NORTH SYDNEY BOYS HIGH SCHOOL

2014

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 1

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 50 minutes
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a new page

Class teacher:
(Please tick or highlight)

- ☐ 12M2A Mr Lin
- ☐ 12M2B Ms Juhn
- ☐ 12M3A Mr Zuber
- ☐ 12M3B Mr Berry
- ☐ 12M3C Mr Lowe
- ☐ 12M4A Ms Ziaziaris
- ☐ 12M4B Mr Lam
- ☐ 12M4C Mr Ireland

Student Number: ....

(To be used by examination markers only)

	Section I	Question 7	Question 8	Question 9	Question 10	Total	%
Mark	$\overline{6}$	$\overline{13}$	$\overline{11}$	$\overline{13}$	$\overline{10}$	$\overline{53}$	

Q10 -- give additional 5 min.

Section I

6 marks
Attempt Questions 1-6

Use the multiple-choice answer sheet for Questions 1-6.

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1 The sequence -2, x, 14, … form an arithmetic sequence. What is the value of x?

(A) 4 (B) 8
(C) -8 (D) 6

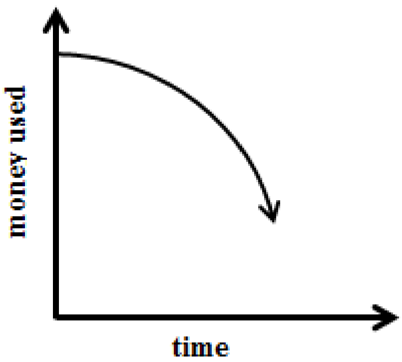
2 For the geometric series $5, -1, \frac{1}{5}, -\frac{1}{25}$ ..., what formula would you use to find the sum of n terms?

(A) $S_n = 2a + (n-1)d$ (B) $S_n = \frac{n}{2}(2a + (n-1)d)$

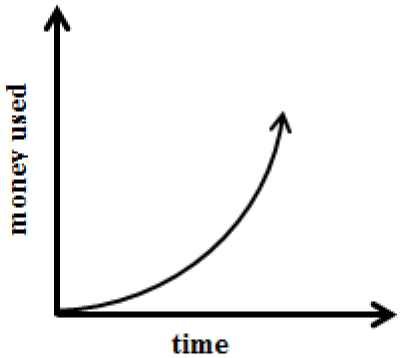
(C) $S_n = \frac{a(1-r^n)}{1-r}$ (D) $S_n = \frac{a}{1-r}$

3 Jamie has been using his money at an increasing rate. Which graph best depicts this information?

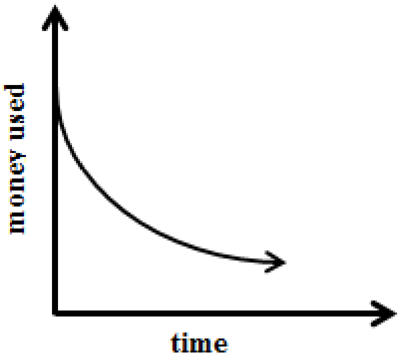
(A)



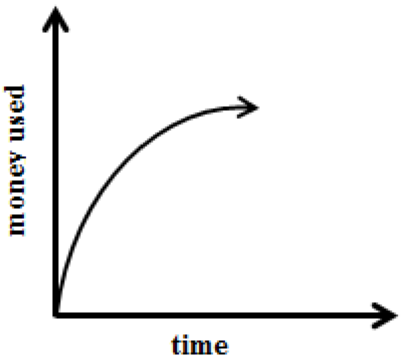
(B)



(C)

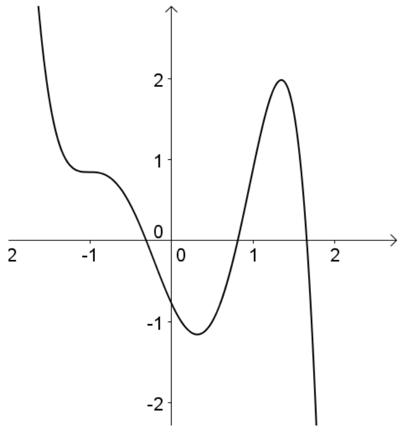


(D)



4 What condition must the discriminant (Δ) satisfy in order for the quadratic equations $ax^2 + bx + c = 0$ have real roots

(A) $\Delta > 0$ (B) $\Delta < 0$
(C) $\Delta = 0$ (D) None of these



Use the diagram above for questions 5 and 6.

