NORTH SYDNEY BOYS HIGH SCHOOL

2010 HSC ASSESSMENT TASK 4

Mathematics

General Instructions

- Working time – 50 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- O Mr Berry
- O Mr Fletcher
- O Mr Rezcallah
- O Mr Lowe
- O Mr Ireland
- O ....

Student Number:_________________________

(To be used by the exam markers only.)

Question No	1	2	3	4	5	Total	Total %
Mark	/7	/13	/12	/12	/6	/50	/100

Question 1 (Start on a new page)

a) The surface area of a balloon is given by the formula:

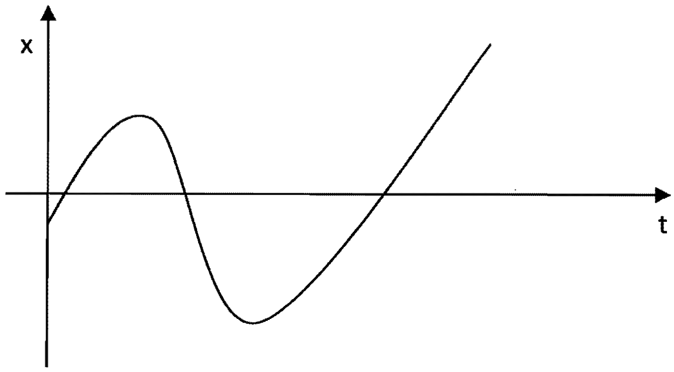
$$SA = 2t^2 + 5t + 7$$

where SA is the surface area in cm^2 and t is the time in seconds:

i) Find the initial surface area of the balloon **1**

ii) How fast is the surface area increasing after 5 seconds? **2**

b) The displacement of a particle is shown in the graph below. Copy the diagram into your answer booklet.



On two different sets of axes draw the graphs for the velocity and acceleration showing all relevant features (i.e. x-intercepts and stationary points) **4**

Question 2 (Start on a new page)

a) A number is selected at random from the numbers 1-100 (inclusive). What is the probability that the number is even or divisible by 3? **2**

b) 5 black balls, 4 white balls and 3 red balls are placed inside a bag. Two balls are drawn without replacement.

i) Draw a probability tree diagram to represent the situation. **1**

ii) What is the probability of:

a) Both the balls being black **1**

b) Neither of the balls being white **1**

c) At least one of the balls being red **1**

c) 150 Year 12 students were asked what they had for breakfast that day.
80 had toast, 60 had cereal and 50 skipped breakfast altogether.

i) Draw a Venn diagram to represent the situation. **2**

ii) If a student is chosen at random, what is the probability that they had cereal but not toast that day? **2**

d) Warren has 4 keys that look the same. Only one of them can unlock the store room door in his office. Each day when he arrives at work he unlocks this door.

i) On a certain day he tries to unlock the door. What is the probability that he will need to try all 4 keys before unlocking the door ? **2**

ii) What is the probability that on 3 consecutive days, Warren will need to try all 4 keys before unlocking the door ? **1**

Question 3 (Start on a new page)

a) The velocity of a particle given by $V = 8t - 16 \text{ ms}^{-1}$. When $t = 0$ the particle is at the origin.

i) What is the acceleration of the particle? **1**

ii) How fast is the particle moving at 2 seconds? **1**

iii) How far does the particle travel in the first 5 seconds? **3**

b) The population N of a certain type of ant is growing according to $N = 400e^{kt}$, where t is the time in weeks after the ants were first counted. At the end of 4 weeks the population has doubled.

i) How many ants were first counted? **1**

ii) Calculate the value of the constant k. **2**

iii) At what rate will the population be increasing after 5 weeks? **2**

iv) Calculate the number of weeks required for this population of ants to reach 12800. **2**

Examination continues over page

Question 4 (Start on a new page)

a) A particle is moving along the x axis. Its displacement x metres from the origin after t seconds is given by

$$x = 1 - 2\cos 4t \quad \text{for} \quad 0 \leq t \leq \frac{\pi}{2}.$$

i) Sketch the graph of the displacement of the particle. **2**

ii) Find when and where the particle first comes to rest after passing through the origin. **2**

iii) Find the maximum speed of this particle. **2**

b) A particle starts to move from the origin along the x axis. Its velocity in ms^{-1} is given by:

$$v = 3 - \frac{1}{2t+1}$$

i) What is the initial velocity of the particle? **1**

ii) Explain why the particle is never at rest. **2**

iii) Find the time taken for the acceleration to reach $0.08ms^{-2}$. **3**

Question 5 (Start on a new page)

a) A biologist wishes to estimate the number of butterflies living in a region. She captures 120 butterflies, tags their wings, and releases them back into the area.

