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NORTH SYDNEY BOYS HIGH SCHOOL

2013 ASSESSMENT TASK 3

Preliminary Mathematics

General Instructions

- Working time – 50 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:

○ Mr Lin

Student Number:_____

(To be used by the exam markers only.)

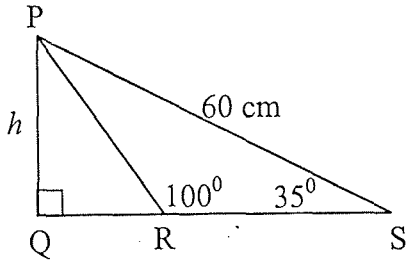
Question No	1	2	3	Total	Total %
Mark	___/13	___/14	___/16	___/43	___/100

Question 1 (13 marks)

(a) Find the exact value of cos 300° **2**

(b) If $\cos\alpha = \frac{2}{7}$ and $\sin\alpha < 0$ find the exact value of $\tan\alpha$ in the domain $0° \leq \alpha \leq 360°$ **3**

(c) In the diagram, $\angle PRS$=100°, $\angle PSR$=35°, PS=60cm, $\angle PQR$=90° and PQ =h cm.



(i) Show that PR= $\frac{60sin35°}{sin100°}$ **1**

(ii) Hence find h correct to 2 decimal places. **2**

(d) Graph the region of the intersection given by the inequations:

$y \geq 0, x \geq 0 \text{ and } y < 4 - x^2$ **3**

(e) Find the exact area of an equilateral triangle with side length of xcm. **2**

Question 2 (14 marks)

(a) Show that $\frac{1}{1-\sin\theta} + \frac{1}{1+sin\theta} = 2sec^2\theta$ **3**

(b) Solve $4cos^2\theta=1$ for $0° \leq \theta \leq 360°$ **3**

(c) Show that the line $3x + 4y + 10 = 0$ is a tangent to the circle:

$$(x-2)^2 + (y+3)^2 = \frac{16}{25}$$ **3**

(d) A plane travels 1380km from A to B on a bearing of 58°T. The plane then travels 985km on a bearing of 135°T to a point C.

(i) Illustrate this information on a sketch and find the distance of CA, correct to one decimal place. **3**

(ii) Find the bearing of A from C to the nearest minute. **2**

Question 3 (16 marks)

In the quadrilateral ABCD, the points A,B,C and D are (3,3), (0,-1) and (6,2) respectively. The line BD bisects the line AC at right angles at the point M.

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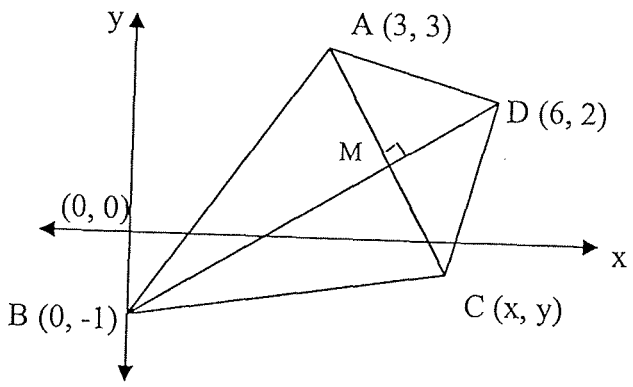


(a) Find the distance BD. **2**

(b) Show that the gradient of BD is $\frac{1}{2}$. **2**

(c) Show that the equation of the line BD is $x - 2y - 2 = 0$. **2**

(d) Show that the equation of AC is $2x + y - 9 = 0$. **2**

(e) Find the coordinates of M. **3**

(f) Show that the coordinates of C are (5,-1). **2**

(g) Find the area of ABCD. **3**

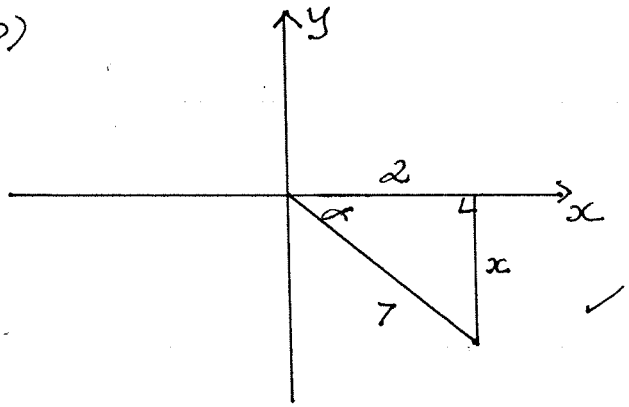
End of paper

Solutions Preliminary Maths task 3
2013

Question 1

(a) $\cos 300° = \cos(360° - 60°)$ ✓
$= \cos 60°$
$= \frac{1}{2}$ ✓

(b)



