# NORMANHURST BOYS HIGH SCHOOL

# MATHEMATICS ADVANCED

2024 Year 11 Course Assessment Task 2

Friday, 28 June 2024

**General instructions**

- Working time – 1 hour.
  (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt **all** questions.

**SECTION I**

- Mark your answers on the answer grid provided (on page 2)

**SECTION II**

- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

**NESA STUDENT #:** .....

**Class** (please ✔)

◯ 11MAA.1 – Ms Quek

Marker's use only.

| **QUESTION** | 1-3 | 4-5 | 6-7 | 8-9 | 10-11 | 12-15 | 16 | **Total** | % |
|---|---|---|---|---|---|---|---|---|---|
| **MARKS** | /3 | /9 | /13 | /7 | /8 | /14 | /6 | /60 | |

# Section I

**3 marks**
**Attempt Question 1 to 3**
**Allow approximately 4 minutes for this section**

Mark your answers on the answer grid.

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**Questions** **Marks**

**1.** Which of the following is equivalent to **1**

$$\frac{\cos(90^\circ - \theta)}{\sin(90^\circ - \theta)}$$

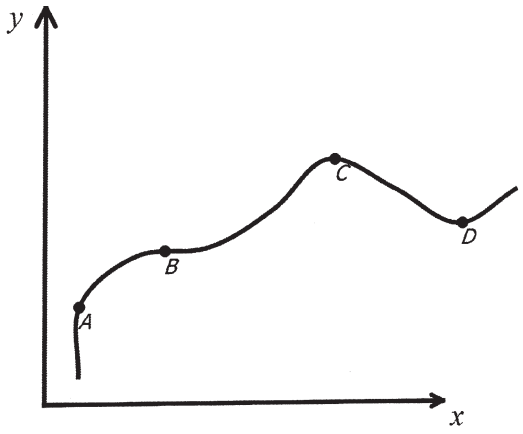
(A) 1 (B) $\cot\theta$ (C) $\tan\theta$ (D) $-\tan\theta$

**2.** Evaluate $\lim\limits_{x\to 1} \dfrac{x^2+2x-3}{x-1}$ **1**

(A) undefined (B) 0 (C) 1 (D) 4

**3.** Which of the following points on the graph of $y = f(x)$ has the following properties simultaneously: **1**

- $f'(x) = 0$
- $f''(x) = 0$




(A) $A$ (B) $B$ (C) $C$ (D) $D$

# Answer grid for Section I

**1** – Ⓐ Ⓑ Ⓒ Ⓓ
**2** – Ⓐ Ⓑ Ⓒ Ⓓ
**3** – Ⓐ Ⓑ Ⓒ Ⓓ

**Examination continues overleaf...**

# Section II

**57 marks**
**Attempt Question 4 to 16**
**Allow approximately 56 minutes for this section**

Write your answers in the space provided.

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**Question 4** (6 marks)

Differentiate with respect to $x$

(a) $x^2\sqrt{x}$ **2**

(b) $(2x^2+1)^6$ **2**

(c) $\dfrac{1+7x}{x^3}$ **2**

**Question 5** (3 marks)

If $f(x) = \dfrac{1}{x-1}$, show that $f'(x) = -\dfrac{1}{(x-1)^2}$ by differentiation from first principles. **3**

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**Examination continues overleaf...**

**Question 6** (5 marks)

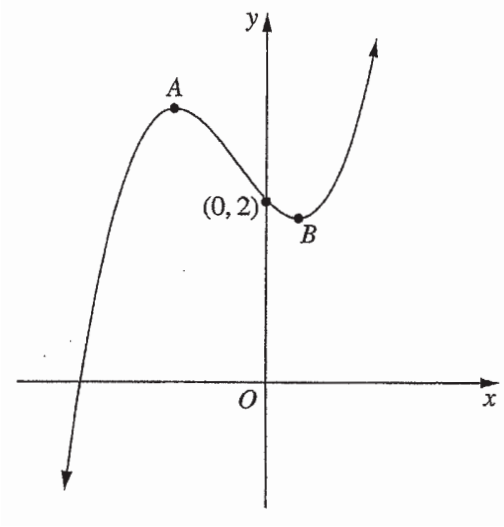
(a) **By using the product rule**, show that the derivative of $y = (3x^3 + x)(x - 1)$ is **2**