The next day she captures 150 butterflies and notes that 20 of them are tagged. Estimate the approximate butterfly population. **2**

b) Ada, Hypatia, Sofia and Shafi go to a restaurant for dinner. The menu at a restaurant has 3 entrees, 5 main courses and 4 desserts.

i) How many different possible meals could be ordered? **1**

ii) What is the probability that

a) Ada and Shafi order the exact same meal? **1**

b) At least two of the women order identical meals? **2**

End of Examination

2 Unit - Task 4 - 2010

Question 1

a) i) $SA = 2t^2 + 5t + 7$

at $t = 0$

$SA = 2(0)^2 + 5(0) + 7$

$= 7 \text{ cm}^2$ ✓

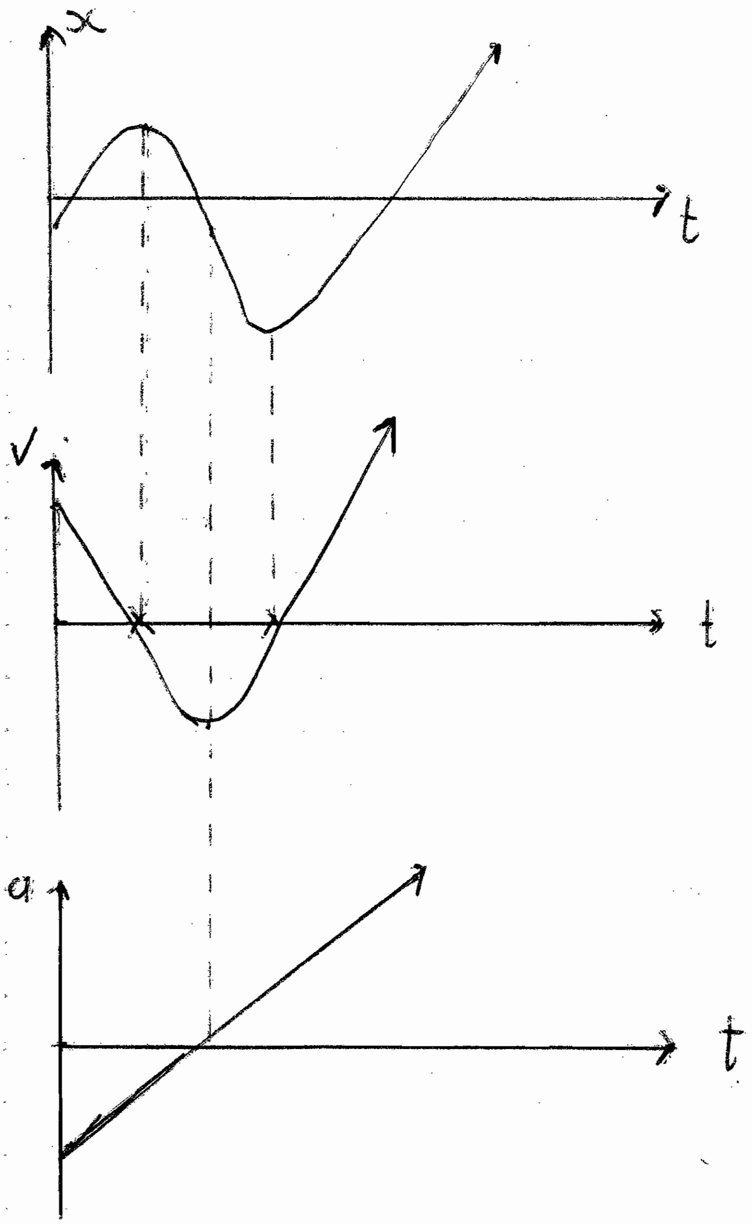
ii) $\frac{dSA}{dt} = 4t + 5$ ✓

at $t = 5$

$\frac{dSA}{dt} = 4(5) + 5$

$= 25 \text{ cm}^2/\text{s}.$ ✓

b)



✓ - correct shape

✓ - x-intercepts & S. P.

