



NORTH SYDNEY BOYS HIGH SCHOOL

2012 HSC ASSESSMENT TASK 4

Mathematics

General Instructions

- Working time – 50 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page**.
- Attempt all questions

Class Teacher:

(Please tick or highlight)

- ☐ Mr Berry
- ☐ Mr Lam
- ☐ Mr Lowe
- ☐ Ms Ziazaris

Student Number: _____

(To be used by the exam markers only.)

Question No	1-5	6	7	8	9	Total	Total %
Mark	5	11	12	8	7	43	100

Question 1

A bag contains 6 green marbles and 5 blue marbles. Two marbles are drawn from the bag (without replacement) what is the probability that both marbles are green?

- a) $\frac{2}{11}$ b) $\frac{3}{11}$ c) $\frac{4}{11}$ d) $\frac{5}{11}$

Question 2

80 rabbits are tagged and released into a forest. Later, a sample of 200 rabbits is captured and 5 of them are found to be tagged. The number of rabbits in the forest is closest to:

- a) 1280 b) 3200 c) 7840 d) 16000

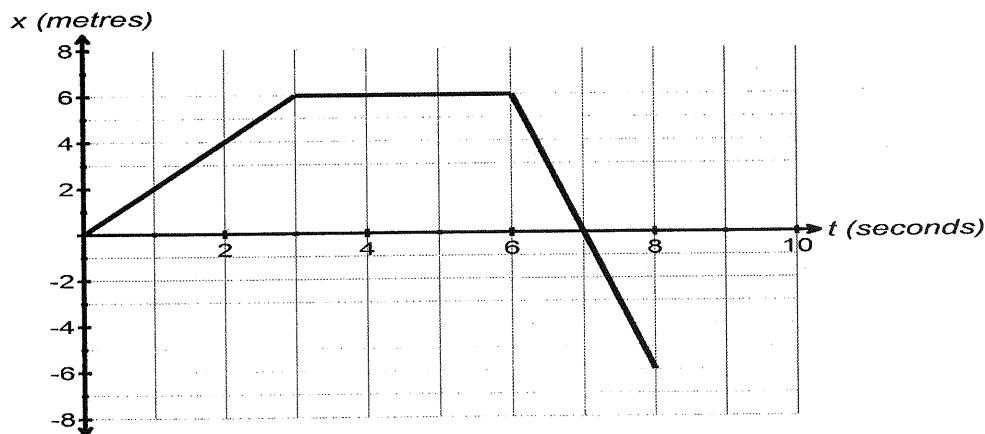
Question 3

Which of the following best describes the motion of a particle that is slowing down?

- a) $x < 0$ & $v < 0$ b) $x > 0$ & $v < 0$ c) $v < 0$ & $a < 0$ d) $v > 0$ & $a < 0$

Question 4

The displacement of a particle is described in the graph below:



What is the average speed of the particle in the first 8 seconds?

- a) 0.75 m/s b) 1.5 m/s c) 2.25 m/s d) 6 m/s

Question 5

Carla invested \$5000 into a bank account that pays 6% interest p.a. compounded monthly. If n represents the number of months, which of the following statements will determine how many months it will take for her investment to exceed \$12 000.

- a) $1.005^n < 2.4$ b) $1.005^n > 2.4$ c) $1.06^n < 2.4$ d) $1.06^n > 2.4$

Question 6 (11 Marks)

- a) In a group of eighty students, 48 watch *Transformers* and 26 watch *My Little Pony*. If 10 of the students watch both shows:
- i) How many only watch *Transformers*? 1
 - ii) What is the probability that a student chosen at random watches neither show? 2
- b) The probability that an archer hits her target is 70%. If she fires three shots find the probability that she hits the target:
- i) Exactly once. 2
 - ii) At least once. 2
- c) Five cards are numbered 0, 1, 2, 2, 3. The cards are placed into a box. Amy randomly selects and retains a card. Brenda then selects a different card. What is the probability that:
- i) The sum of the numbers is 4. 2
 - ii) Both numbers are the same. 1
 - iii) The first number is greater than the second. 1

Question 7 (12 Marks)