$$\frac{dy}{dx} = 12x^3 - 9x^2 + 2x - 1$$

(b) Hence, find the equation of the normal to the curve $y = (3x^3 + x)(x - 1)$ at the point where $x = 1$. Express your answer in general form $ax + by + c = 0$. **3**

**Question 7** (8 marks)

The graph of $y = x^3 + x^2 - x + 2$ is sketched below.




The points $A$ and $B$ are the turning points.

(a) Find the coordinates of $A$ and $B$. **4**

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(b) For what values of $x$ is the curve concave up? **2**

Show all mathematical reasons for your answer.

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(c) For what value(s) of $k$ does the equation $x^3 + x^2 - x + 2 = k$ only has one real solution? **2**

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**Examination continues overleaf...**

**Question 8** (4 marks)

Given $\log_7 2 = 0.36$ and $\log_7 5 = 0.83$, find the values of the following.

(a) $\log_7 0.4$ **2**

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(b) $\log_7 50$ **2**

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**Question 9** (3 marks)

In the space below, sketch the graph of $y = \log_5(5x - 5)$, showing all key features including: **3**

- any $x$ intercept(s)
- any asymptote(s)

**Examination continues overleaf...**

**Question 10** (4 marks)

(a) Solve the following equation for $x$, correct to 2 decimal places. **2**
*Full working out must be shown to gain full marks.*

$$0.4^{x+6} = 128$$

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(b) By considering the graph of $y = 0.4^{x+6}$ and also part (a), find the values of $x$ such that **2**

$$0.4^{x+6} < 128$$

(*Hint:* draw a rough sketch of the graph of $y = 0.4^{x+6}$)

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**Examination continues overleaf...**

**Question 11** (4 marks)

A drug is used to control a medical condition. It is known that the quantity $Q$ of drug remaining in the body after $t$ hours satisfies an equation of the form

$$Q = Q_0 e^{-kt} \quad \text{where } Q_0 \text{ and } k \text{ are constants}$$

The initial dose is a 6 milligrams and after 15 hours the amount remaining in the body is half the initial dose.

(a) Find the value of $Q_0$. **1**

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(b) Show that the value of $k$ is 0.046, correct to 3 decimal places. **1**

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(c) Show (with full working out) that it takes approximately 45 hours 12 minutes for one-eighth of the initial dose to remain. **2**

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**Examination continues overleaf...**

**Question 12** (4 marks)

Sydney is 694 nautical miles (M) south and 761 M west of Noumea. **4**

A plane must fly to make a trip from Noumea to Sydney. Calculate the distance and the true bearing of the flight to the nearest nautical miles and degree respectively.

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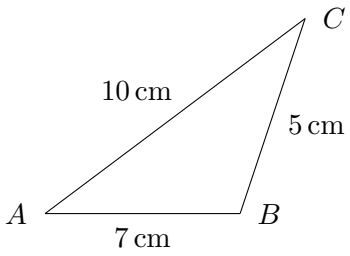
**Question 13** (3 marks)

The diagram shows $\triangle ABC$ with sides $AB = 7$cm, $BC = 5$cm and $AC = 10$cm.