✓ - correct shape

✓ - x-intercept

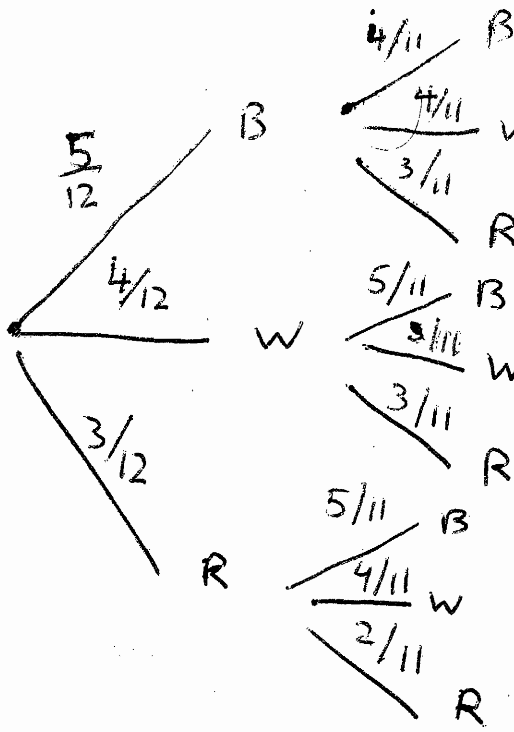
Question 2

a) $P(2 \cup 3) = P(2) + P(3) - P(2 \cap 3)$ ✓

$= \frac{50}{100} + \frac{33}{100} - \frac{16}{100}$

$= \frac{67}{100}$ ✓

b) i)



BB $\left(\frac{20}{132}\right)$

BW $\left(\frac{20}{132}\right)$

BR $(15/132)$ ✓ (tree only)

WB $(20/132)$

WW $(12/132)$

WR $(12/132)$

RB $(15/132)$

RW $(12/132)$

RR $(6/132)$

ii) a) $\frac{20}{132} = \frac{5}{33}$ ✓

~~b) $1 - \frac{12}{132} = \frac{10}{11}$~~

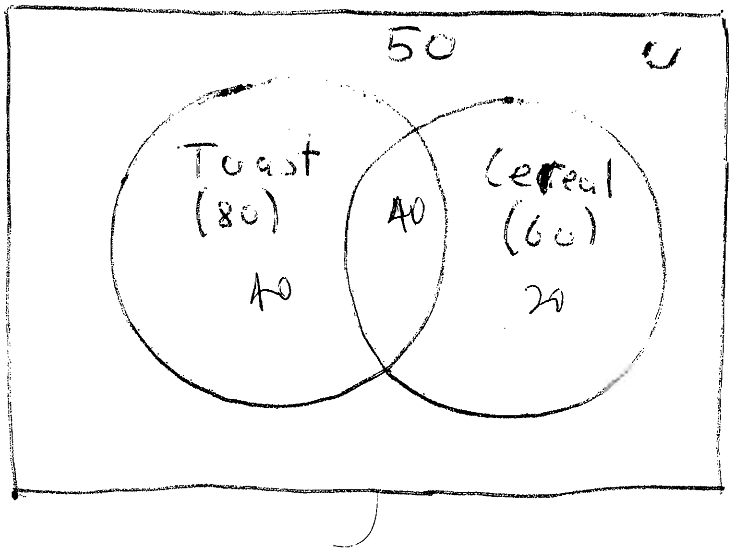
$\frac{5}{12} \times \frac{4}{11} + \frac{5}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{5}{11} + \frac{3}{12} \times \frac{2}{11} = \frac{14}{33}$ ✓

c) $\frac{15}{132} + \frac{12}{132} + \frac{15}{132} + \frac{12}{132} + \frac{6}{132}$

$= \frac{5}{11}$ ✓

OR $1 - \left(\frac{5}{12} \times \frac{4}{11} + \frac{5}{12} \times \frac{4}{11} + \frac{4}{12} \times \frac{5}{11} + \frac{4}{12} \times \frac{3}{11}\right)$

c) i)



✓ - intersecting circles

✓ - universal set

ii) $|T \cup C| = 150 - 50$
$= 100$

$\therefore |T \cap C| = 40$

$\therefore |C \cap {}^{-}T| = 60 - 40$
$= 20$ ✓

$\therefore P(C \cap {}^{-}T) = \frac{20}{150}$ ✓

$= \frac{2}{15}$

d) i) $\frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{6}{24}$ ✓✓

$= \frac{1}{4}$

ii) $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$ ✓

Question 3.

