# NORTH SYDNEY BOYS HIGH SCHOOL

## 2015 HSC ASSESSMENT TASK 1

# Mathematics

### General Instructions

- Working time – 55 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

**Class Teacher:**
(Please tick or highlight)

- O Ms Lee
- O Mr Berry
- O Ms Ziaziaris
- O Mr Zuber
- O Mr Ireland
- O Mr Lam
- O Mr Lin

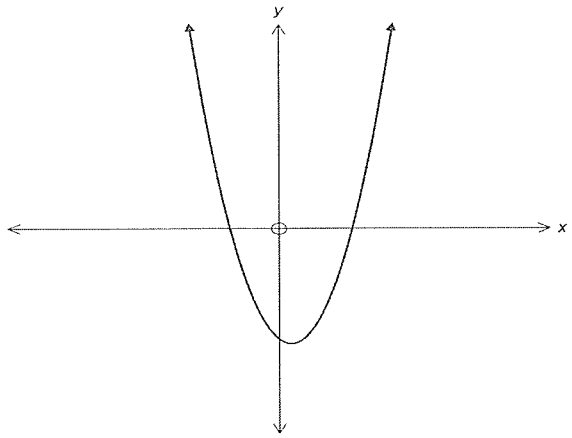
Student Number:_____

(To be used by the exam markers only.)

| Question No | 1-5 | 6 | 7 | 8 | Total | Total % |
|---|---|---|---|---|---|---|
| Mark | /5 | /26 | /9 | /11 | /51 | /100 |

QUESTION 1-5 MULTIPLE CHOICE *(5 Marks)*

1.




The graph above shows $y = ax^2 + bx + c$. Which of the following is true?

(a) $a > 0$ & $\Delta > 0$ (b) $a > 0$ & $\Delta < 0$

(c) $a < 0$ & $\Delta > 0$ (d) $a < 0$ & $\Delta < 0$

2. P$(x, y)$ moves such that P is equidistant from the line $y = -1$ and the point (0,1). What is the equation of the locus of P?

(a) $y = \frac{x^2}{4}$ (b) $y = \frac{x^2}{2}$

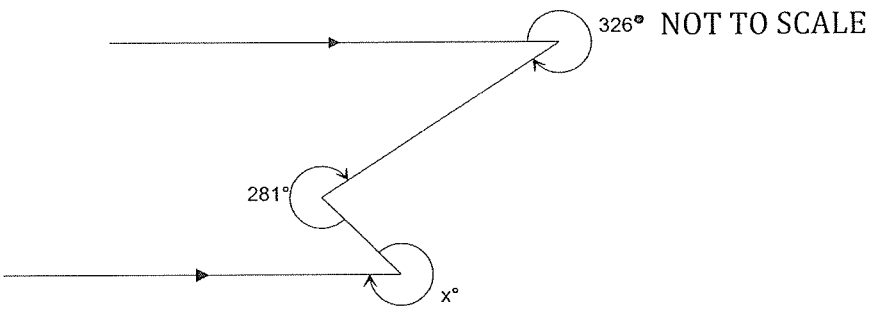
(c) $y = x^2$ (d) $y = 2x^2$

3. Which of the following is the length of the latus rectum of $y = x^2$ ?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) 2

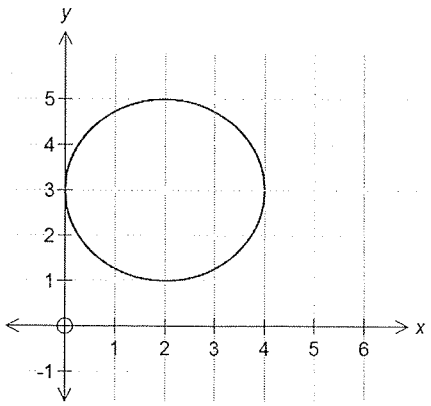
4. Calculate the value of the pronumeral $x$




(a) 310° (b)315°

(c) 320° (d) 325°

5.




NOT TO SCALE

What is the equation of the figure shown above?

