NORTH SYDNEY BOYS HIGH SCHOOL

2013 HSC ASSESSMENT TASK 1

Mathematics

General Instructions

Section A contains 5 multiple choice questions to be answered on the booklet provided.

Reading Time – 5 minutes
Working Time – 50 minutes
Total Time – 55 minutes

Section B is to be answered in the booklet provided, showing ALL necessary working.

- Write using blue or black pen
- Board approved calculators may be used
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher: Please tick

- O Mr Lowe
- O Mr Berry
- O Mr Lin
- O Mr Lucas
- O Mr Lam
- O Mr Fletcher
- O Ms Ziaziaris

Student Number:____________________________

(To be used by the exam markers only.)

Question No	1-5	6	7	8	9	10	Total	Total %
Mark	/5	/13	/~~10~~	/10	/8	/5	/~~51~~	/100

8 marks

Total: 49 marks

SECTION A

Question 1

What is the value of $\frac{dy}{dx}$ if $y = 2\sqrt{x}$?

(A) $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$

(B) $\frac{dy}{dx} = \frac{2}{\sqrt{x}}$

(C) $\frac{dy}{dx} = \frac{\sqrt{x}}{2}$

(D) $\frac{dy}{dx} = 2$

Question 2

What is the gradient of the curve $y = x^2 - x - 6$ at $(6, 24)$?

(A) 11

(B) 12

(C) 23

(D) 24

Question 3

What values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

(A) $x < -\frac{1}{6}$

(B) $x > -\frac{1}{6}$

(C) $x < -6$

(D) $x > 6$

Question 4

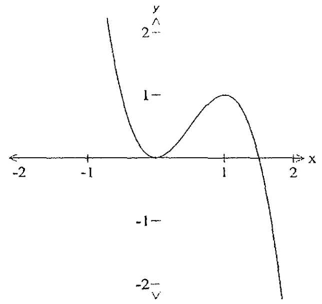
In the first week of the snow season 5 cm of snow falls. In each following weeks the snowfalls increase by 2 cm, so in the second week there is 7 cm, in the third week there is 9 cm. How much snow falls in the 11th week?

(A) 23 cm
(B) 25 cm
(C) 27 cm
(D) 29 cm

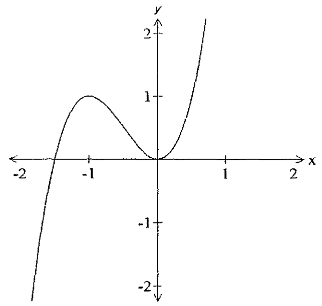
Question 5

Which of the following is the graph of $f(x) = 2x^3 - 3x^2$?

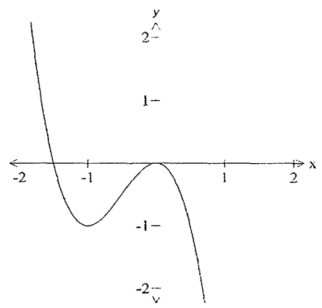
(A)



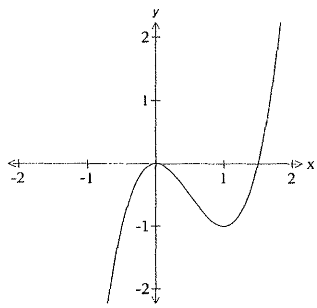
(B)



(C)



(D)



Section B

Question 6 (13 Marks) Commence a NEW page. **Marks**

(a) Differentiate with respect to x:

(i) $1 + 2x - 4x^2 - x^3$ **1**

(ii) $\sqrt{1 - x^2}$ **1**

(iii) $(x - 1)^5(3x^2 + 1)$ **3**

(iv) $\dfrac{x+4}{x+5}$ **2**

(b) Find the limit of the following:

(i) $\lim\limits_{x \to 4} \dfrac{x}{x+4}$ **1**

(ii) $\lim\limits_{x \to -4} \dfrac{x^2 + 3x - 4}{x+4}$ **1**

(iii) $\lim\limits_{x \to \infty} \dfrac{3x^2 - x + 1}{x^2 - 1}$ **1**

(c) Find the coordinates of the point(s) on the curve $y = 3x^2 + 4x + 2$ where the tangent is perpendicular to the line $x + 2y - 10 = 0$ **3**

Question 7 (10 Marks) Commence a NEW page. **Marks**

(a) An arithmetic series has a seventh term of 8 and the common difference is -4

(i) What is the first term a? **1**

(ii) Calculate the sum of the first 17 terms. **2**

(b) Evaluate $\displaystyle\sum_{2}^{5} (-1)^k (\frac{1}{k})$ **2**

(c) The third term of a geometric series is $\frac{3}{4}$ and the seventh term is 12.