(a) (i) $\ddot{x} = 8 \, m/s^2$

(ii) @ $t = 2$ $\quad V = 8(2) - 16 = 0 \, m/s$

Particle is stationary.

(iii) First Method:

$x = \int (8t - 16)\,dt = 4t^2 - 16t + c$

@ $t = 0$, $x = 0$ $\quad \therefore \quad c = 0$

$x = 4t^2 - 16$.

@ $t = 2 \longrightarrow x = -16 \, m$

@ $t = 5 \longrightarrow x = 20 \, m$.

Distance $= (0 - -16) + (20 - (-16))$

$= 16 + 36 = 52m$

Note: write x = distance = 20 only 1 mark is allocated!!
students had to use the stationary pt. to get the distance.

2ND Method:

$$\text{Distance} = \left|\int_0^2 (8t - 16)\,dt\right| + \left|\int_2^5 (8t - 16)\,dt\right|$$

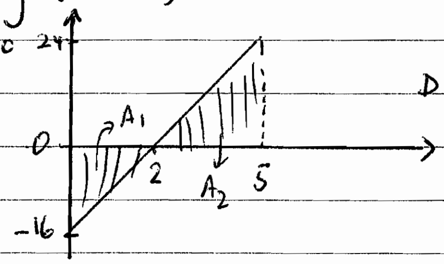
$$= \left|\left[4t^2 - 16t\right]_0^2\right| + \left[4t^2 - 16t\right]_2^5$$

$$= \left|4 \times 4 - 16 \times 2 - 0\right| + \left|\left[4 \times 25 - 16 \times 5 - (4 \times 4 - 32)\right]\right|$$

$$= 16 + 20 + 16 = 52 \, m.$$

Note: Writing $\int_0^5 (8t - 16)\,dt = 20m$ only gets 1 mark.

3rd method:



Distance $= A_1 + A_2$

$= \frac{1}{2} \times 2 \times 16 + \frac{24 \times 3}{2}$

$= 16 + 36$

$= 52m.$

(b) (i) $N = 400e^0 = 400$ ants.

(ii) $800 = 400\,e^{4k}$

$2 = e^{4k}$

$4k = \ln 2$. $\Rightarrow k = \frac{\ln 2}{4} = 0.1733\cdots$

(iii) $\frac{dN}{dt} = 400k\,e^{kt}$

$= 400 \times \frac{\ln 2}{4} \, e^{\frac{\ln 2}{4} \times 5}$ $= 164.86$ ants/week

$= 165$ ants / week

(iv) $12800 = 400e^{kt}$

$32 = e^{\frac{\ln 2}{4} t}$

$\ln 32 = \frac{\ln 2}{4} \cdot t$ $\quad \therefore t = \frac{4 \ln 32}{\ln 2} = 4 \times 5$