(a) $(x-2)^2+(y-3)^2=2$ (b) $(x-2)^2+(y-3)^2=4$

(c) $(x+2)^2+(y+3)^2=2$ (d) $(x+2)^2+(y+3)^2=4$

QUESTION 6 - QUADRATICS (*26 Marks)*

(a) (i) Sketch the graph of:

$y=2x^2+7x-4$

clearly showing the vertex and intercepts with the axes. 3

(ii) Hence or otherwise, solve:

$2x^2+7x\leq 4$ 1

(b) Solve for *x:*

$2x^2-8x+1=0$ 2

(c) Solve for *x:*

$9^x-10\times 3^x+9=0$ 3

(d) For what values of $k$ will the equation

$x^2-(k+3)x+4k=0$

have:

(i) Equal roots? 2

(ii) One root equal to 5? 1

(iii) Roots which are reciprocals of each other? 1

(iv) Roots of equal magnitude but opposite sign? 1

(v) Calculate the value of the roots in (iv). 1

(e) If α and β are the roots of:

$2x^2 + 5x - 1 = 0$

Without solving the equation, find the values of:

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ 2

(iv) $\alpha^2 + \beta^2$ 2

(v) $|\alpha - \beta|$ 2

(f) Express $2x^2 - 7x - 4$ in the form $a(x+2)^2 + b(x+2) + c$ 3

## QUESTION 7 – LOCUS AND THE PARABOLA *(9 Marks)*

(a) A point P$(x, y)$ moves so that it is always equidistant from the points $A(-3, 5)$ and $B(2, -1)$.

(i) Write the equation for the locus of P. 2

(ii) Sketch the equation of the locus of P on the number plane 2

(iii) Hence or otherwise, shade the region on the number plane where $PA \leq PB$ 1

(b) Sketch on the number plane:

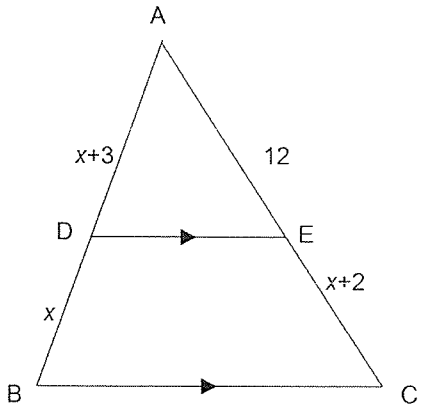
$y^2 + 4y - 24x + 4 = 0$

clearly showing the focus, vertex and directrix 4

*Examination continues on the next page*

## QUESTION 8 – PLANE GEOMETRY *(11 Marks)*

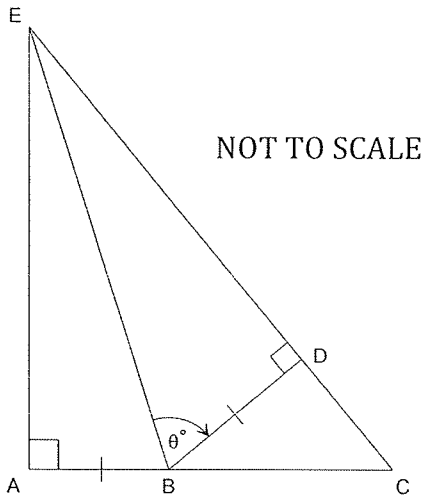
(a)




NOT TO SCALE

In the diagram above, find the value(s) of $x$. 3

(b) In the diagram, $\Delta ABE$ is a right-angled triangle. The point B lies on AC and the point D lies on CE. Also $\angle BDE = 90°$, $AB = BD$ and $\angle DBE = \theta$.




