NORTH SYDNEY BOYS HIGH SCHOOL

2013 YEAR 12 ASSESSMENT TASK 4

Mathematics

General Instructions

- Working time – 60 minutes
- **Reading time – 5 minutes**

- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher: Please colour

- O Mr Lowe
- O Ms Juhn
- O Mr Lam
- O Mr Berry
- O Mr Lin

Student Number:_____

(To be used by the exam markers only.)

Question No	1-2	3	4	5	6	7	8	Total	Total
Mark	/6	/4	/11	/6	/9	/6	/8	/50	/100

1. A box has 5 Psychology books and 3 Astronomy books in it.
   Two books are selected at random from the box.
   (a) Draw a probability tree to show all the possible outcomes. **2**
   (b) Find the probability that :
      (i) the two books are Astronomy books. **1**
      (ii) at least one of the books is a Psychology book. **2**

2. In a game of Yahtzee, 5 standard dice are rolled.
   Find the probability of rolling all the same number. **1**

3. David and Ryan play a game where they each take turns at throwing two ordinary dice. The winner is the first person to throw a double.
   David throws first.
   (a) Show that the probability that Ryan wins the game on his first throw is $\frac{5}{36}$. **2**
   (b) Show that the probability Ryan wins the game on his first or second throw is given by $\frac{5}{36}+\frac{5^3}{6^4}$. **2**

4. A super ball is thrown into the air and its path of motion is described by the equation $x = 24t - 3t^2$. Time is measured in seconds and distance travelled in metres.
   (a) At what times is the displacement zero? **2**
   (b) Find the maximum height reached by the ball. **2**
   (c) Find the velocity at $t = 1$ **2**
   (d) Find the average velocity in the first 5 seconds **2**
   (e) Find the average speed in the first 5 seconds. **2**

exam continues overleaf.....→

5. A particle moves in a straight line. At time t seconds , its displacement, x metres from a fixed point O on the line is given by $x = 1 - \cos \pi t$

(a) What is the initial displacement of the particle? **1**

(b) Find an expression for the velocity of the particle at any time t. **1**

(c) What is the velocity of the particle at time $t = \frac{1}{6}$? **1**

(d) Find an expression for the acceleration of the particle at any time t. **1**

(e) At what time does the particle **first** reach its maximum speed? **2**

6. In a fish hatchery, the fish population, N , satisfies the equation $N = N_0 e^{kt}$ where N_0 and k are constants and t is measured in months.

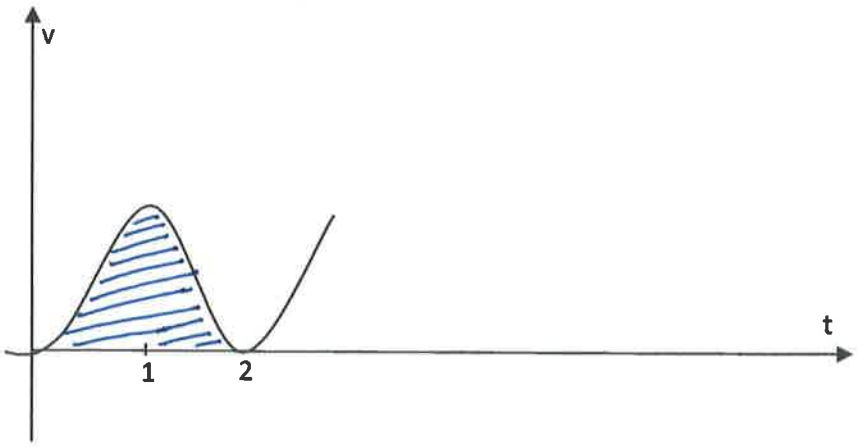
(a) Initially there were 1000 fish in the hatchery and at the end of 5 months there were 5000. Find the value of k correct to 3 decimal places. **2**

(b) Find the number of fish in the hatchery at the end of 8 months. (Give your answer correct to the nearest hundred.) **2**

(c) At the end of which month will the fish population exceed 50 000 for the first time? **3**

(d) At what rate is the population increasing at the end of six months? (Give your answer correct to the nearest hundred fish per month.) **2**

7.