(i) Find the 14th term of the series. **3**

(ii) Find the sum of the first 14 terms. **2**

Question 8 (10 Marks) Commence a NEW page. **Marks**

Consider the curve given by $y = \frac{1}{4}x^4 - x^3$

(i) Find any turning points and determine their nature. **4**

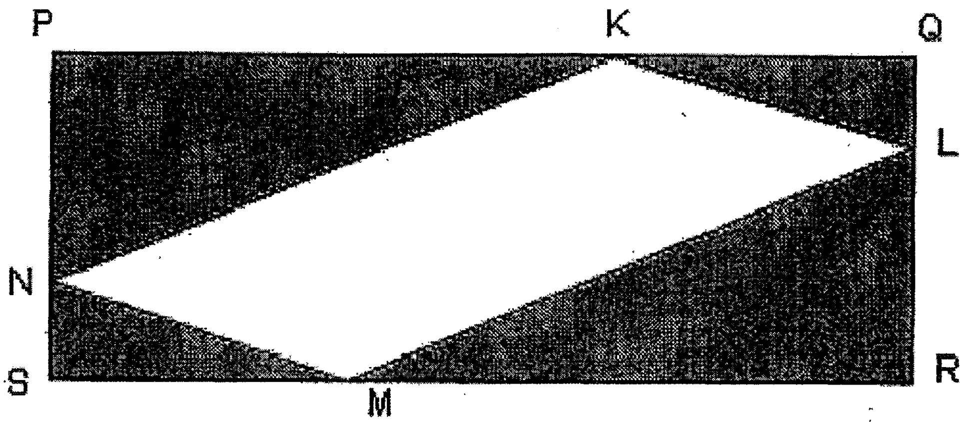
(ii) Find any points of inflexion. **3**

(iii) Sketch the curve for $-1 \leq x \leq 5$, indicating where the curve crosses the x axis. **2**

(iv) For what values of x is the curve concave down? **1**

Question 9 (8 Marks) Commence a NEW page. **Marks**

(a) In the diagram below, PQRS is a rectangle with PQ = 40cm and SP = 10 cm. The shaded portions are cut away, leaving away a parallelogram KLMN. QL = SN = x cm and QK = SM = $4x$ cm.



(i) Show that the area of the parallelogram KLMN is given by $A = 80x - 8x^2$ **2**

(ii) What is the domain of x? **1**

(iii) Find the maximum area of the parallelogram KLMN **2**

(b) Sketch the graph of the function that satisfies the given conditions: **3**

$$\lim_{x \to 3} f(x) = -\infty, \qquad f''(x) < 0 \text{ if } x \neq 3, \qquad f'(0) = 0$$

$$f'(x) > 0 \text{ if } x < 0 \text{ or } x > 3, \qquad f'(x) < 0 \text{ if } 0 < x < 3$$

Question 10 (5 Marks) Commence a NEW page. **Marks**

A_n and B_n are two series given by

$A_n = 1^2 + 5^2 + 9^2 + 13^2 + \cdots + (4n-3)^2$

$B_n = 3^2 + 7^2 + 11^2 + 15^2 + \cdots$

(i) Find the nth term of B_n **1**

(ii) If $S_{2n} = A_n - B_n$ Prove that $S_{2n} = -8n^2$ **2**

(iii) Hence or otherwise evaluate **2**

$$101^2 - 103^2 + 105^2 - 107^2 + \cdots + 2001^2 - 2003^2$$

END OF EXAMINATION

MC Answers: 1. A 2. A 3. A 4. B 5. D.

Question 6

a) i) $2 - 8x - 3x^2$ (1 mark)

ii) $f(x) = \sqrt{1-x^2}$

$= (1-x^2)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot -2x$

$= -\frac{x}{\sqrt{1-x^2}}$ (1 mark)

iii) $f(x) = (x-1)^5(3x^2+1)$

$f'(x) = (x-1)^5 \cdot \frac{d}{dx}(3x^2+1) + (3x^2+1) \cdot \frac{d}{dx}(x-1)^5$

$= (x-1)^5 \cdot 6x + (3x^2+1) \cdot 5(x-1)^4$ (*)

$= (x-1)^4 [(x-1) \cdot 6x + (3x^2+1) \cdot 5]$

$= (x-1)^4 [6x^2 - 6x + 15x^2 + 5]$

$= (x-1)^4 [21x^2 - 6x + 5]$ (3 marks)

iv) $f(x) = \frac{x+4}{x+5}$

$f'(x) = \frac{(x+5) \cdot \frac{d}{dx}(x+4) - (x+4) \cdot \frac{d}{dx}(x+5)}{(x+5)^2}$