NOT TO SCALE

(i) Show that $\Delta ABE \equiv \Delta DBE$ 3

(ii) Show that $\angle ACE = (2\theta - 90)°$ 2

(iii) Show that $\Delta ACE \,|||\, \Delta DCB$ 2

(iv) Hence show that $\frac{EA}{AB} = \frac{CE}{CB}$ 1

END OF EXAMINATION

Q1./ A

Q2./ A

Q3./ C

Q4./ B

Q5./ B

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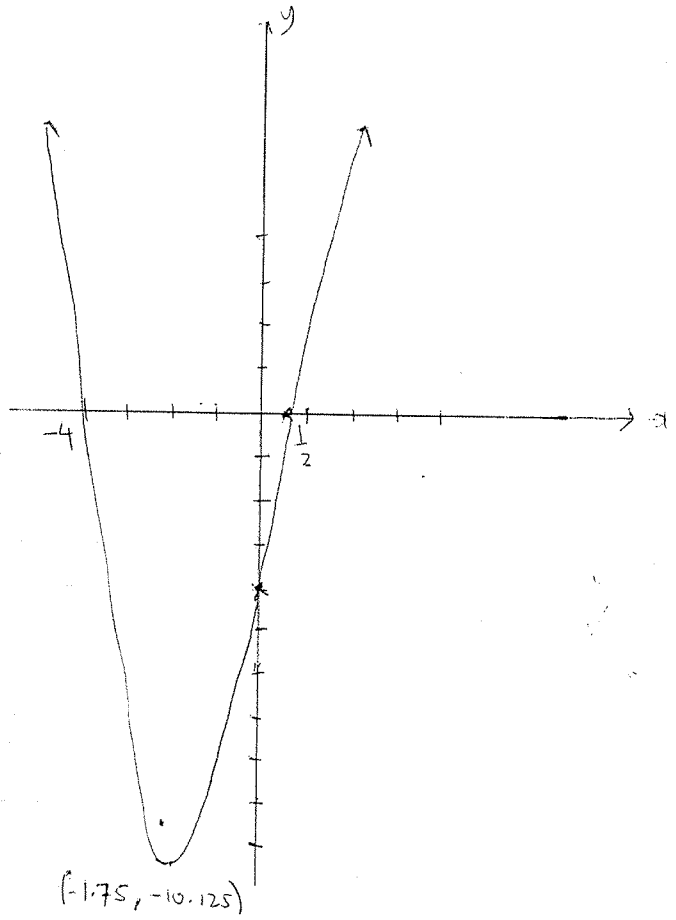
Q6 a i/




Q6 a ii/ $-4 \leq x \leq \frac{1}{2}$

Q6 b./ $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{8 \pm \sqrt{64 - 4 \times 2 \times 1}}{2 \times 2}$

$= \frac{8 \pm \sqrt{56}}{4}$

$= \frac{8 \pm 2\sqrt{14}}{4}$

$= \frac{4 \pm \sqrt{14}}{2}$

6 c/ let $u = 3^x$

$u^2 - 10u + 9 = 0$

$(u-9)(u-1) = 0$

$\therefore u = 9$ or $u = 1$

$3^x = 9$ or $3^x = 1$

$\therefore x = 2$ or $x = 0$

(d) i/ $\Delta = 0$

$b^2 - 4ac = 0$

$(-(k+3))^2 - 4 \times 1 \times 4k = 0$

$k^2 + 6k + 9 - 16k = 0$

$k^2 - 10k + 9 = 0$

$(k-9)(k-1) = 0$

$\therefore k = 1$ or $k = 9$

dii/ $5^2 - (k+3) \times 5 + 4k = 0$

$25 - 5k - 15 + 4k = 0$

$\therefore -k + 10 = 0$

$\therefore k = 10$

diii/ $\alpha \times \frac{1}{\alpha} = 1$

$\therefore \frac{4k}{1} = 1$

$\therefore k = \frac{1}{4}$

div/ $\alpha + (-\alpha) = 0$

$\therefore \frac{k+3}{1} = 0$

$\therefore k = -3$

dv/ $\alpha \times (-\alpha) = \frac{4k}{1}$

$-\alpha^2 = 4 \times (-3)$

$\alpha^2 = 12$

$\alpha = \pm\sqrt{12}$

$\therefore$ roots are $\pm 2\sqrt{3}$

ei/ $\alpha + \beta = -5/2$

eii/ $\alpha\beta = -1/2$

eiii/ $\frac{1}{\alpha} + \frac{1}{\beta}$

$= \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{-5/2}{-1/2}$

$= 5$

ev/ $(\alpha+\beta)^2 = \alpha^2+2\alpha\beta+\beta^2$

$\therefore \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(-\frac{5}{2}\right)^2 - 2\left(-\frac{1}{2}\right)$

