



NORTH SYDNEY BOYS HIGH SCHOOL

2011 HSC ASSESSMENT TASK 1

Mathematics

General Instructions

- Working time – 50 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page**.
- Attempt all questions

Class Teacher:

(Please tick or highlight)

- ☐ Mr Berry
- ☐ Mr Lowe
- ☐ Mr Trenwith
- ☐ Mr Weiss
- ☐ Mr Lam
- ☐ Mr Ireland
- ☐ Mr Fletcher
- ☐ Mr Rezcallah

Student Number: _____

(To be used by the exam markers only.)

Question No	1	2	3	4	5	Total	Total %
Mark	$\frac{9}{9}$	$\frac{11}{11}$	$\frac{11}{11}$	$\frac{7}{7}$	$\frac{12}{12}$	$\frac{50}{50}$	$\frac{100}{100}$

Question 1 (9 marks) *Start a new page***Marks**(a) Differentiate with respect to x :

(i) $5x^{-3}$ 1

(ii) $\frac{x+3}{x^2-1}$ 2

(b) For the function: $f(x) = (7x - x^4)^5$. Find:

(i) $f'(x)$ 2

(ii) $f'(2)$ 1

(c) Find the equation of the normal to the curve $y = 3 + 4x^2 - x^5$ at the point $(-1, 8)$. 3**Question 2 (11 marks)** *Start a new page*

(a) The third term of an arithmetic sequence is -1 and the eighth term is 24:

(i) Find the first term and the common difference. 3

(ii) Find the sum of the first 20 terms. 2

(iii) Is 1007 a term in this sequence? Justify your answer with working. 2

(b) Evaluate $\sum_{n=2}^5 \frac{n}{3}$ 2

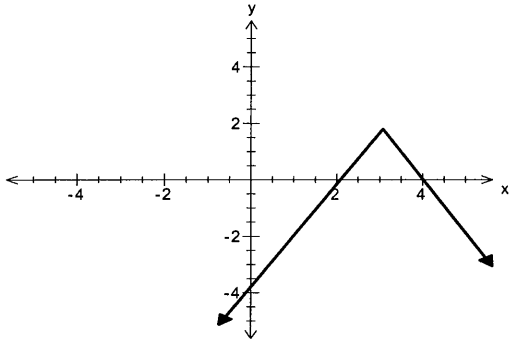
(c) If the sum of the first n terms of a sequence is given by: $S_n = \frac{n^2+n}{2}$, find an expression for T_n . 2

Question 3 (11 marks) *Start a new page*

Marks

- (a) Draw a neat sketch of the first derivative of the following function.

2



- (b) For the curve $y = 2x^3 + 3x^2 - 12x + 2$:

- (i) Find any stationary points and determine their nature.

4

- (ii) Find any points of inflexion.

2

- (iii) Sketch a neat graph of the curve showing all relevant features (NOT x -intercepts).

2

- (iv) For what values of x is the curve concave up?

1

Question 4 (7 marks) *Start a new page*

- (a) For the GP: $x + 2x^2 + 4x^3 + 8x^4 + \dots$

- (i) For what values of x does a limiting sum exist?

1

- (ii) Find an expression for the limiting sum.

1

- (b) Bright Spark light globes manufactures 5000 light globes in their first year of operation. Each year they manufacture 20% more than they did the previous year.

- (i) State how many light globes Bright Spark will manufacture in their first, second and third years of operation.

2

- (ii) Prove that this forms a GP with a common ratio of 1.2

1

- (iii) Find the total number of light globes Bright Spark will produce in their first decade of operation.

2

Examination continues over the page

Question 5 (12 marks) *Start a new page***Marks**

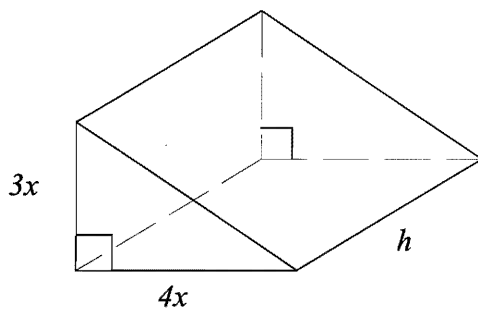
(a) The displacement of a particle from the origin is given by the formula:

$$x = 8 + 2t - t^2.$$

Where x is the displacement in metres and t is the time in seconds.