(a) Show that $\cos A = \dfrac{31}{35}$ **1**

(b) Hence by finding the exact value of $\sin A$, determine the exact area of $\triangle ABC$. **2**

**Examination continues overleaf...**

**Question 14** (2 marks)

Given $\sin\theta = \dfrac{3}{4}$ and $\tan\theta < 0$, find the exact value of $\cos\theta$ **2**

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**Question 15** (5 marks)

(a) Prove that $\dfrac{2\cos^2\theta}{1-\sin\theta} = 2 + 2\sin\theta$ **2**

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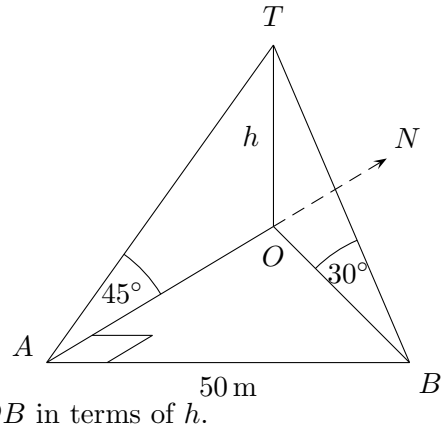
(b) Hence draw a sketch of a function $y = \dfrac{2\cos^2\theta}{1-\sin\theta}$ in the domain $-90° < \theta < 90°$ **3**

**Examination continues overleaf...**

**Question 16** (6 marks)

A person is standing at point $A$, which is due south of a tower $OT$ that is $h$ metres tall. The person walks 50 metres east to point $B$. The angle of elevation at points $A$ and $B$ to the top of the tower are 45° and 30° respectively.




(a) Express $OA$ and $OB$ in terms of $h$. **2**

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(b) Show that $h = 25\sqrt{2}$ metres. **2**

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(c) Find the bearing of $B$ from the base of the tower $O$, correct to the nearest minute. **2**

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**End of paper.**

## Additional writing space

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**NESA STUDENT #:** ..Sample..solution...

**Class** (please ✔)

◯ 11MAA.1 – Ms Quek

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# Section I

**3 marks**
**Attempt Question 1 to 3**
**Allow approximately 4 minutes for this section**

Mark your answers on the answer grid.

---

**Questions** **Marks**

**1.** Which of the following is equivalent to **1**

$$\frac{\cos(90^\circ - \theta)}{\sin(90^\circ - \theta)} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

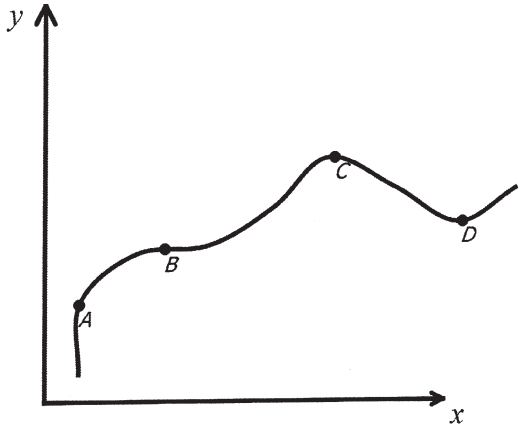
(A) 1 (B) $\cot\theta$ (C) ✓ $\tan\theta$ (D) $-\tan\theta$

**2.** Evaluate $\lim_{x\to 1} \dfrac{x^2+2x-3}{x-1}$ $= \lim_{x\to 1} \dfrac{(x+3)(x-1)}{x-1} = \lim_{x\to 1} x+3 = 4$ **1**

(A) undefined (B) 0 (C) 1 (D) ✓ 4

**3.** Which of the following points on the graph of $y = f(x)$ has the following properties simultaneously: **1**

- $f'(x) = 0$ → stationary point
- $f''(x) = 0$ → Horizontal point of inflection




(A) $A$ (B) ✓ $B$ (C) $C$ (D) $D$

# Answer grid for Section I

**1** – (A) (B) (C) (D)

**2** – (A) (B) (C) (D)

**3** – (A) (B) (C) (D)

**Examination continues overleaf...**

# Section II

**57 marks**
**Attempt Question 4 to 16**
**Allow approximately 56 minutes for this section**

Write your answers in the space provided.

---

**Question 4** (6 marks)

-1 setting out overall.