$= \frac{25}{4} + 1$

$= \frac{29}{4}$

ev/ $|\alpha-\beta| = \sqrt{(\alpha-\beta)^2}$

$= \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{\left(-\frac{5}{2}\right)^2 - 4\left(-\frac{1}{2}\right)}$

$= \sqrt{\frac{25}{4} + 2}$

$= \sqrt{\frac{33}{4}}$

$= \frac{\sqrt{33}}{2}$

f/ $a(x+2)^2 + b(x+2) + c \equiv 2x^2 - 7x - 4$

$a(x^2+4x+4) + bx + 2b + c \equiv 2x^2 - 7x - 4$

$ax^2 + 4ax + 4a + bx + 2b + c \equiv 2x^2 - 7x - 4$

$\therefore ax^2 + (4a+b)x + 4a + 2b + c \equiv 2x^2 - 7x - 4$

$\therefore \boxed{a = 2}$

$4 \times 2 + b = -7$

$\boxed{b = -15}$

$4 \times 2 + 2 \times (-15) + c = -4$

$c = -4 - 8 + 30$

$\therefore \boxed{c = 18}$

equating co-efficients

Q7ai/ $\sqrt{(x+3)^2 + (y-5)^2} = \sqrt{(x-2)^2 + (y+1)^2}$

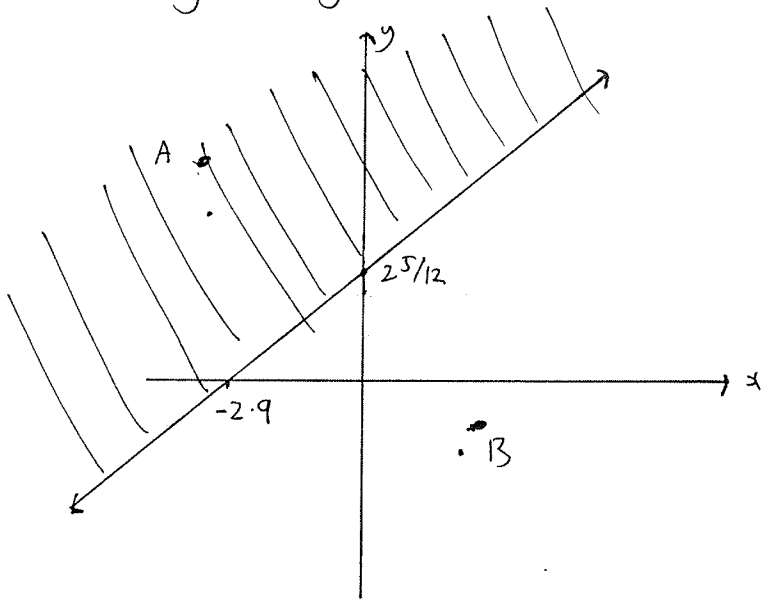
$\therefore x^2 + 6x + 9 + y^2 - 10y + 25 = x^2 - 4x + 4 + y^2 + 2y + 1$

$\therefore 10x - 12y + 29 = 0$

✓ some progress with a correct method
✓ correct answer.