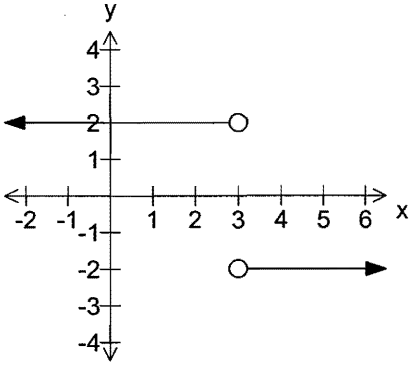
- | | |
|--|----------|
| (i) What is the initial position of the particle? | 1 |
| (ii) Write an equation for the velocity of the particle. | 1 |
| (iii) When is the particle stationary? | 1 |
| (iv) What is the acceleration of the particle? | 1 |
| (v) What is the total distance travelled by the particle over the first 4 seconds? | 2 |

(b) The figure below is a right triangular prism.
All the units are in cm. The volume is 96 cm^3 .



- | | |
|--|----------|
| (i) Find an expression for h in terms of x . | 1 |
| (ii) Show that the surface area, $A \text{ cm}^2$, can be expressed as: | |
| $A = 12x^2 + \frac{192}{x}$ | 2 |
| (iii) Find the value of x which will minimise the surface area. | 3 |

	Suggested Solutions – 2011 HSC Assessment Task 1	Marks
2 a) i)	$T_3 = -1$ $T_8 = 24$ $24 = a + (8 - 1)d \dots \textcircled{1}$ $-1 = a + (3 - 1)d \dots \textcircled{2}$ $\textcircled{2} - \textcircled{1}$ $25 = 7d - 2d$ $5d = 25$ $\therefore d = 5$ <i>substitute into</i> $\textcircled{2}$ $-1 = a + (2 \times 5)$ $\therefore a = -11$	✓ Simultaneous equations ✓ Common difference ✓ First term
2 a) ii)	$S_n = \frac{n}{2}(2a + (n - 1)d)$ $= \frac{20}{2}(2 \times (-11) + (20 - 1) \times 5)$ $= 730$	✓ Correct use of formula ✓ Correct answer
2 a) iii)	$T_n = a + (n - 1)d$ $1007 = -11 + (n - 1) \times 5$ $5(n - 1) = 1018$ $n - 1 = 203.6$ $n = 204.6$ $\therefore 1007$ is not a term since n is not a whole number	✓ Answer (not a term) ✓ Reason
2 b)	$\sum_{n=2}^5 \frac{n}{3} = \frac{2}{3} + \frac{3}{3} + \frac{4}{3} + \frac{5}{3}$ $= \frac{14}{3}$	✓ Expansion of sigma ✓ Correct answer
2 c)	$T_n = S_n - S_{n-1}$ $= \frac{n^2 + n}{2} - \frac{(n-1)^2 + (n-1)}{2}$ $= \frac{n^2 + n}{2} - \frac{n^2 - 2n + 1 + n - 1}{2}$ $= \frac{n^2 + n - n^2 + 2n - 1 - n + 1}{2}$ $= \frac{2n}{2}$ $= n$	✓ Substitution into formula ✓ Correct answer

	Suggested Solutions – 2011 HSC Assessment Task 1	Marks
3 a)		✓ Two horizontal lines ✓ Point of discontinuity
3 b) i)	$y' = 6x^2 + 6x - 12$ $y'' = 12x + 6$ when $y' = 0$ $6x^2 + 6x - 12 = 0$ $6(x^2 + x - 2) = 0$ $6(x + 2)(x - 1) = 0$ $\therefore$ stationary points occur when: $x = -2 \text{ and } x = 1$ when $x = -2$ $y = 2(-2)^3 + 3(-2)^2 - 12(-2) + 2$ $= 22$ $y'' = (12)(-2) + 6$ $= -18 (< 0)$ $\therefore$ there is a maxima at $(-2, 22)$ when $x = 1$ $y = 2(1)^3 + 3(-1)^2 - 12(1) + 2$ $= -5$ $y'' = 12(1) + 6$ $= 18 (> 0)$ $\therefore$ there is a minima at $(1, -5)$	✓ Derivative ✓ x values ✓ y values ✓ Testing nature