Differentiate with respect to $x$

(a) $x^2\sqrt{x}$ **2**

$= x^{2+\frac{1}{2}}$
$= x^{\frac{5}{2}}$ (1)

$\frac{d}{dx}\left(x^{\frac{5}{2}}\right) = \frac{5}{2}x^{\frac{3}{2}}$ (1)

(b) $(2x^2+1)^6$ **2**

$\frac{d}{dx}(2x^2+1)^6 = 6(2x^2+1)^5 \times 4x$ (1)

$= 24x(2x^2+1)^5$ (1)

(c) $\frac{1+7x}{x^3}$ **2**

$= \frac{1}{x^3} + \frac{7x}{x^3}$

$= x^{-3} + 7x^{-2}$ (1)

$\frac{d}{dx}\left(x^{-3} + 7x^{-2}\right) = -3x^{-4} + 7x - 2x^{-3}$

$= -\frac{3}{x^4} - \frac{14}{x^3}$ (1)

**Question 5** (3 marks)

If $f(x) = \dfrac{1}{x-1}$, show that $f'(x) = -\dfrac{1}{(x-1)^2}$ by differentiation from first principles. **3**

$f(x+h) = \dfrac{1}{x+h-1}$

$f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ (1) for keeping $\lim\limits_{h\to 0}$ till the end

$= \lim\limits_{h\to 0} \dfrac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h}$ (1)

$= \lim\limits_{h\to 0} \dfrac{\frac{x-1-(x+h-1)}{(x+h-1)(x-1)}}{h}$

$= \lim\limits_{h\to 0} \dfrac{(x-1-x-h+1)}{h(x+h-1)(x-1)}$

$= \lim\limits_{h\to 0} \dfrac{-h}{h(x+h-1)(x-1)}$

$= -\dfrac{1}{(x-1)(x-1)}$

$= -\dfrac{1}{(x-1)^2}$ (1)

**Examination continues overleaf...**

**Question 6** (5 marks)

(a) **By using the product rule**, show that the derivative of $y = (3x^3 + x)(x - 1)$ is **2**

$$\frac{dy}{dx} = 12x^3 - 9x^2 + 2x - 1$$

$u = 3x^3 + x \quad v = x - 1$

$u' = 9x^2 + 1 \quad v' = 1$

$\frac{dy}{dx} = (9x^2+1)(x-1) + (3x^3+x)$ (1)

$= 9x^3 - 9x^2 + x - 1 + 3x^3 + x$

$= 12x^3 - 9x^2 + 2x - 1$ (1)

(b) Hence, find the equation of the normal to the curve $y = (3x^3 + x)(x - 1)$ at the point where $x = 1$. Express your answer in general form $ax + by + c = 0$. **3**

at $x = 1$, $y = (3(1)^3 + 1)(1-1)$

$= 0$

$m_{tangent} = 12(1)^3 - 9(1)^2 + 2(1) - 1$

$= 4$ (1)

$\therefore m_{normal} = -\frac{1}{4}$

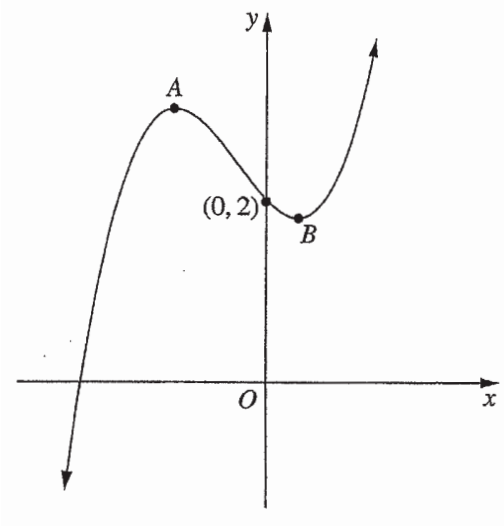
Equation of normal: $y - 0 = -\frac{1}{4}(x - 1)$

$-4y = x - 1$ (1)

$\therefore x + 4y - 1 = 0$ (1) – For leaving answer in general form.