The velocity - time graph of a particle starting from rest at the origin is shown in the diagram.

(a) When is the particle at rest again? **1**

(b) Does the particle change its direction during the motion? Give reasons. **2**

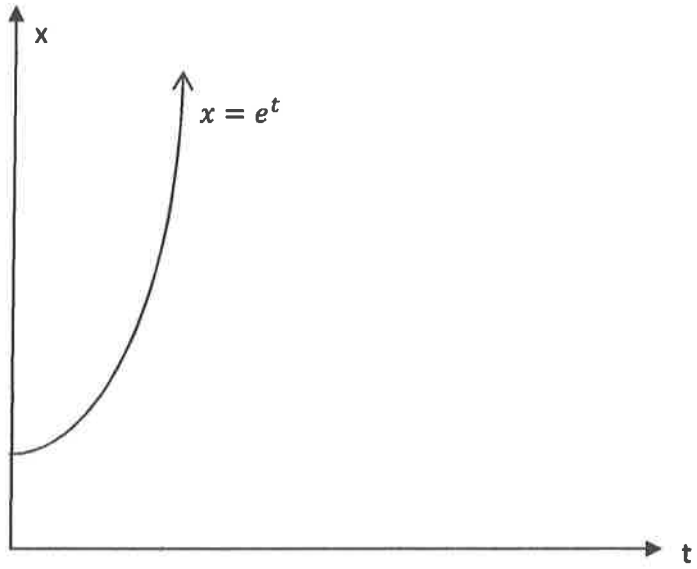
(c) Give the first time that the acceleration of the particle is zero. **1**

(d) Give a physical interpretation of the shaded area bounded by the t axis and the curve. **1**

(e) Sketch the displacement-time graph of the particle. **2**

exam continues overleaf.....→

8. Two particles P and Q are moving along a horizontal line. At any time t seconds, the position of the particle P is given by $x = e^t$ and the position of particle Q is given by $x = 1 + 6e^{-t}$. The diagram below shows the position of particle P, x metres, at any time t seconds.



(a) As time increases indefinitely, what position does the particle Q approach? **1**

(b) On a pair of coordinate axes, sketch the path of particle Q. **2**

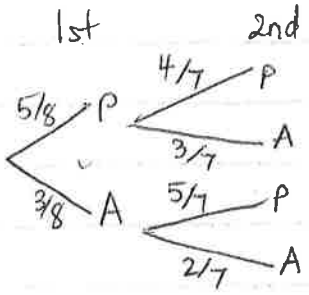
(c) Calculate the position where the two particles meet. **4**

(d) Explain why P and Q will never travel at the same velocity. **1**

END OF EXAMINATION

2UNIT SOLUTIONS - ASSESS. TSK 4 2013

1. a)



b)(i) $P(AA) = \frac{3}{8} \times \frac{2}{7}$

$= \frac{3}{28}$

(ii) $P(\text{at least one P}) = 1 - P(AA)$

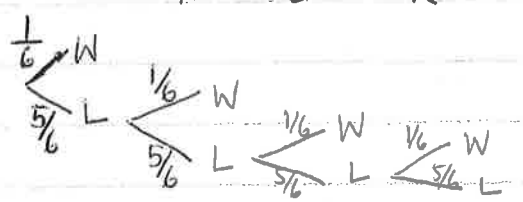
$= 1 - \frac{3}{28}$

$= \frac{25}{28}$

2. $P(\text{same number}) = \left(\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}\right) \times 6$

$= \frac{1}{6^4}$

3.



a) $P(\text{Ryan wins}) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

b) $P(\text{Ryan wins}) = \frac{5}{36} + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$