	Suggested Solutions – 2011 HSC Assessment Task 1	Marks
4 a) i)	$-1 < r < 1$ $-1 < 2x < 1$ $-\frac{1}{2} < x < \frac{1}{2}$	✓ Correct answer
4 a) ii)	$S_n = \frac{a}{1-r}$ $= \frac{x}{1-2x}$	✓ Correct answer
4 b) i)	$Year\ 1 = 5000$ $Year\ 2 = 5000 \times 1.2$ $= 6000$ $Year\ 3 = 6000 \times 1.2$ $= 7200$	 ✓ Year 2 value ✓ Year 3 value
4 b) ii)	$\frac{T_3}{T_2} = \frac{7200}{6000}$ $= 1.2$ $\frac{T_2}{T_1} = \frac{6000}{5000}$ $= 1.2$ $\therefore \frac{T_3}{T_2} = \frac{T_2}{T_1}$ $\therefore$ this is a GP where $r = 1.2$	 ✓ Correct proof
4b) iii)	$S_n = \frac{a(r^n - 1)}{r - 1}$ $= \frac{5000(1.2^{10} - 1)}{1.2 - 1}$ $= 129\ 793$	 ✓ Substitution into formula ✓ Correct answer

	Suggested Solutions – 2011 HSC Assessment Task 1	Marks
5 a) i)	<i>initially</i> $t = 0$ $\therefore x = 8 + 2(0) - 0^2$ $= 8m$	✓ Correct answer
5 a) ii)	$v = \frac{dx}{dt}$ $= 2 - 2t \text{ ms}^{-1}$	✓ Correct answer
5 a) iii)	<i>when</i> $v = 0$ $\therefore 2 - 2t = 0$ $t = 1 \text{ second}$	
5 a) iv)	$a = \frac{dv}{dt}$ $= -2 \text{ ms}^{-2}$	✓ Correct answer
5 a) v)	<i>direction change at 1 second</i> $\therefore \text{when } t = 0$ $x = 8$ <i>when</i> $t = 1$ $x = 8 + 2(1) - (1)^2$ $= 9$ <i>when</i> $t = 4$ $x = 8 + 2(4) - (4)^2$ $= 0$ $\therefore \text{distance} = 9 - 8 + 0 - 9 $ $= 1 + 9$ $= 10 \text{ m}$	✓ Recognising change of direction ✓ Correct answer

	Suggested Solutions – 2011 HSC Assessment Task 1	Marks
5 b) i)	$V = Ah$ $96 = \left(\frac{1}{2}\right)(3x)(4x)(h)$ $96 = 6x^2h$ $\therefore h = \frac{16}{x^2}$	✓ Correct answer
5 b) ii)	$\text{hypotenuse} = 5x \text{ cm (by Pythagoras' Theorem)}$ $\therefore A = (2)\left(\frac{1}{2}\right)(3x)(4x) + 3xh + 4xh + 5xh$ $= 12x^2 + 12xh$ $= 12x^2 + 12x\left(\frac{16}{x^2}\right) \text{ (from part (i))}$ $= 12x + \frac{192}{x}$	✓ Addition of areas ✓ Substitution from i)
5 b) iii)	$A = 12x^2 + 192x^{-1}$ $\therefore A' = 24x - 192x^{-2}$ $\text{when } A' = 0$ $24x - \frac{192}{x^2} = 0$ $24x = \frac{192}{x^2}$ $24x^3 = 192$ $x^3 = 8$ $x = 2$ $A'' = 24 + \frac{384}{x^3}$ $\text{when } x = 2$ $A'' = 24 + \frac{384}{2^3}$ $= 72 (> 0 \text{ which implies a minima})$ $\therefore x = 2 \text{ cm will minimise the surface area}$	✓ Derivative ✓ Solving for x ✓ Proof of minima