5 How many stationary points are there (if any) in the graph?

(A) 2 (B) 3

(C) 4 (D) There are no stationary points.

6 How many points of inflexion are there (if any) in the graph?

(A) 2 (B) 3

(C) 4 (D) There are no points of inflexion.

Section II

49 marks
Attempt Questions 7-10

Question 7 (13 marks) Use a NEW page.

(a) Consider the sequence 98, 92, 86, 80, …

(i) Find an expression for the n^{th} term in simplest form. **2**

(ii) Find the sum of the first 50 terms. **2**

(iii) What is the value of the first negative term of the sequence? **3**

(b) Consider the sequence $1, (3x+2), (3x+2)^2$ ...

(i) For what values of x does the sequence have a limiting sum? **2**

(ii) If $x = -\frac{1}{2}$, find the limiting sum. **2**

(c) Evaluate $$\sum_{k=6}^{8}(2k+1)(1-k)$$ **2**

Question 8 (11 marks) Use a NEW page.

(a) For what values of x is the function $f(x) = -4x^3 + 27x^2 + 30x - 18$ decreasing? **3**

(b) A function is given by $f(x) = 3x^4 - 16x^3 + 24x^2$.

(i) Find any stationary points and determine their nature. **2**

(ii) Find any points of inflexion. **2**

(iii) Sketch the curve, clearly showing the intercepts with axes, turning points and points of inflexion. **3**

(iv) What is the minimum value of $f(x)$? **1**

Question 9 continues over the page

Question 9 (13 marks) Use a NEW page.

(a) For the equation $12(2^{2x}) + (2^x) - 1 = 0$, solve for x. **3**

(b) α and β are the roots to the quadratic equation $2x^2 - 4x + 6 = 0$. Find:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ **2**

(iv) $\alpha^2 + \beta^2$ **2**

(c) For what value(s) of b will the equation $x^2 + 4x - 2b = 0$ have:

(i) one of the roots equal to 3. **1**

(ii) roots which are reciprocals of each other. **1**

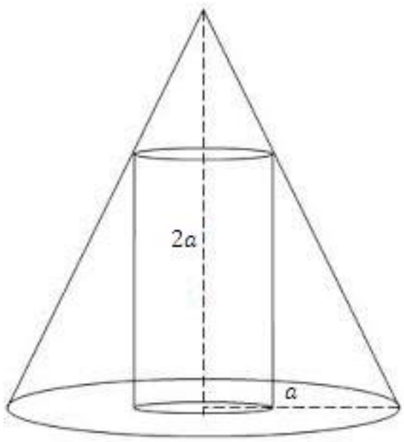
(iii) the sum of the roots equal to their product. **2**

Question 10 (10 marks) Use a NEW page.

(a) Sketch the curve satisfying the following conditions, clearly showing the important features. **3**

- $f(x) > 0$ for all values of x and $f(0) = 1$
- $f'(x) = 0$ at $x = 2, f'(x) > 0$ for every other values $of\ x$
- $f''(x) > 0$ for $x < 1$ or $x > 2, f''(x) < 0$ for $1 < x < 2$
- $f''(x) = 0$ at $x = 1\ and\ x = 2$

(b)



A cylinder is inscribed in a cone of base radius a and height $2a$. The radius of the cylinder is given by r and h is the height of the cone.

(i) Show that $r = a - \frac{h}{2}$. **1**

(ii) Show that the volume of the cylinder is $V = \pi(a^2h - ah^2 + \frac{h^3}{4})$ **1**

(iii) Find the maximum volume of the cylinder in term of a in simplest form. **3**

(c) Find the sum of the following infinite series. **2**

$$\frac{1}{6} + \frac{13}{6^2} + \frac{19}{6^3} + \frac{97}{6^4} + \cdots + \frac{3^n + (-2)^n}{6^n} + \cdots$$

End of paper

SUGGESTED SOLUTIONS.

Multiple choice

1. D
2. C
3. B
4. D
5. B
6. B

7. 98, 92, 86, 80 ...