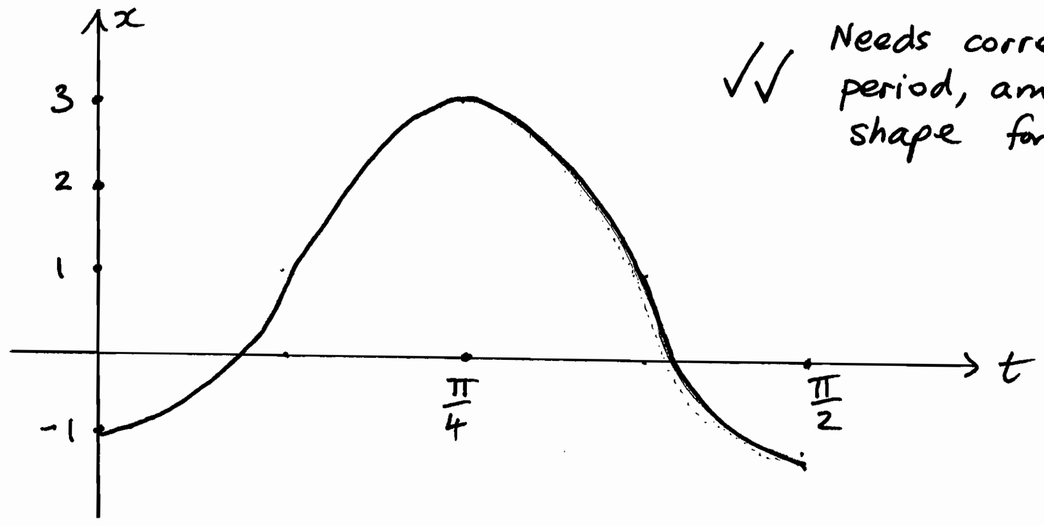
$= 20$ weeks

To get the 2ND mark students should use the exact value of k to get 20.

QUESTION 4

(a)

(i)



✓✓ Needs correct period, amplitude, shape for full marks

(ii) From the graph, particle is at rest at $t = \frac{\pi}{4}$ seconds, when $x = 3$ m.

✓ correct time

✓ correct x value

(iii) $x = 1 - 2\cos 4t$

$\therefore\ v = 8 \sin 4t$ ✓ v equation

Since $|\sin 4t| \leq 1$ $\therefore$ maximum speed $= 8$ m/s ✓ answer

(b) (i) at $t = 0$, $v = 3 - \frac{1}{2(0)+1} = 2$ m/s ✓

(ii) 'at rest' $\Rightarrow v = 0$ $\therefore\ 3 - \frac{1}{2t+1} = 0$

$\therefore\ 2t + 1 = \frac{1}{3}$

$\therefore\ t = -\frac{1}{3}$ ✓

but $t \geq 0$, $\therefore$ v cannot be 0

$\therefore$ particle is never at rest. ✓

* Solutions need mathematical reasoning, and words.
- Clearly demonstrating that $\frac{1}{2t+1}$ is >0 and ≤ 1, and making a conclusion, is appropriate.
- Calculating +ve acceleration is not by itself sufficient, unless linked with $v(0) = +2$.

Question 4 (cont'd)

(iii) $V = 3 - \dfrac{1}{2t+1}$

$= 3 - (2t+1)^{-1}$

$\therefore a = (-1)\cdot -(2t+1)^{-2}\cdot 2$

$\therefore a = \dfrac{2}{(2t+1)^2}$ ✓ correct acceleration

Solving, $\dfrac{2}{(2t+1)^2} = 0\cdot 08$

$\therefore \dfrac{1}{(2t+1)^2} = 0\cdot 04$

$\therefore (2t+1)^2 = 25$ ✓ correct working

$(2t+1) = \pm 5$

$\therefore 2t = -6, 4$

$t = -3, 2$

But $t > 0$ $\therefore t = 2$ seconds. ✓ $t = 2$ secs.

* If answers give $a = \dfrac{1}{(2t+1)^2}$, then answers ending in $t \doteqdot 1.268$ secs or $t = \dfrac{-2+5\sqrt{2}}{4}$ get max. 2 marks.

* If solutions give the integral, not derivative, ie $-\ln(2t+1)$, or similar, then no marks given [it is a terminating error].

* If solutions lose the square on the $(2t+1)^2$, then max. of 1 mark, as solution is too easy.

Question 5

a) Let x be the total number

$$\frac{20}{150} = \frac{120}{x}$$ ✓

$\therefore 20x = 18000$

$\therefore x = 900$ ✓

b) i) $3 \times 5 \times 4 = 60$ ✓

ii) a) $P(\text{same}) = 1 \times \frac{1}{60}$

$= \frac{1}{60}$ ✓

iii) $P(\text{all different}) = 1 \times \frac{59}{60} \times \frac{58}{60} \times \frac{57}{60}$

$= \frac{195054}{216000}$

$P(\geq 2 \text{ same}) = 1 - P(\text{All Different})$ ✓

$= \frac{20946}{216000}$ ✓

$= \frac{3491}{36000}$

(answers in decimal or percentage format also okay)