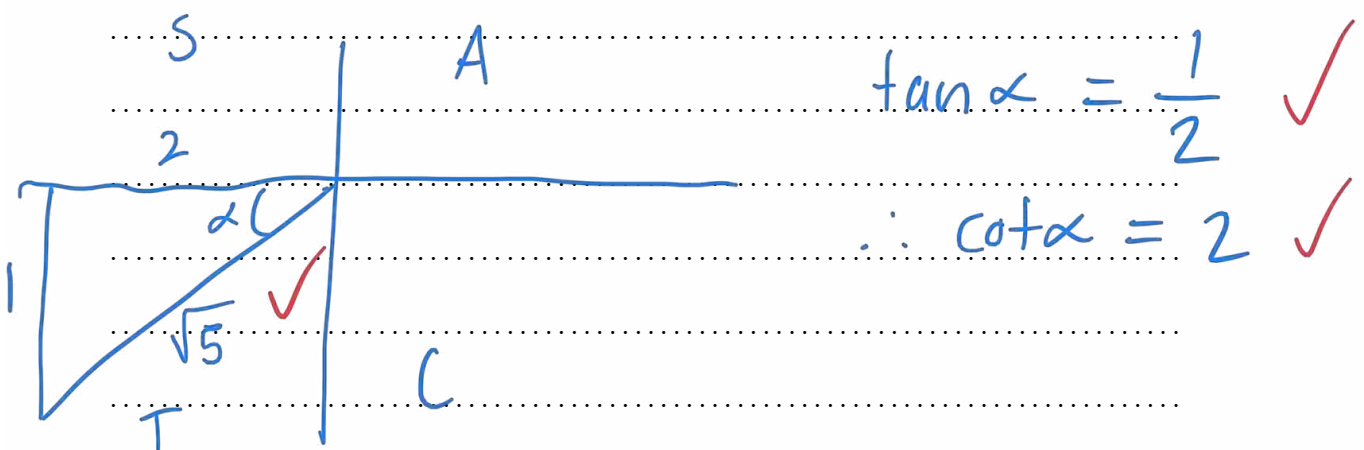
- (e) Hence, calculate the value of  $h$ , correct to the nearest metre. 1

put in calculator :  $h = 149.0402 \dots$   
 $\approx 149 \text{ m}$  ✓

**Question 10** (3 marks)

Given that  $\cos \alpha = -\frac{2}{\sqrt{5}}$  and  $180^\circ \leq \alpha \leq 360^\circ$ , find the exact value of  $\cot \alpha$ . 3

Show all relevant working.



Examination continues overleaf...

**Question 11** (4 marks)

A newly bought car is worth \$10 000. After  $t$  years, the value of the car (in dollars) is given by the formula:

$$V = Ae^{-0.2t}$$

- (a) Find the value of  $A$ .

1

when  $t = 0$ ,  $V = 10\ 000$   
 $10\ 000 = Ae^0 \Rightarrow A = 10\ 000$  ✓

- (b) Find the value of the car after 6 years, correct to the nearest cent.

1

$V = 10\ 000 e^{-0.2 \times 6} = \$3\ 011.94$  ✓

- (c) How many years (correct to 2 decimal places) will it take for the car's value to reach \$1 000?

2

$1000 = 10\ 000 e^{-0.2t}$  ✓  
 $e^{-0.2t} = 0.1$   
 $-0.2t = \ln(0.1)$   
 $t = \frac{\ln(0.1)}{-0.2} = 11.51292 \dots$   
 $= 11.51 \text{ years}$  ✓

**Question 12** (3 marks)

Express:

$$\log(x^2 \sqrt{y^3 z})$$

in the form of  $a \log x + b \log y + c \log z$ .

3

$\log(x^2 y^{\frac{3}{2}} z^{\frac{1}{2}})$  ✓  
 $= \log x^2 + \log y^{\frac{3}{2}} + \log z^{\frac{1}{2}}$  ✓  
 $= 2 \log x + \frac{3}{2} \log y + \frac{1}{2} \log z$  ✓

End of paper.