$= \frac{(x+5) - (x+4)}{(x+5)^2}$

$= \frac{1}{(x+5)^2}$ (2 marks)

b) (i) $\lim_{x \to 4} \frac{x}{x+4}$

$= \frac{4}{4+4}$

$= \frac{1}{2}$ (1 mark)

(ii) $\lim_{x \to -4} \frac{(x+4)(x-1)}{(x+4)}$

$= \lim_{x \to -4} x - 1$

$= -5$ (1 mark)

(iii) $\lim_{x \to \infty} \frac{3x^2 - x + 1}{x^2 - 1}$

$= \lim_{x \to \infty} \frac{3 - \frac{1}{x} + \frac{1}{x^2}}{1 - \frac{1}{x^2}}$

$= 3$ (1 mark)

c) $y = 3x^2 + 4x + 2$

radient nction of tangent: $\frac{dy}{dx} = 6x + 4$

$x + 2y - 10 = 0 \Rightarrow y = -\frac{x}{2} + 10$

$\therefore$ gradient perpendicular to line has gradient = 2

$\frac{dy}{dx} = 2 \Rightarrow$ $6x + 4 = 2$

$6x = -2$

$x = -\frac{1}{3}$

$\therefore$ $y = 3(-\frac{1}{3})^2 + 4(-\frac{1}{3}) + 2$

$= \frac{1}{3} - \frac{4}{3} + \frac{6}{3}$

$= 1$

$\therefore$ coordinate $(-\frac{1}{3}, 1)$ (3 marks)

Question 7

a) (i) $T_n = a+(n-1)d$

$T_7 = 8 \qquad d = -4$

$\therefore \quad T_7 = a + 6\cdot(-4)$

$\therefore \quad a - 24 = 8$

$a = 32$ (1 mark)

(ii) $S_{17} = \frac{n}{2}\left[2a+(n-1)d\right]$

$= \frac{17}{2}\left[64 + (16\times -4)\right]$

$= 0$ (2 marks)

b)

$\sum_{2}^{5} (-1)^k \left(\frac{1}{k}\right)$

$= (-1)^2\left(\frac{1}{2}\right) + (-1)^3\left(\frac{1}{3}\right) + (-1)^4\left(\frac{1}{4}\right) + (-1)^5\left(\frac{1}{5}\right)$

$= \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5}$

$= \frac{13}{60}$ (2 marks)

c) $T_n = ar^{n-1}$

$T_3 = \frac{3}{4} \qquad T_7 = 12$

$ar^2 = \frac{3}{4}$ — ① $\quad ar^6 = 12$ — ②

$\therefore$ ② $\div$ ①

$r^4 = 16$

$\therefore r = \pm 2$ sub into ①

$\therefore 4a = \frac{3}{4}$

$\therefore a = \frac{3}{16}$

$\therefore T_{14} = ar^{13}$

$= \frac{3}{16}(-2)^{13}$ or $\frac{3}{16}(2)^{13}$

$= -1536$ or 1536

(3 marks)

c) ii) $S_{14} = \dfrac{a(r^{14}-1)}{r-1}$

$= \dfrac{\frac{3}{16}\left[(-2)^{14}-1\right]}{-3}$ or $\dfrac{\frac{3}{16}\left[(2)^{14}-1\right]}{1}$

$= \dfrac{16383}{16}$ $= \dfrac{49149}{16}$ (2 marks)

Question 8

$y = \frac{1}{4}x^4 - x^3$

i) $\frac{dy}{dx} = x^3 - 3x^2$

stationary points occur when $\frac{dy}{dx} = 0$

$\Rightarrow x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$\therefore \quad x = 0 \quad$ or $\quad x = 3$

$\frac{d^2y}{dx^2} = 3x^2 - 6x$

at $x = 0$

$\frac{d^2y}{dx^2} = 0$

possible point of inflexion.

at $x = 3$

$\frac{d^2y}{dx^2} = 9 > 0$

$\therefore$ minimum at $x = 3$

$\therefore$ minimum turning point at $(3, -\frac{27}{4})$

x	-1	0	1
$\frac{d^2y}{dx^2}$	9	0	-3
	$\cup$	$\cdot$	$\cap$

$\therefore$ point of inflexion at $x = 0$

$\therefore$ horizontal point of inflexion at $x = 0$

i.e. $(0, 0)$

(4 marks)

ii) Points of inflexion occurs when $\frac{d^2y}{dx^2} = 0$

From (i) $\frac{d^2y}{dx^2} = 3x^2 - 6x$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$\therefore x = 0$ or $x = 2$

From (i) $x = 0$ is a horizontal point of inflexion.

testing for concavity.