(a) (i) $a = 98 \quad d = -6$

$$T_n = 98 + (n-1)(-6)$$
$$= 98 - 6n + 6$$
$$= 104 - 6n$$

(ii) $S_{50} = 25(2 \times 98 + (49)(-6))$
$$= -2450.$$

(iii) $104 - 6n < 0$
$$6n > 104$$
$$n > 17.\dot{3}$$

18^{th} term.

(b) (i) $-1 < 3x + 2 < 1$
$$-3 < 3x < -1$$
$$-1 < x < -\frac{1}{3}$$

(ii) when $x = -\frac{1}{2}$, $S_\infty = \dfrac{1}{1 - (-\frac{3}{2} + 2)}$
$$= 2$$

(c) $(-65) + (-90) + (-119)$
$$= -274$$

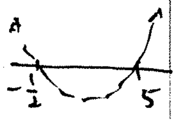
8. (a) $f(x) = -4x^3 + 27x^2 + 30x - 18$

$f'(x) = -12x^2 + 54x + 30$

decreasing when $f'(x) < 0$

$-12x^2 + 54x + 30 < 0$

$2x^2 - 9x - 5 > 0$



$(2x+1)(x-5) > 0$

$x < -\frac{1}{2}$ or $x > 5$

(b) $f(x) = 3x^4 - 16x^3 + 24x^2$

(i) $f'(x) = 12x^3 - 48x^2 + 48x$

$f''(x) = 36x^2 - 96x + 48$

Stationary points occur when $f'(x) = 0$

$x(x^2 - 4x + 4) = 0$

$x(x-2)^2 = 0$

$\therefore x = 0 \quad x = 2$

To determine the nature of stationary points

$f''(0) = 48$ which is > 0

$\therefore$ minimum turning point at $(0,0)$

$f''(2) = 0$

$\therefore$ possible point of inflexion at $(2, 16)$

(ii) Points of inflexion occur when $f''(x) = 0$

$36x^2 - 96x + 48 = 0$

$3x^2 - 8x + 4 = 0$

$(3x-2)(x-2) = 0$

$x = \frac{2}{3} \quad x = 2$

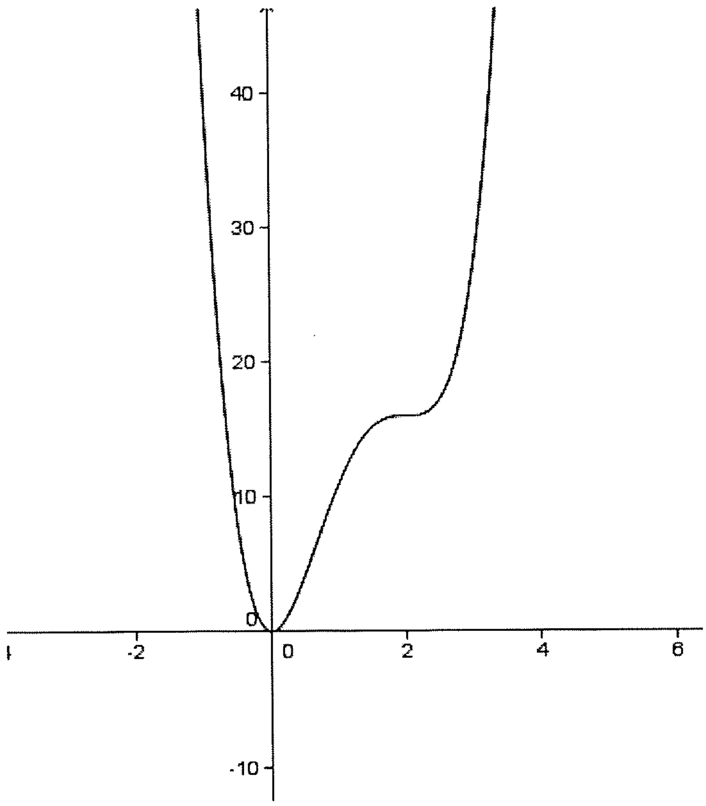
To see if there is a change of concavity

x		$\frac{2}{3}$		2	
$f''(x)$					

(various solutions)

$\therefore$ points of inflexion at $(\frac{2}{3}, \frac{176}{27})$ and $(2, 16)$