**Question 7** (8 marks)

The graph of $y = x^3 + x^2 - x + 2$ is sketched below.




The points $A$ and $B$ are the turning points.

(a) Find the coordinates of $A$ and $B$. **4**

$\frac{dy}{dx} = 3x^2 + 2x - 1$

When $\frac{dy}{dx} = 0$;

$3x^2 + 2x - 1 = 0$ (1)

$(3x - 1)(x + 1) = 0$

$x = \frac{1}{3}, -1$ (1)

When $x = \frac{1}{3}$, $y = (\frac{1}{3})^3 + (\frac{1}{3})^2 - (\frac{1}{3}) + 2$ When $x = -1$, $y = 3$ (1) – For finding both y values

$= \frac{49}{27}$ $\therefore A(-1, 3)$ & $B(\frac{1}{3}, \frac{49}{27})$ (1)

(b) For what values of $x$ is the curve concave up? **2**

Show all mathematical reasons for your answer.

Concave up when $\frac{d^2y}{dx^2} > 0$

$\frac{d^2y}{dx^2} = 6x + 2$ $\therefore 6x + 2 > 0$ (1)

$x > -\frac{1}{3}$ (1)

(c) For what value(s) of $k$ does the equation $x^3 + x^2 - x + 2 = k$ only has one real solution? **2**

$k > 3$ (1) or $k < \frac{49}{27}$ (1)

**Examination continues overleaf...**

**Question 8** (4 marks)

Given $\log_7 2 = 0.36$ and $\log_7 5 = 0.83$, find the values of the following.

(a) $\log_7 0.4$ **2**

$\log_7 0.4 = \log_7 \frac{2}{5}$

$= \log_7 2 - \log_7 5$ (1)

$= 0.36 - 0.83$

$= -0.47$ (1)

(b) $\log_7 50$ **2**

$\log_7 50 = \log_7 (25 \times 2)$

$= \log_7 5^2 + \log_7 2$ (1)

$= 2\log_7 5 + \log_7 2$

$= 2 \times 0.83 + 0.36$

$= 2.02$ (1)

**Question 9** (3 marks)

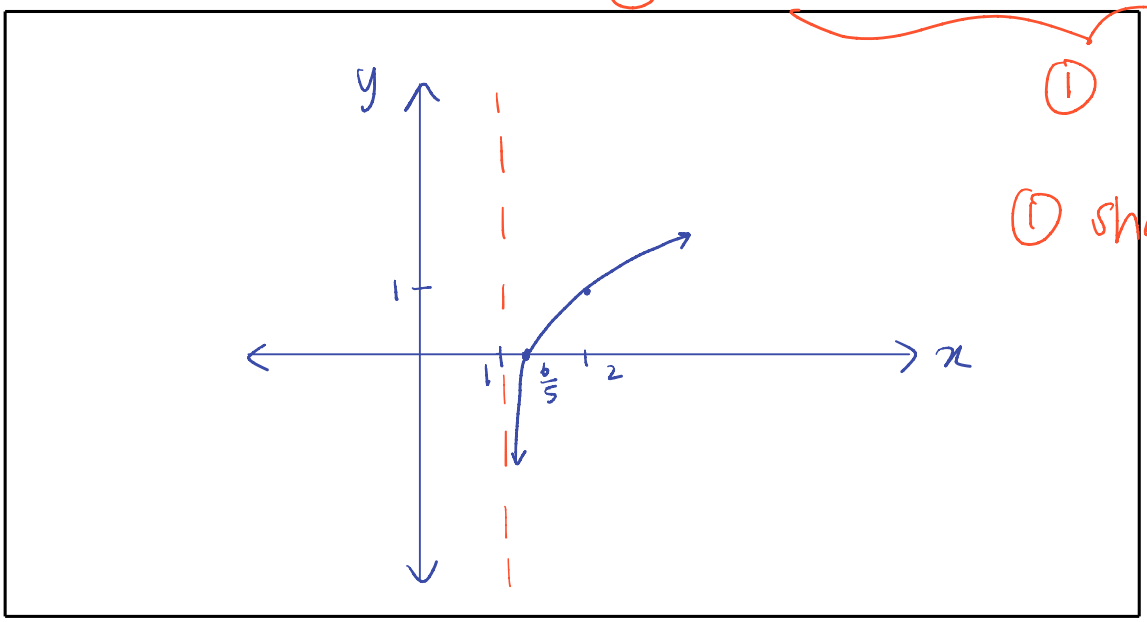
In the space below, sketch the graph of $y = \log_5(5x - 5)$, showing all key features including: **3**