$= \frac{5}{36} + \frac{5^3}{6^4}$

(4.) $x = 24t - 3t^2$

a) $0 = 24t - 3t^2$

$0 = 3t(8-t)$

$t = 0, \quad t = 8.$

b) Max height : $t = 4$

$\therefore x = 24 \times 4 - 3 \times 16$

$= 48 \text{ m}$

OR Max height : $\dot{x} = 0.$

$0 = 24 - 6t$

$6t = 24$

$t = 4$

$\therefore x = 48 \text{ m}.$

c) $\dot{x} = 24 - 6t$

When $t = 1$

$\dot{x} = 24 - 6$

$\dot{x} = 18 \text{ m/s}$

d) Average Velocity $= \frac{x_2 - x_1}{t_2 - t_1}$

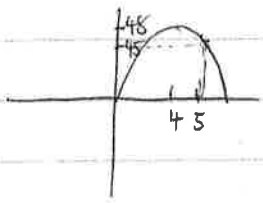
$= \frac{45 - 0}{5 - 0}$

$= 9 \text{ m/s}$

e) Average speed $= \frac{48 + 3}{5}$

$= \frac{51}{5} \text{ m/s}$

$= 10.2 \text{ m/s}.$



(5.) $x = 1 - \cos \pi t$

a) When $t = 0$

$x = 1 - \cos 0$

$x = 0$

b) $\dot{x} = \pi \sin \pi t$

c) When $t = 1/6$

$\dot{x} = \pi \sin \frac{\pi}{6}$

$\dot{x} = \frac{\pi}{2} \text{ m/s}.$

d) $\ddot{x} = \pi^2 \cos \pi t.$

e) Max Speed : $\ddot{x} = 0$

$\pi^2 \cos \pi t = 0$

$\cos \pi t = 0.$

$\pi t = \frac{\pi}{2}$

$t = \frac{1}{2} \text{ sec}.$

6. $N = N_0 e^{kt}$

a) $N = 1000 e^{kt}$

When $t=5$, $N=5000$

$5000 = 1000 e^{5k}$

$5 = e^{5k}$

$\ln 5 = 5k$

$k = \frac{1}{5} \ln 5$

$k = 0.322$

b) When $t=8$.

$N = 1000 e^{0.322 \times 8}$

$N = 13132.639$ (using exact k)

$N = \underline{13100}$ (using exact k)

$N = 13144.455$ (using $0.322 = k$)

$= \underline{13100}$

c) $1000 e^{0.322t} > 50000$

$e^{0.322t} > 50$

$0.322t > \ln 50$

$t > \frac{\ln 50}{0.322}$

$t > 12.149$

* At the end of the 13th month

d) $\frac{dN}{dt} = k N_0 e^{kt}$

$= 0.322 \times 1000 e^{0.322 \times 6}$

$= 2200$ fish/mth

7,

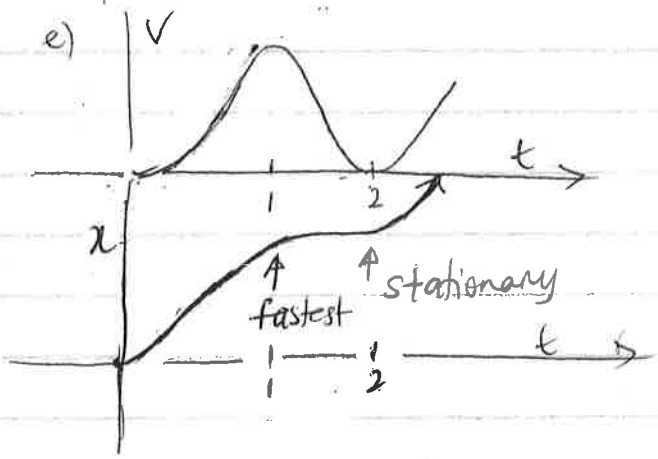
a) At $t = 1$

b) No as the velocity is always positive

c) $t = 1$

d) distance travelled by the particle in the first 2 seconds.

e)



8. P: $x = e^t$

Q: $x = 1 + 6e^{-t}$

a) As $t \to \infty$

$x = 1 + \frac{6}{e^t}$

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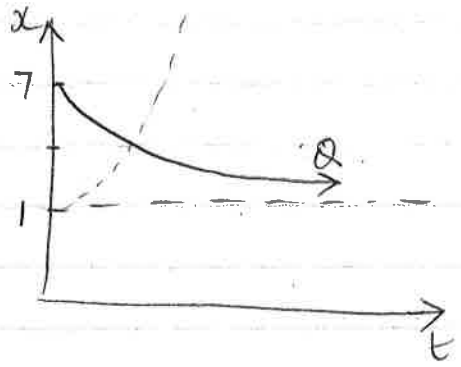
$e^t \to \infty$

$\frac{6}{e^t} \to 0$.