(iii)



(iv) minimum value is 0

9\. (a) $12(2^{2x}) + (2^x) - 1 = 0$

let $u = 2^x$

$12u^2 + u - 1 = 0$

$(3u+1)(4u-1) = 0$

$u = -\frac{1}{3}$ $\quad u = \frac{1}{4}$

$2^x = -\frac{1}{3}$

no real solutions

$2^x = \frac{1}{4}$

$\therefore x = -2$

(b) $2x^2 - 4x + 6 = 0$

(i) $\alpha + \beta = 2$

(ii) $\alpha\beta = 3$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{2}{3}$

(iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 4 - 6$

$= -2$

(c) $x^2 + 4x - 2b = 0$

(i) let $x = 3$ $\quad 9 + 12 - 2b = 0$

$b = \frac{21}{2}$

(ii) $\alpha, \frac{1}{\alpha}$ $\quad \alpha \times \frac{1}{\alpha} = -2b$

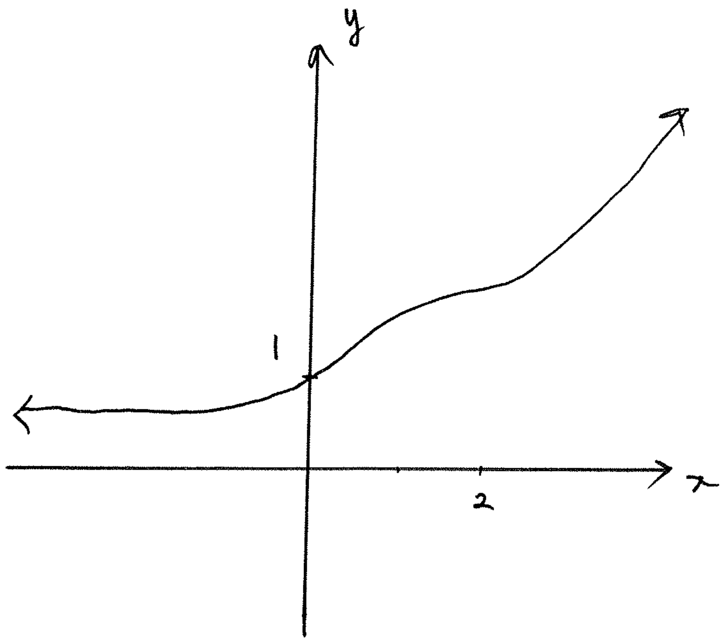
$\therefore b = \frac{1}{-2}$

(iii) $\alpha + \beta = \alpha\beta$

$-4 = -2b$

$\therefore b = 2$

10, (b)



(c) (i) $\frac{r}{2a-h} = \frac{a}{2a}$

$\therefore r = a - \frac{h}{2}$

(ii) $V = \pi (a - \frac{h}{2})^2 h$

$= \pi h (a^2 - ah + \frac{h^2}{4})$

$= \pi (a^2 h - ah^2 + \frac{h^3}{4})$

(iii) $\frac{dV}{dh} = a^2\pi - 2a\pi h + \frac{3\pi h^2}{4}$

let $\frac{dV}{dh} = 0$

$a^2 - 2ah + \frac{3h^2}{4} = 0$

$4a^2 - 8ah + 3h^2 = 0$

$(2a - 3h)(2a - h) = 0$

$h = \frac{2a}{3}$ $\quad h = 2a$

$\frac{d^2V}{dh^2} = -2a\pi + \frac{3\pi h}{2}$

when $h = \frac{2a}{3}$, $\frac{d^2V}{dh^2} = -2a\pi + \pi a$

$= -a\pi$ which is < 0 $\quad \therefore$ maximum at $h = \frac{2a}{3}$

when $h = 2a$, $\frac{d^2V}{dh^2} = -2a\pi + 3\pi a$

$= \pi a$ which is > 0 $\quad \therefore$ minimum at $h = 2a$

$\therefore$ maximum volume occurs when $h = \frac{2a}{3}$

$V = \frac{8\pi a^3}{27}$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{6}\right)^n + \left(\frac{-2}{6}\right)^n$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n + \left(-\frac{1}{3}\right)^n$$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}} + \frac{-\frac{1}{3}}{1+\frac{1}{3}}$$

$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$