✓

$x^2 + 2^2 = 7^2$
$x^2 = 45$ ✓
$x = \sqrt{45}$

$\therefore \tan\alpha = -\frac{3\sqrt{5}}{2}$ ✓
$(\tan\alpha < 0)$

(c) (i) Using sine rule in $\triangle PRS$,

$$\frac{PR}{\sin 35°} = \frac{60}{\sin 100°}$$

$$PR = \frac{60 \sin 35°}{\sin 100°}$$ ✓

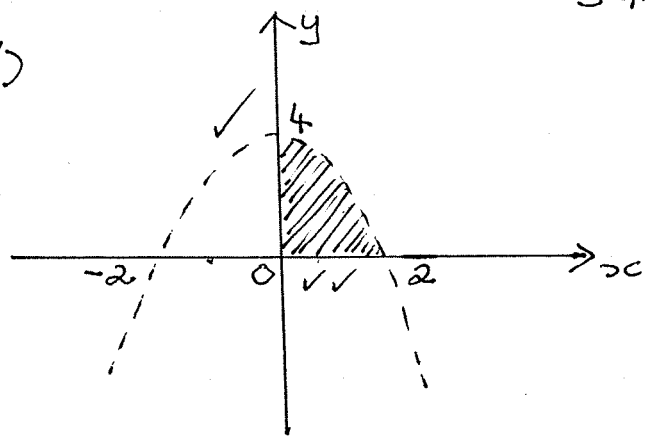
(ii) $\angle PQR = 180° - 100° = 80°$

In $\triangle PQR$, $\frac{h}{PR} = \sin 80°$ ✓

$h = PR \sin 80° = \frac{60 \sin 35° \sin 80°}{\sin 100°}$

$= 34.41$ (2 d.pl.) ✓

(d)



✓ ✓ ✓

(e) $A = \frac{1}{2} ab \sin C$ ✓
$= \frac{1}{2} \times x \times x \times \sin 60°$
$= \frac{x^2}{2} \times \frac{\sqrt{3}}{2}$
$= \frac{\sqrt{3} x^2}{4}$ ✓ cm^2

Question 2

(a) Show $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$

$$LHS = \frac{1+\sin\theta + 1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)} \quad \checkmark$$

$$= \frac{2}{1-\sin^2\theta}$$

$$= \frac{2}{\cos^2\theta} \quad \checkmark$$

$$= 2\sec^2\theta \quad \checkmark$$

$$= RHS$$

(b) $4\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{4}$ ✓

$\cos\theta = \pm\frac{1}{2}$ ✓

$\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$ ✓

(c) $(x-2)^2 + (y+3)^2 = \frac{16}{25}$ Centre $(2,-3)$ $r = \frac{4}{5}$

Perpendicular distance from centre to line: $3x+4y+10=0$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}} \quad \checkmark$$

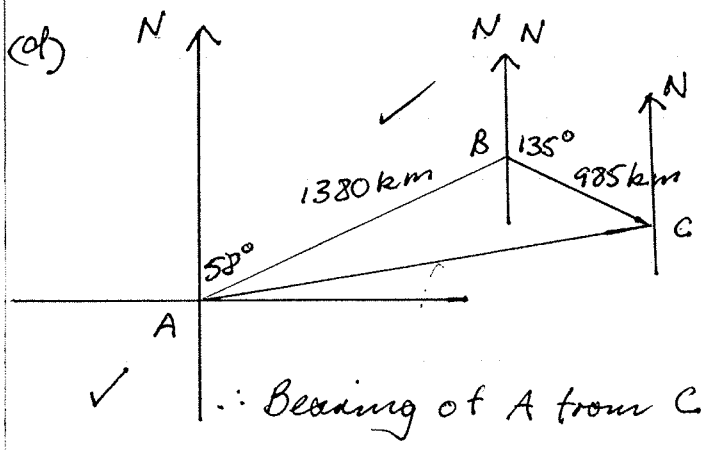
$$= \frac{|3\times 2 + 4\times -3 + 10|}{\sqrt{9+16}}$$

$$= \frac{4}{5} \quad \checkmark$$

$$= r \quad \checkmark$$

$\therefore$ the line is a tangent to the circle

(d)



(i) $\angle ABC = 58^\circ + (180^\circ - 135^\circ) = 103^\circ$ ✓

✓ $CA^2 = 1380^2 + 985^2 - 2.1380.985\cos 103^\circ$

✓ $CA = 1867.1$ km (1 d.p.)

(ii) $\sin\angle BCA = \frac{1380\sin 103^\circ}{1867.1}$ ✓

$\angle BCA = 46^\circ 4'$

✓ $\therefore$ Bearing of A from C $= 360^\circ - (46^\circ 4' + 45^\circ) = 268^\circ 56'$

Question 3

(a) $d = \sqrt{(2+1)^2 + (6-0)^2}$ ✓

$= \sqrt{45}$

$= 3\sqrt{5}$ units ✓

(b) $m_{BD} = \frac{2+1}{6-0}$ ✓

$= \frac{1}{2}$ ✓

(c) $y = mx + b$

BD: $y = \frac{1}{2}x - 1$ ✓

$2y = x - 2$

$\therefore x - 2y - 2 = 0$ ✓

(d) $m_{AC} = -2$

$y - 3 = -2(x - 3)$ ✓

$= -2x + 6$

$\therefore 2x + y - 9 = 0$ ✓

(e) $x - 2y - 2 = 0$ } —①

$2x + y - 9 = 0$ } —② ✓

② − ① $5y - 5 = 0$

$y = 1$ ✓

$x = 4$ ✓

$\therefore M(4, 1)$

(f) $4 = \frac{x+3}{2}$ $\qquad 1 = \frac{y+3}{2}$ ✓

$x = 5$ $\qquad y = -1$ ✓

$\therefore C(5, -1)$

(g) $A = \frac{1}{2} BD \cdot AC$ ✓

$= \frac{1}{2} \times 3\sqrt{5} \times 2\sqrt{5}$

$= 15 \text{ unit}^2$ ✓

* $AC = \sqrt{(-1-3)^2 + (5-3)^2}$

$= 2\sqrt{5}$ ✓