- a) The displacement of a particle is given by the formula $x = 2t - \log_e(t + 1)$.
- i) Find the initial position and velocity of the particle. 3
 - ii) Find the acceleration of the particle at 5 seconds. 2
 - iii) Explain why this particle is always speeding up. 1
- b) The velocity, V , of a particle is given by:
- $$v = 2\sin\left(\frac{\pi}{2}t\right), \text{ where } v \text{ is measured in metres per second}$$
- i) Sketch a velocity time graph for $0 \leq t \leq 4$. 3
 - ii) Find the total distance travelled by the particle in the first 3 seconds. 3

Question 8 (8Marks)

- (a) A radioactive element used in a hospital decays according to $y = Ae^{-kt}$, where k is a constant and t is time in years. If the element initially has a mass of 200g and one year later has a mass of 182g:

a) Show that $k = -\ln\left(\frac{91}{100}\right)$ 2

b) Find its mass in 20 years. Answer to 1 decimal place. 1

c) Find its half-life (that is, the length of time elapsed when it has lost half of its original mass). 2

- (b) The populations of two towns, A and B are given by: $P_A = 15\,000e^{0.05t}$ and $P_B = 50\,000e^{0.02t}$, where t is the number of years elapsed since January 2010.

In what year will the two towns have the same rate of change? 3

Question 9 (7 Marks)

Tiffany is in the process of applying for a \$380 000 home-loan. She approaches a bank that charges 8.4% p.a. interest, where the interest is compounded quarterly.

Each quarter she is expected to pay \$Q towards the home-loan.

i) Find A_1 , the amount owing after 1 quarter, in terms of Q. 1

ii) Show that the amount owing, A_n , after n quarters is given by:

$$A_n = 1.021^n \left(380000 - \frac{1000}{21} Q \right) + \frac{1000}{21} Q$$
 3

iii) If Tiffany's quarterly repayments are \$12 000, calculate how long it will take her to pay off the loan. 3

End of Examination.



Student Name : _____

HSC MATHEMATICS ASSESSMENT TASK 4

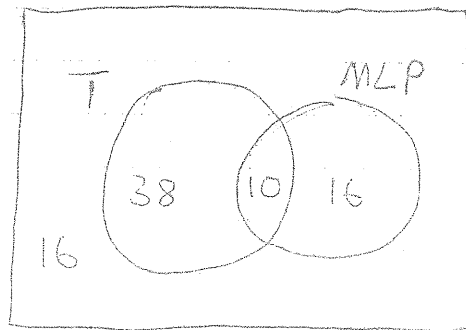
QUESTIONS 1 - 5

Select the alternative A, B, C or D that best answers the question and indicate your choice by shading the appropriate letter.

1.	☐ A	☒	☐ C	☐ D
2.	☐ A	☒	☐ C	☐ D
3.	☐ A	☐ B	☐ C	☒
4.	☐ A	☐ B	☒	☐ D
5.	☐ A	☒	☐ C	☐ D

Q6 a) i) $48 - 10 = 38$ ✓

ii)



✓ relevant working

$$\frac{16}{80} = \frac{1}{5}$$

or 20% ✓

b) i) $P(1 \text{ hit}) = P(HMM) + P(MHM) + P(MMH)$ ✓

$$= 3 \times 0.7 \times (0.3)^2$$

$$= 0.189 \text{ or } \frac{189}{1000}$$
 ✓

ii) $P(>1H) = 1 - P(MMM)$ ✓

$$= 1 - (0.3)^3$$

$$= 0.973 \text{ or } \frac{973}{1000}$$
 ✓

1st card

c)	0	1	2 _a	2 _b	3
0	0,0	1,0	2,0	2,0	3,0
1	0,1	1,1	2,1	2,1	3,1
2	0,2	1,2	2,2	2,2	3,2
a	2,2	0,2	1,2	2,2	3,2
b	2,3	0,3	1,3	2,3	3,3