aii/ $12y = 10x + 29$

$\therefore y = \frac{5}{6}x + 2\frac{5}{12}$

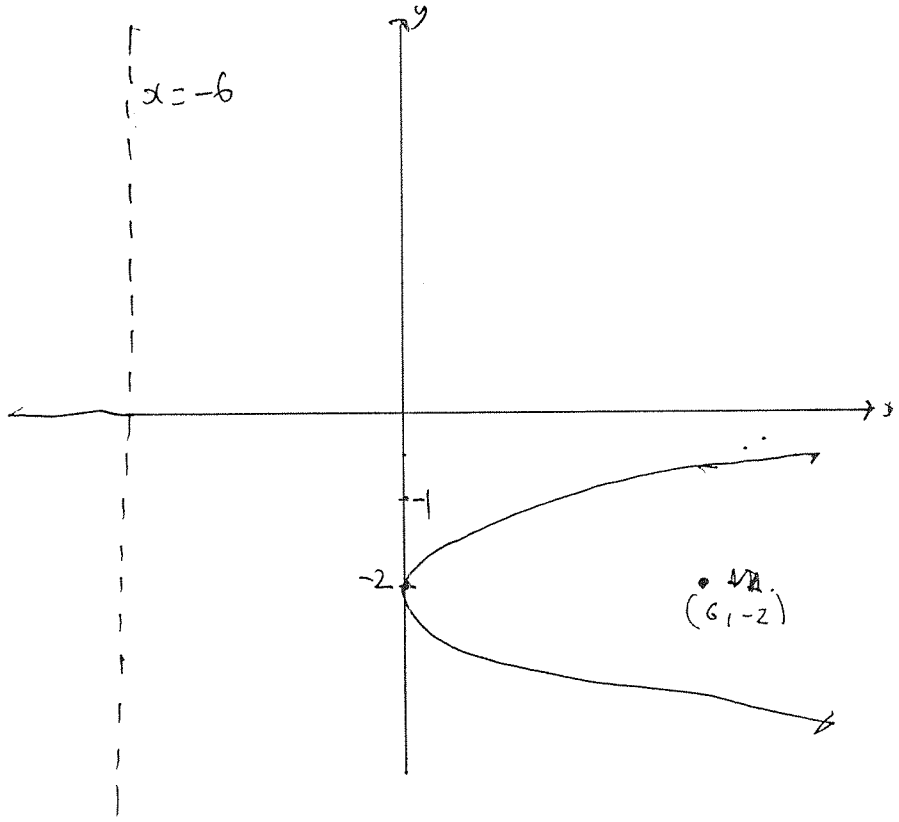



aiii/ region shaded on graph

b/ $y^2 + 4y = 24x - 4$

$y^2 + 4y + 4 = 24x$

$(y+2)^2 = 4 \times 6(x - 0)$



8a/

$\frac{x+3}{x} = \frac{12}{x+2}$ (ratio of intercept b)

$(x+3)(x+2) = 12x$

$x^2 + 2x + 3x + 6 = 12x$

$x^2 + 5x + 6 = 12x$

$x^2 - 7x + 6 = 0$

$(x-6)(x-1) = 0$

$\therefore x = 6$ or $x = 1$

8bi/ In $\Delta$s ABE & $\Delta$ DBE

EB is common

$\angle EAB = \angle EDB$ (given $-90°$)

AB = DB (given)

$\therefore \Delta ABE \equiv \Delta DBE$ (RHS)

bii/ $\angle ABE = \angle DBE = \theta$ (corresponding angles in congruent triangles $\Delta ABE$ & $\Delta DBE$)

$\therefore \angle BDC + \angle BCD = \angle ABD$ (exterior angle of $\Delta BCD$)

$\therefore 90 + \angle BCD = 2\theta$

$\therefore \angle BCD = (2\theta - 90)°$

$\therefore \angle ACE = (2\theta - 90°)$

(as $\angle BCD = \angle ACE$)

biii/ In $\Delta$s ACE & $\Delta$ DCB

$\angle ACE = \angle DCB$ (common to both)

$\angle EAC = \angle BDC$ ($90°$ - given)

$\therefore \Delta ACE \,|||\, \Delta DCB$ (equiangular)

biv./ $\frac{EA}{DB} = \frac{CE}{CB}$ (corresponding sides in similar triangles $\Delta ACE$ & $\Delta DCB$)

$\therefore \frac{EA}{AB} = \frac{CE}{CB}$ (since DB = AB - given)

shortcut $EA \times CB = AB \times CE$