- any $x$ intercept(s)
- any asymptote(s)

asymptote;
when $5x - 5 = 0$
$x = 1$ (1)

when $x = 2$;
$y = \log_5(5 \times 2 - 5)$
$= 1$.

$x$-intercept;
when $5x - 5 = 1$
$5x = 6$
$x = \frac{6}{5}$ (1)




(1) shape

**Examination continues overleaf...**

**Question 10** (4 marks)

(a) Solve the following equation for $x$, correct to 2 decimal places. **2**
*Full working out must be shown to gain full marks.*

$$0.4^{x+6} = 128$$

$\log 0.4^{x+6} = \log 128$ (1)

$(x+6)\log 0.4 = \log 128$

$x+6 = \dfrac{\log 128}{\log 0.4}$

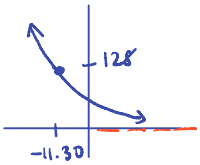
$x = \dfrac{\log 128}{\log 0.4} - 6$

$= -11.30$ (2dp) (1)

(b) By considering the graph of $y = 0.4^{x+6}$ and also part (a), find the values of $x$ such that **2**

$$0.4^{x+6} < 128$$

(*Hint:* draw a rough sketch of the graph of $y = 0.4^{x+6}$)



(1)

$x > -11.30$ (1)

**Examination continues overleaf...**

**Question 11** (4 marks)

A drug is used to control a medical condition. It is known that the quantity $Q$ of drug remaining in the body after $t$ hours satisfies an equation of the form

$$Q = Q_0 e^{-kt} \quad \text{where } Q_0 \text{ and } k \text{ are constants}$$

The initial dose is a 6 milligrams and after 15 hours the amount remaining in the body is half the initial dose.

(a) Find the value of $Q_0$. **1**

when $t=0$, $Q=6$

$6 = Q_0 e^{-k\times 0}$

$\therefore Q_0 = 6$

(b) Show that the value of $k$ is 0.046, correct to 3 decimal places. **1**

when $t=15$, $Q = \frac{1}{2}Q_0$

$\frac{1}{2}Q_0 = Q_0 e^{-15k}$

$\ln \frac{1}{2} = \ln e^{-15k}$

$\therefore -15k = \ln \frac{1}{2}$

$k = \frac{\ln \frac{1}{2}}{-15}$

$= 0.046$ (3dp)

(c) Show (with full working out) that it takes approximately 45 hours 12 minutes for one-eighth of the initial dose to remain. **2**

$t=?$ $Q = \frac{1}{8}Q_0$

$\frac{1}{8}Q_0 = Q_0 e^{-0.046t}$ } (1)

$\ln \frac{1}{8} = \ln e^{-0.046t}$

$-0.046t = \ln \frac{1}{8}$

$t = \frac{\ln \frac{1}{8}}{-0.046}$

$= 45^\circ 12' 18.9''$

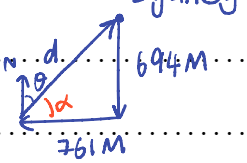
$\therefore$ takes approx. 45hrs 12mins. (1)

**Examination continues overleaf...**

**Question 12** (4 marks)

Sydney is 694 nautical miles (M) south and 761 M west of Noumea. **4**

A plane must fly to make a trip from Noumea to Sydney. Calculate the distance and the true bearing of the flight to the nearest nautical miles and degree respectively.