i) $\frac{4}{20} = \frac{1}{5}$ /

ii) $\frac{2}{20} = \frac{1}{10}$ /

iii) $\frac{9}{20}$ /

Q7 a) i) $x = 2t - \log_e(t+1)$

when $t = 0$

$$x = 2 \times 0 - \log_e(0+1)$$

$$\therefore x = 0 \quad \therefore \text{at the origin} \quad \checkmark$$

$$v = 2 - \frac{1}{t+1} \quad \checkmark$$

when $t = 0$

$$v = 2 - \frac{1}{0+1} \\ = 1$$

$$v = 1 \text{ m/s in the positive direction} \quad \checkmark$$

ii) $a = \frac{1}{(t+1)^2} \quad \checkmark$

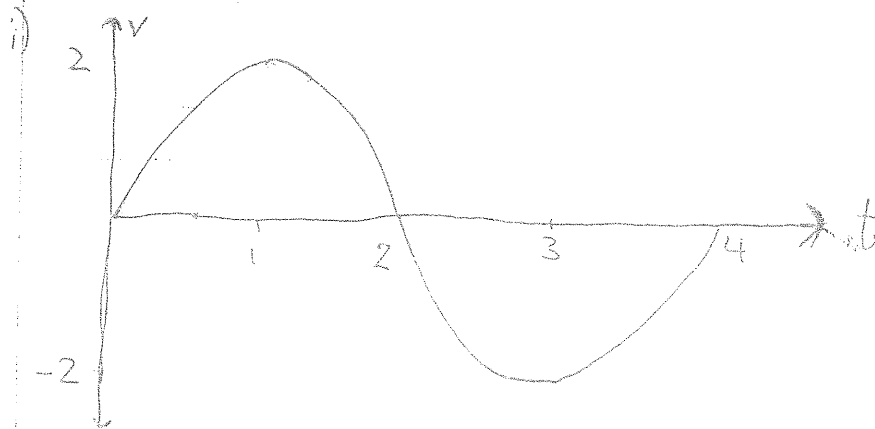
when $t = 5$

$$a = \frac{1}{(5+1)^2}$$

$$\therefore a = 36 \text{ m/s}^2 \quad \checkmark$$

iii) Because velocity and acceleration are positive for all ~~positive~~ values of t . ✓

$$b) v = 2 \sin\left(\frac{\pi}{2} t\right)$$



✓ ✓ period / ampl 1/2

✓ shape

$$ii) \int_0^2 2 \sin\left(\frac{\pi}{2} t\right) dt + \left| \int_2^3 2 \sin \frac{\pi}{2} dt \right| \quad \checkmark$$

$$= \left[-\frac{4}{\pi} \cos \frac{\pi}{2} t \right]_0^2 + \left| \left[-\frac{4}{\pi} \cos \frac{\pi}{2} t \right]_2^3 \right| \quad \checkmark$$

$$= \left[\left(-\frac{4}{\pi} \cos \pi \right) - \left(-\frac{4}{\pi} \cos 0 \right) \right] + \left| \left(-\frac{4}{\pi} \cos \frac{3\pi}{2} \right) - \left(-\frac{4}{\pi} \cos \pi \right) \right|$$

$$= \left[-\frac{4}{\pi} (-1) + \frac{4}{\pi} \right] + \left| (0) - \frac{4}{\pi} \right|$$

$$= \frac{8}{\pi} + \frac{4}{\pi}$$

$$= 12/\pi \text{ m} \quad \checkmark$$

Suggested Solutions

Question 8 (Lam)

(a) i. (2 marks)

✓ [1] for $A = 200$.✓ [1] for showing all working to obtain $k = -\ln \frac{91}{100}$.

$$y = Ae^{-kt}$$

$$t = 0 \quad y = 200$$

$$t = 1 \quad y = 182$$

When $t = 0$,

$$200 = Ae^0$$

$$\therefore A = 200$$

When $t = 1$,

$$182 = 200e^{-k}$$

$$e^{-k} = \frac{182}{200}$$

$$-k = \ln \frac{91}{100}$$

$$\therefore k = -\ln \frac{91}{100}$$

ii. (1 mark)

When $t = 20$,

$$y = 200e^{20 \times \ln \frac{91}{100}}$$

$$= 30.3289 \dots$$

$$= 30.3 \text{ g}$$

iii. (2 marks)