$\therefore x \to 1$. m.

b)



c) Two particles meet at same x value.

$1 + 6e^{-t} = e^t$

$1 + \frac{6}{e^t} = e^t$

$e^t + 6 = e^{2t}$

$e^{2t} - e^t - 6 = 0$

Let $a = e^t$

$\therefore a^2 - a - 6 = 0$

$(a-3)(a+2) = 0$

$a = 3$ or -2

ie. $e^t = 3$ or $e^t = -2$ no sol

$t = \ln 3$.

ie. $x = e^{\ln 3}$

$x = 3$.

$\therefore$ They meet at $x = 3$m.

d) Never travel at the same velocity because Q is always decreasing and hence velocity is negative. P is always increasing and hence velocity positive.

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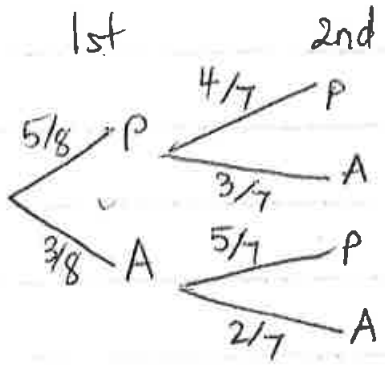

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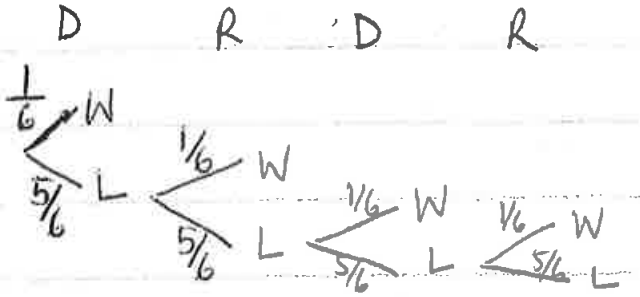

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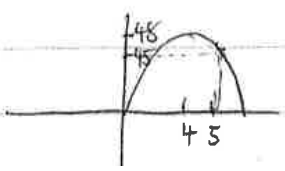

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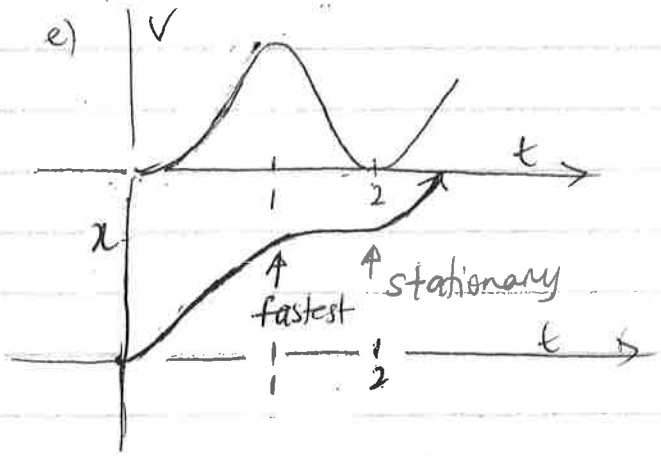

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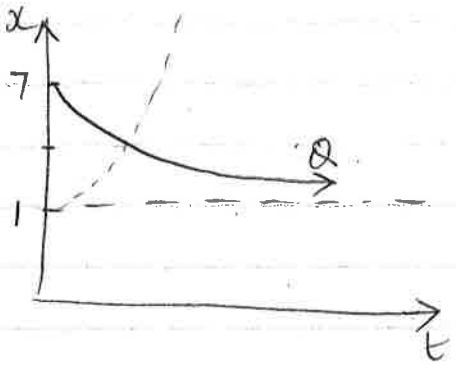

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