$d = \sqrt{694^2 + 761^2}$ $(d > 0)$ → (1)

$= 1029.93\ldots$

$= 1030$ M (1)

$\tan\alpha = \frac{694}{761}$

$\alpha = 42°21'48.6''$

$= 42°$ (nearest degree)

$\therefore \theta = 90° - 42°$

$= 48°$ (1)

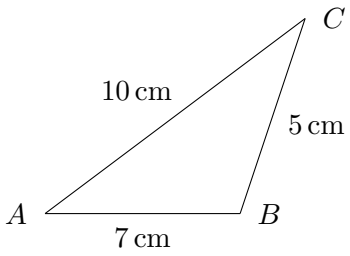
∴ bearing from Noumea to Sydney is

048°T (1)

**Examination continues overleaf...**

**Question 13** (3 marks)

The diagram shows $\triangle ABC$ with sides $AB = 7$cm, $BC = 5$cm and $AC = 10$cm.



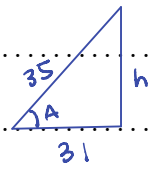

(a) Show that $\cos A = \dfrac{31}{35}$ **1**

$\cos A = \dfrac{7^2+10^2-5^2}{2\times 7\times 10}$ (1)

$= \dfrac{124}{140}$

$= \dfrac{31}{35}$

(b) Hence by finding the exact value of $\sin A$, determine the exact area of $\triangle ABC$. **2**




$h = \sqrt{35^2-31^2}$

$= \sqrt{264}$

$= 2\sqrt{66}$

$\therefore \sin A = \dfrac{2\sqrt{66}}{35}$ (1)

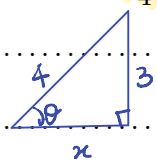
$A_{\triangle ABC} = \frac{1}{2} \times 7 \times 10 \times \dfrac{2\sqrt{66}}{35}$

$= 2\sqrt{66}$ units$^2$ (1)

**Examination continues overleaf...**

**Question 14** (2 marks)

Given $\sin\theta = \frac{3}{4}$ and $\tan\theta < 0$, find the exact value of $\cos\theta$ **2**




$x = \sqrt{4^2-3^2}$

$= \sqrt{7}$

$\therefore \cos\theta = -\frac{\sqrt{7}}{4}$

① for $\cos\theta = \frac{\sqrt{7}}{4}$

① for −ve sign.