✓ [1] for obtaining $e^{\ln \frac{91}{100} t} = \frac{1}{2}$.✓ [1] for $t = 7.3496 \dots$

$$100 = 200e^{\ln \frac{91}{100} t}$$

$$e^{\ln \frac{91}{100} t} = \frac{1}{2}$$

$$\ln \frac{91}{100} t = \ln \frac{1}{2}$$

$$t = \frac{\ln \frac{1}{2}}{\ln \frac{91}{100}} = 7.3496 \dots$$

(b) (3 marks)

✓ [1] for differentiating both expressions correctly. As this is a rate of change question, if P_A and P_B are not differentiated, (0 marks) awarded.

✓ [1] for equating both derivatives.

✓ [1] for final answer.

$$P_A = 10\,000e^{0.05t}$$

$$P_B = 50\,000e^{0.02t}$$

$$\frac{dP_A}{dt} = 10\,000 \times 0.05 \times e^{0.05t}$$

$$= 750e^{0.05t}$$

$$\frac{dP_B}{dt} = 50\,000 \times 0.02 \times e^{0.02t}$$

$$= 1\,000e^{0.02t}$$

When the rates of change are equal,

$$\frac{dP_A}{dt} = \frac{dP_B}{dt}$$

$$750e^{0.05t} = 1\,000e^{0.02t}$$

$$\frac{e^{0.05t}}{e^{0.02t}} = \frac{1\,000}{750} = \frac{4}{3}$$

$$e^{0.03t} = \frac{4}{3}$$

$$0.03t = \ln \frac{4}{3}$$

$$t = \frac{\ln \frac{4}{3}}{0.03} = 9.59 \dots \text{ years}$$

Q9) i) interest = 2.1% per quarter

$$A_1 = 380\,000(1.021) - Q \quad \checkmark$$

$$\begin{aligned} \text{ii) } A_2 &= A_1(1.021) - Q \\ &= ((380\,000)(1.021) - Q)(1.021) - Q \\ &= 380\,000(1.021)^2 - Q(1.021) - Q \\ &= 380\,000(1.021)^2 - Q(1 + 1.021) \quad \checkmark \end{aligned}$$

~~Q3~~

$$\begin{aligned} A_3 &= A_2(1.021) - Q \\ &= (380\,000(1.021)^2 - Q(1 + 1.021))(1.021) - Q \\ &= 380\,000(1.021)^3 - Q(1 + 1.021)(1.021) - Q \\ &= 380\,000(1.021)^3 - Q(1.021 + 1.021^2) - Q \\ &= 380\,000(1.021)^3 - Q(1 + 1.021 + 1.021^2) \end{aligned}$$

and so on until

$$A_n = 380\,000(1.021)^n - Q(1 + 1.021 + 1.021^2 + \dots + 1.021^{n-1}) \quad \checkmark$$

↑
this part is a GP
 $a=1$ & $r=1.021$

$$A_n = 380\,000(1.021)^n - Q \left(\frac{1(1.021^n - 1)}{0.021} \right)$$

$$= 380\,000(1.021)^n - Q \left(\frac{1000}{21} \right) (1.021^n - 1)$$

$$= 380\,000(1.021)^n - \left(\frac{1000}{21} \right) (Q)(1.021)^n + \frac{1000}{21} Q$$

$$= 1.021^n \left(380\,000 - \frac{1000}{21} Q \right) + \frac{1000}{21} Q, \quad \checkmark$$

as required.

Q9 iii) let $A_n = 0$

$$\therefore 1.021^n \left(380\,000 - \frac{1000}{21} Q \right) + \frac{1000}{21} Q = 0$$

$$\therefore 1.021^n \left(380\,000 - \frac{1000}{21} Q \right) = -\frac{1000}{21} Q$$

$$\therefore 1.021^n = \frac{-1000/21 Q}{380\,000 - 1000/21 Q}$$

$$\therefore n = \frac{\log \left(\frac{-1000/21 Q}{380\,000 - 1000/21 Q} \right)}{\log 1.021}$$

$$\therefore n = 52.622$$

$\therefore 53$ quarters.