x	1	2	3
$\frac{d^2y}{dx^2}$	-3	0	9

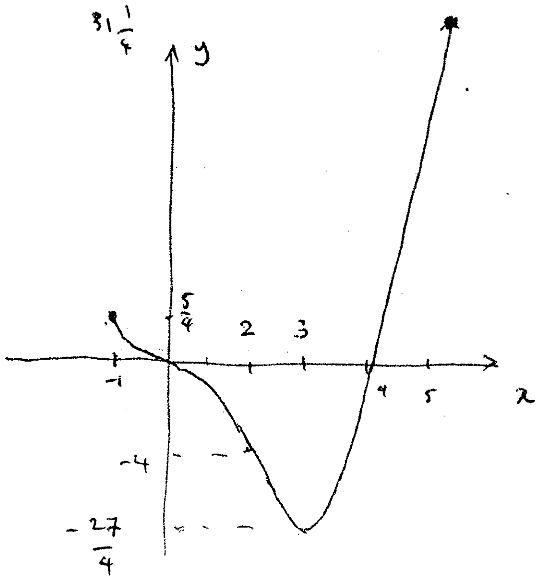
$\frown \qquad , \qquad \smile$

$\therefore$ change in concavity

$\therefore$ point of inflexion at $x=2$.

$\therefore$ points of inflexion are at $(0,0)$ and $(2,-4)$ (3 marks)

iii)

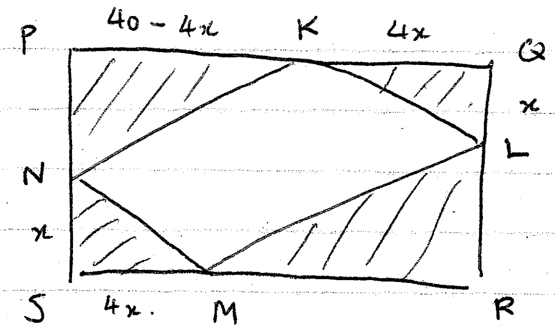


(2 marks)

iv) $0 < x < 2$. (1 mark)

Question 9

a) i)



$$\text{Area of parallelogram} = 400 - 2\left(\tfrac{1}{2}\times(10-x)(40-4x)\right) - 2\left(\tfrac{1}{2}\times 4x\times x\right)$$

$$= 400 - \left[400 - 80x + 4x^2\right] - 4x^2$$

$$= 80x - 8x^2$$

(2 marks)

ii) $0 < x < 10$ (1 mark)

iii) $A = 80x - 8x^2$

$$\frac{dA}{dx} = 80 - 16x$$

$$\frac{dA}{dx} = 0 \Rightarrow 80 - 16x = 0$$

$$16x = 80$$

$$x = 5$$

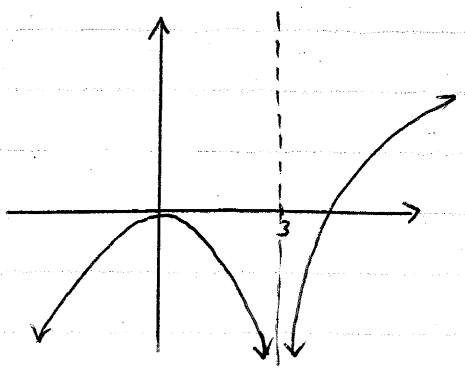
$$\frac{d^2A}{dx^2} = -16 < 0 \quad \text{for all values}$$

$\therefore$ maximum occurs at $x = 5$

$\therefore$ maximum area $= (80\times 5) - (8\times 5^2)$

$= 200 \text{ cm}^2$ (2 marks)

b



(3 marks)

Question 10

i) $(3+(n-1)4)^2$

$= (3+4n-4)^2$

$= (4n-1)^2$ (1 mark)

ii) $S_{2n} = A_n - B_n$

$= (1^2+5^2+9^2+13^2+\ldots+(4n-3)^2) - (3^2+7^2+11^2+15^2+\ldots+(4n-1)^2)$

$= (1^2-3^2)+(5^2-7^2)+(9^2-11^2)+\ldots+[(4n-3)^2-(4n-1)^2]$

$= -8-24-40+\ldots+(-16n+8)$

this is a AP, $a=-8$ $n=-16$

$\therefore S_{2n} = \frac{n}{2}[a+l]$

$= \frac{n}{2}[-8+(-16n+8)]$

$= -8n^2$ (2 marks)

iii) $101^2-103^2+105^2-107^2+\ldots+2001^2-2003^2$

$= S_{1002} - S_{50}$

$= -8(501)^2-(-8\times 25^2)$

$= -2003008$ (2 marks)

Alternatively they can do

(101^2-103^2) as first term of AP

(2001^2-2003^2) as last term and

find the number of terms in between