**Question 15** (5 marks)

(a) Prove that $\frac{2\cos^2\theta}{1-\sin\theta} = 2 + 2\sin\theta$ **2**

$LHS = \frac{2\cos^2\theta}{1-\sin\theta}$

$= \frac{2(1-\sin^2\theta)}{1-\sin\theta}$ ①

$= \frac{2(1-\sin\theta)(1+\sin\theta)}{1-\sin\theta}$

$= 2 + 2\sin\theta$ ①

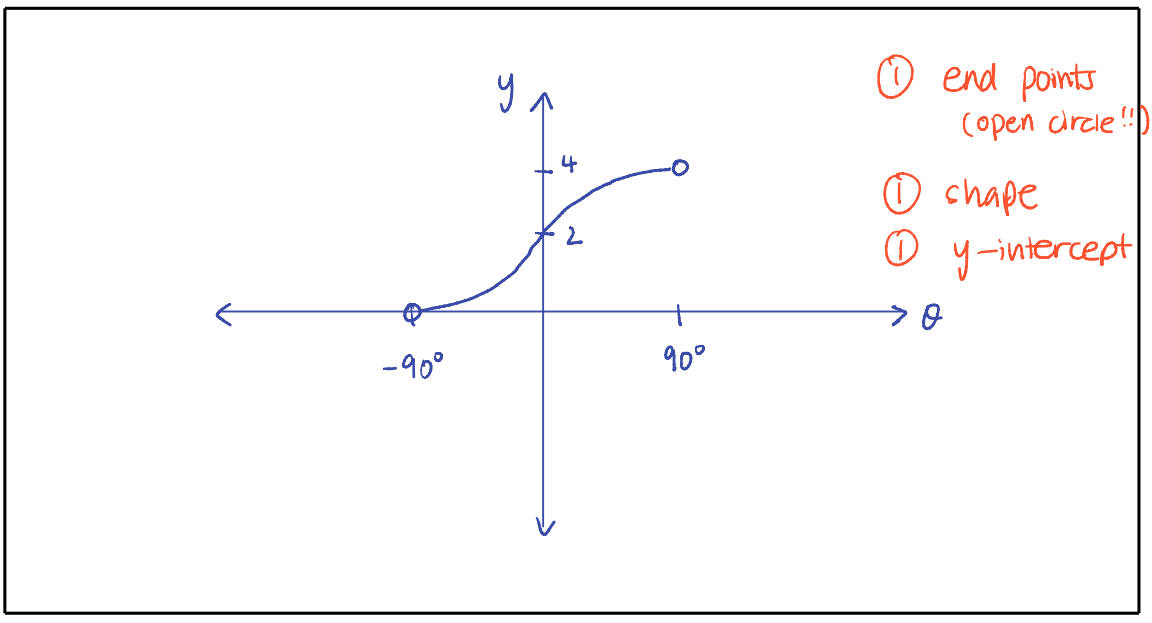
$= RHS$

(b) Hence draw a sketch of a function $y = \frac{2\cos^2\theta}{1-\sin\theta}$ in the domain $-90° < \theta < 90°$ **3**

$y = \frac{2\cos^2\theta}{1-\sin\theta}$

$= 2 + 2\sin\theta$

$a = 2 \quad n = 1 \quad T =$




① end points (open circle!!)

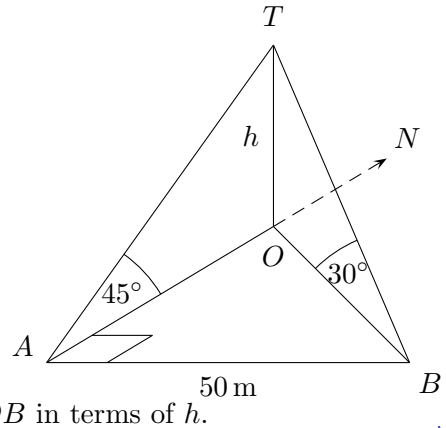
① shape

① y-intercept

**Examination continues overleaf...**

**Question 16** (6 marks)

A person is standing at point $A$, which is due south of a tower $OT$ that is $h$ metres tall. The person walks 50 metres east to point $B$. The angle of elevation at points $A$ and $B$ to the top of the tower are 45° and 30° respectively.




(a) Express $OA$ and $OB$ in terms of $h$. **2**

$\tan 45° = \frac{h}{OA}$ $\quad \tan 30° = \frac{h}{OB}$

$OA = \frac{h}{\tan 45°}$ $\quad OB = \frac{h}{\tan 30°}$

$= h$ (1) $\quad = \sqrt{3}h$ (1)

(b) Show that $h = 25\sqrt{2}$ metres. **2**

In $\triangle AOB$, $OB^2 = OA^2 + AB^2$

$(\sqrt{3}h)^2 = h^2 + 50^2$ (1)

$2h^2 = 50^2$ $\quad \therefore h = \sqrt{1250}$

$h = \sqrt{\frac{50^2}{2}}$ $\quad h > 0$ $\quad = 25\sqrt{2}$ metres (1)

(c) Find the bearing of $B$ from the base of the tower $O$, correct to the nearest minute. **2**

$\tan \angle AOB = \frac{AB}{OA}$

$= \frac{50}{25\sqrt{2}}$

$\angle AOB = 54° 44'$ (nearest mins) (1)

$\therefore$ bearing of B from O is $125° 16'$ (nearest min) (1)

**End of paper.**